CONFLICTING PRIORITIES: A THEORY OF COVENANTS AND COLLATERAL*

Jason Roderick Donaldson† Denis Gromb‡ Giorgia Piacentino§

October 6, 2018

Abstract

Debt secured by collateral has absolute priority in the event of default—it is paid ahead of unsecured debt, even if unsecured debt is protected by negative pledge covenants prohibiting new secured debt. We develop a model of how this priority rule leads to conflicts among creditors, but can be optimal nonetheless: borrowers' option to use collateral in violation of covenants allows for the dilution of existing debt, and hence prevents under-investment , whereas creditors' option to accelerate debt following a covenant violation deters dilution, and hence prevents over-investment. The optimal outcome is implementable via a mix of different types of debt, including secured and unsecured debt with tight and loose covenants. The model is consistent with a number of stylized facts about debt structure, covenants, and their violations.

---

*For valuable comments, we thank Patrick Bolton, Song Ma, Chris Mayer, Suresh Sundaresan and seminar participants at Columbia and Washington University in St. Louis.

†Washington University in St. Louis and CEPR; jasonrdonaldson@gmail.com.

‡HEC Paris; gromb@hec.fr.

§Columbia University; g.piacentino@gsb.columbia.edu.

1 Introduction

Firms finance themselves mainly via debt,[1] often using several different types of debt at once, including debt protected by covenants and debt backed by collateral (see Barclay and Smith (1995), Rauh and Sufi (2010), and Colla, Ippolito, and Li (2013)). But these debts are not treated the same way in default—they are prioritized: some are paid first, others later. In practice, the absolute priority rule dictates that debt secured by collateral be paid in full before unsecured debt is paid at all. Hence, if a borrower takes on new secured debt, it may dilute existing unsecured debt. To protect unsecured debt against dilution, borrowers use so-called anti-dilution covenants. Indeed, the explicit promise not to take on new secured debt, called a negative pledge covenant, is among the most common covenants.[2] Negative pledge covenants supposedly protect creditors by giving them the right to accelerate their debt in the event of a covenant violation: if a borrower takes on new secured debt, they can demand payment of interest and principle immediately. Yet, legal scholars still express skepticism about their effectiveness. The reason is that the new secured debt has priority over the accelerated debt, even when this new debt is taken on in violation of the covenant. As Bjerre (1999) puts it,

> [negative pledge covenants] are enforceable only against the borrower, and not against third parties who take security interests in violation of the covenant. Hence, when a borrower breaches a negative pledge covenant, the negative pledgee generally has only a cause of action against a party whose assets are, by hypothesis, already encumbered.

This leads him to conclude that "negative pledge covenants may be of little practical comfort."[3]

But if negative pledge covenants are not a credible way to promise priority, why do borrowers rely on them so much, rather than simply using secured debt? Why do borrowers prefer to use secured debt as just one part of a multi-layered debt structure, which includes unsecured debts with and without negative pledge covenants as well? Moreover, why is one type of debt given absolute priority, so that it can expressly undermine other contracts?

Model preview. To address these questions, we present a model in which a borrower, B, finances two projects sequentially subject to two frictions. First, pledgeability is limited,

[1] For example, debt, including convertibles, public bonds, private loans, and private debt placements accounts for 95.6% of financing in Erel, Julio, Kim, and Weisbach's (2012) sample of public firms.

[2] For example, negative pledge covenants are included in 44% of the debt in Billett, King, and Mauer's (2007) sample and 92% in Ivashina and Vallée's (2018) sample.

[3] Some finance theory papers use this conclusion to justify abstracting from these covenants altogether (e.g., Bolton and Oehmke (2015) and Donaldson, Gromb, and Piacentino (2018)).

so B cannot borrow against the full present value of his projects. As a result, B can be financially constrained, and unable to finance even some positive NPV projects. Second, contracts are non-exclusive, so B cannot commit not to enter into contracts with different creditors in the future. As a result, B can enter into conflicting contracts. In particular, if B signs one contract promising not to sign any other contracts, nothing stops him from breaking the promise. Hence, there must be rules for how conflicts among contracts are resolved. We assume that, as in practice, collateral plays the role of establishing priority among debt contracts: debt secured by collateral overrides unsecured debt. Hence, secured debt is always paid first, even if it is taken on in violation of a covenant and even if unsecured debt is accelerated in response to such a violation.

Results preview. Because secured debt overrides other contracts, B can use secured debt to dilute existing unsecured debt, even that protected by covenants. This dilution can lead to over-investment. Indeed, it subsidizes new investments forcing existing creditors to bear part of their funding costs. But dilution can also prevent under-investment , by loosening financial constraints due to limited pledgeability. These two sides of dilution underly our main results.

We first consider what happens if B finances his first project via unsecured debt without covenants. Our first main result is that this can lead to over-investment. The reason is that B can finance his second project with secured debt, diluting the existing unsecured debt. This effectively transfers financing costs onto existing creditors, making B want to invest in some negative NPV projects.

We then consider what happens if B finances his first project via secured debt. Our second main result is that this can lead to under-investment . Since secured debt has absolute priority, it cannot be diluted at all. This can prevent inefficiunder-investment ent dilution, limiting over-investment. However, it can also prevent efficient dilution, causing under-investment. This "collateral-overhang problem" cannot be resolved through financial restructuring (i.e. by renegotiation). Indeed, given limited pledgeability, even positive-NPV projects may generate too little pledgeable cash flow to compensate existing creditors. Secured creditors prefer to keep their claims to B's existing assets. This resonates with practitioners' intuition that secured borrowing "encumbers assets":

> Asset encumbrance not only poses risks to unsecured creditors...but also has wider...implications since encumbered assets are generally not available to obtain...liquidity (Deloitte Blogs (2014)).

B can mitigate this inefficiency by mixing secured debt with unsecured debt, and hence allowing for some dilution. Indeed, if little dilution is needed to finance positive-NPV projects, B will choose a fraction of secured debt that allows him to undertake them, but still prevents him from doing negative NPV projects. However, if a lot of dilution is needed to finance efficient projects, under-investment persists.

Hence, we consider what happens if B finances his first project via unsecured debt with negative pledge covenants, i.e. he borrows without collateral but promises not to borrow with collateral in the future. Although this unsecured debt can be diluted by new secured debt, the threat of acceleration could still deter dilution, since forcing B to pay early could cause him to default, destroying his continuation value. But if all existing debt has negative pledge covenants, then creditors have nothing to gain from acceleration—they are paid after the new secured debt whether they accelerate or not. As a result, their acceleration threat is not credible, and B does not invest efficiently.

But what happens if not all of B's initial debt has negative pledge covenants, but some does, and some does not? Our third main result is that, in this case, creditors can gain from acceleration and, hence, acceleration can be a credible threat to deter dilution. The reason is that debt acceleration allows some unsecured creditors to be paid ahead of others.

Like for secured debt, this acceleration threat might deter not only inefficient dilution, but also efficient dilution. However, unlike secured debt, this threat deters B from doing projects that need little dilution to finance (not a lot of dilution as for secured debt). The reason is that accelerating creditors have more to gain from acceleration when covenant violations are less severe. To see why, observe that if B violates a covenant by financing a new project with secured debt, the existing unsecured debt is ipso facto junior. It is paid after new secured debt, but ahead of equity. Hence, it is both debt-like and equity-like. And the more it is diluted, the closer it is to a residual claim—the more it resembles equity, a call option on B's assets that creditors are reluctant to exercise early—and the less credible the acceleration threat is. Hence, although covenants can effectively deter minor violations, they cannot effectively deter severe ones. As a result, a mix of unsecured debt with and without negative pledge covenants can be efficient when a lot of dilution is needed to finance positive NPV projects, i.e. exactly when secured debt leads to an inefficient collateral overhang.

Our fourth main result is that, in our baseline specification, B can choose a debt structure that implements the efficient investment strategy. The optimal debt structure typically mixes different types of debt, including debt with negative pledge covenants, secured debt issued in violation of those covenants, and unsecured debt without negative pledge covenants.

This result rationalizes the absolute priority of secured debt: understanding this priority rule, B can choose debt instruments appropriately to commit to doing all and only efficient investments—using only non-contingent (debt) instruments, he implements the optimal state-contingent policy. To do so, he uses non-exclusivity to his advantage, exploiting the option to dilute usecured debt with new secured debt. Absolute priority is useful: its power to defeat other claims, even those with negative pledge covenants, facilitates contingent dilution.

Policy. The absolute priority rule is a subject of debate in the law literature; e.g., Bebchuk and Fried (1996) challenge

> the desirability of a fundamental and longstanding feature of bankruptcy law: the principle that a secured creditor is entitled to receive the entire amount of its secured claim...before any unsecured claims paid (p. 859),

arguing that the absolute priority of secured facilitates dilution. Whereas our model affirms this conclusion, it reveals that (i) relaxing the absolute priority rule could block dilution too much, reducing financial flexibility, and that (ii) given the current priority rules, borrowers may be able to structure their debt to block inefficient dilution but allow for efficient dilution.

Realism. The optimal debt structure in our model resembles real-world debt structure. B uses a mix of simple instruments—indeed, in our baseline specification, B can rely on realistic debt instruments even with complete contingent contracts available. Moreover, our model explains the following facts:

(i) Borrowers frequently use negative pledge covenants despite their weakness, when they could simply rely on secured debt (e.g., Billett, King, and Mauer (2007) and Ivashina and Vallée (2018)). Thus responding to the puzzle stressed by, e.g., Bjerre (1999):

> Some may wonder why, given their weakness, costs, and difficulties, lenders bother with negative pledge covenants at all.... [B]orrowers have strong incentives to breach the covenant if necessary financing is available only on a secured basis. When a debtor subject to a negative pledge covenant finds its financial condition worsening and feels the need to borrow on a secured basis, it must either breach the covenant or approach the negative pledgee for a waiver.... The foregoing simply raises, however, the broader question of why lenders ever agree to lend on an unsecured basis, with or without a negative pledge covenant, if collateral is available. [...] All things being equal, any lender would obviously prefer to be secured; a security interest protects the lender against dilution by unsecured creditors (pp. 338–339).

(ii) Debt secured by collateral and debt protected by covenants are two parts of a multi-tiered debt structure (Rauh and Sufi (2010)).

(iii) Only a fraction of available assets are used to secure debt and not all unsecured debt has tight covenants—there is not a pecking order of debt structure, in which borrowers use collateral first, then covenants; rather, they mix different instruments, seeming to exploit complementarities among them (Rauh and Sufi (2010) and Ivashina and Vallée (2018)).

(iv) Covenants are frequently violated (e.g., Chava and Roberts (2008), Dichev and Skinner (2002), and Roberts and Sufi (2009)).

(v) Covenants are typically waived following violations, rarely leading to default or acceleration (e.g., Beneish and Press (1993, 1995), Gopalakrishnan and Prakash (1995), Nini, Smith, and Sufi (2012), and Sweeney (1994)).

(vi) Borrowers have public and private debt at the same time, and private debt has tighter covenants than public debt (Gopalakrishnan and Prakash (1995)).

Good dilution may be most important for growth firms, which have good investment opportunities but limited pledgeable assets/cash flow, whereas bad dilution may be most important for distressed firms, which have incentive to undertake even bad investment opportunities, e.g., tunneling, asset stripping, or risk shifting. If so, our model also explains the following:

(v) Covenant use increases in growth opportunities (Billett, King, and Mauer (2007)).

(vi) Firms do "priority spreading," using secured and subordinated debt, when they near distress (Badoer, Dudley, and James (2018) and Rauh and Sufi (2010)).

Literature. Our paper contributes to the large finance theory literature on collateral and the small one on covenants (e.g., Gârleanu and Zwiebel (2009), Park (2002), Rajan and Winton (1995)). In this literature, covenants and collateral typically mitigate conflicts of interest between borrowers and creditors.[4] We focus on how they mitigate conflicts of interest among creditors instead, which is arguably the main legal role of collateral and the express intention of anti-dilution covenants.[5] Bolton and Oehmke (2015) and Donaldson, Gromb, and Piacentino (2018) do explore how collateral establishes priority among creditors, but both papers rule out negative-pledge covenants, our main focus here. Ayotte and Bolton (2011) do not. Hence, this is probably the closest paper to ours. Unlike us, however, they do not allow for good dilution, and they do not rationalize covenant violations (and subsequent waivers) or the existing priority structure. They also abstract from acceleration and renegotiation, two of the most important features of covenants, both in our model and in practice.

[4]See, e.g., Tirole (2006) on collateral and Smith (1993) on covenants.

[5]Attar, Casamatta, Chassagnon, and Décamps (2015) show, however, that some covenants can help creditors to collude.

Our paper is also related to the law literature on secured debt and priority (e.g., Bebchuk and Fried (1996), Hansmann and Kraakman (2002), Hansmann and Santilli (1997), Kronman and Jackson (1979), Schwarcz (1997), and Schwartz (1984, 1994, 1997). And to papers on contracting subject to legal rules (e.g., Aghion and Hermalin (1990) and Gennaioli (2006)).

Layout. Section 2 presents the model. Section 3 presents the first- and second-best benchmarks. Section 4 contains the analysis of unsecured debt and secured debt. Section 5 contains the analysis of negative pledge covenants. Section 6 includes a full characterization of the equilibrium and a discussion of our results. Section 7 is the Conclusion.

2 Model

We consider a model with one good, "cash," three dates $t \in \{0, 1, 2\}$, universal risk neutrality, and no discounting, in which a borrower finances two projects sequentially subject to frictions.

2.1 Projects

A penniless borrower B has access to two investment projects.

Project 0 costs I_0 at Date 0 and generates a risky payoff at Date 2 when B consumes: with probability q, the project succeeds and pays off $X_0 + Y_0$, where $X_0 \geq 0$ is pledgeable and $Y_0 \geq 0$ is not; otherwise, it fails and pays nothing. Pledgeable cash flow can be promised to third parties (i.e., creditors) by contract, while non-pledgeable cash flow cannot.

Project 1 can be high- or low quality, its quality $\tau \in \{H, L\}$ being revealed at Date 1, with $\mathbb{P}[\tau = H] =: p$. The project costs I_1 at Date 1 and pays off at Date 2, when it succeeds or fails with Project 0. If it succeeds, it pays off $X_1^\tau + Y_1^\tau$, where $X_1^\tau \geq 0$ is pledgeable and $Y_1^\tau \geq 0$ is not. If it fails, it pays nothing.

Projects mature at Date 2 but can be liquidated early for the expected value of their pledgeable cash flows. Liquidation is assumed to be inefficient in that it destroys all (but only) non pledgeable cash flows, representing any cost of terminating a project before completion, due to, e.g., the loss of B's inalienable human capital.

We use the notation $X_{\text{tot.}}$ for the total pledgeable cash flow of all projects undertaken given success:

$$X_{\text{tot.}} := \mathbb{1}_0 X_0 + \mathbb{1}_1 X_1, \tag{1}$$

where $\mathbb{1}_t$ is the indicator variable,

$$\mathbb{1}_t := \begin{cases} 1 & \text{if Project } t \text{ is undertaken,} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

2.2 Financing

2.2.1 Frictions

We assume that at each date B can borrow from competitive creditors under two frictions.

First, cash flow pledgeability is limited so B cannot borrow against its projects' full value, and may be unable to finance positive NPV projects.

Second, contracts are non-exclusive, so B cannot commit not to contract with new creditors in the future.

2.2.2 Instruments

We focus on three debt instruments: secured debt and unsecured debt with or without negative pledge covenants. These instruments reflect practice and we will show that, in our model, restricting attention to them is without loss of generality.

1. **Secured debt** is a promise to repay a fixed face value F_0 at Date 2 which is collateralized by a fraction σ of Project 0's Date-2 cash flow X_0 as collateral. (The role of the collateral depends on the priority rule, as described below.)

2. **Unsecured debt** is a promise to repay a fixed face value F_0 which is not backed by collateral.

3. **Unsecured debt with negative pledge covenants** is unsecured that comes with the option to accelerate, i.e. to demand repayment at Date 1, if the borrower takes on new secured debt (i.e. if he violates the negative pledge covenant), although this is only an option (i.e. covenants can be waived).

2.2.3 Priority Rules

Contracts being non-exclusive, B can enter into different contracts with different creditors that need not be consistent: they can conflict. In particular, B can take on debt at Date 1 promising new creditors repayments he cannot make unless he defaults on payment promised to existing creditors (i.e. on debt taken at Date 0), or he may even violate negative pledge covenants.

Given this, priority rules must be specified to resolve potential conflicts among mutually inconsistent contracts. We consider the following priority rules.

1. **Secured debt vs. unsecured debt.** Secured debt is paid ahead of unsecured debt, irrespective of acceleration.
2. **Secured debt vs. secured debt.** Secured debt is paid in order of issuance: secured debt issued at Date 0 is prioritized ahead of secured debt issued at Date 1.
3. **Unsecured debt vs. unsecured debt.** Here there are two cases. If debt is not accelerated, unsecured debt is paid pro rata in the event of default. If debt is accelerated, its maturity is shortened so that it can be repaid before default. Hence, accelerated debt has effective priority over other unsecured debt (but not over secured debt).

These priority rules reflect practice; as Schwartz (1989) puts it,

> Current law regulating these priorities rests on three "priority principles": First, if the first creditor to deal with the debt makes an unsecured loan, it shares pro rata with later unsecured creditors in the debtor's assets on default. Second, if this initial creditor makes an unsecured loan and a later creditor takes security, the later creditor has priority over the initial creditor in the assets subject to the security interest. Third, if the initial creditor makes a secured loan, it generally has priority over later creditors in the assets in which it has security (p. 209);

see also Barclay and Smith (1995). Hahn (2010) details how acceleration can dilute unsecured debt but not secured debt:

> [Acceleration] facilitates collection by the speedy...creditors [who accelerate their debt] with the potential of harming the less fortunate ones [who do not]..... Moreover, in the case of a debtor who is also indebted to secured creditors acceleration by unsecured creditors...seems somewhat futile (p. 240).

We also show that beyond being realistic, these priority rules are (weakly) optimal in our model.

2.3 Timeline

The timeline is as follows:

Date 0: B funds Project 0 from competitive creditors[6] or does not.

[6] By "fund from competitive creditors" we mean that raises the cost of its investment from (possibly multiple) creditors, and these creditors break even.

Date 1: The quality τ of Project 1 is revealed.

B funds Project 1 from competitive creditors or does not.

If a covenant is violated, creditors accelerate (causing liquidation) or do not.

Date 2: If projects were not liquidated at Date 1, projects succeed or fail (together) with probability p, and B makes repayments or defaults.

2.4 Assumptions

We impose three restrictions on parameters.

Assumption 1. Project 0 is efficient and Project 1 is efficient if and only if it is high quality:

$$q(X_0 + Y_0) > I_0, \tag{3}$$

$$q(X_1^H + Y_1^H) > I_1 > q(X_1^L + Y_1^L). \tag{4}$$

The next two assumptions imply that there is enough pledgeable cash flow for the efficient strategy to be implementable with an exclusive contract. This ensures that our results are driven by non-exclusivity, not limited pledgeability (see Section 3).

Assumption 2. In the efficient strategy—undertaking Project 0 and undertaking Project 1 only if it is high quality—the expected pledgeable cash flow exceeds the expected investment cost:

$$q(X_0 + pX_1^H) \geq I_0 + pI_1. \tag{5}$$

Assumption 3. Irrespective of Project 1's quality, the total liquidation value of Project 0 and Project 1 exceeds the face value needed to finance Project 1, i.e. for $\tau \in \{H, L\}$,

$$q\big(X_0 + X_1^\tau\big) > \frac{I_1}{q}. \tag{6}$$

Note that to finance Project 1, B would have to promise creditors a repayment of F_1 in case the projects succeed, which occurs with probability q. Hence the smallest value of F_1 for which creditors break even is I_1/q. Therefore the assumption ensures that the projects' liquidation value $q\big(X_0 + X_1^\tau\big)$ is sufficient to cover that face value.

3 First Best and Second Best

In this section, we consider two benchmarks.

The first best strategy follows immediately from Assumption 1.

Lemma 1. (First best) *The efficient strategy is to undertake Project 0 and to undertake Project 1 if and only if it is high quality.*

We define the second best outcome as the best that can be implemented with exclusive contracts, i.e. the outcome that maximizes net output subject only to the limited pledgeability constraint. Since creditors are competitive and risk-neutral, exclusive contracting is equivalent to contracting with a single creditor. Thus, the second-best outcome is (first-best) efficient if B and a single creditor can commit to an exclusive contract at Date 0 with Date-2 repayments F^H and F^L in states H and L if the projects succeed such that:

1. Irrespective of Project 1's quality, B's pledgeable cash flow suffices to meet the promised repayments given success at Date 2:

$$X_0 + X_1^H \geq F^H, \tag{7}$$

$$X_0 \geq F^L. \tag{8}$$

2. Given B's repayments F^H and F^L, creditors are willing to lend I_0 at Date 0 and I_1 at Date 1 if Project 1's quality is high:

$$q\big(pF^H + (1-p)F^L\big) \geq I_0 + pI_1. \tag{9}$$

Lemma 2. (Second best) *The first best is implementable with exclusive contracts.*

This result implies that borrowing with exclusive, state-contingent contracts yields the first best outcome. With non-exclusive contracts, however, this need not be the case, as we study next.

4 Unsecured and Secured Debt

In this section, we study the implications of the non-exclusivity friction focusing on purely unsecured and purely secured debt to start with. We find that it may not be possible to attain the first best with these instruments.

4.1 Unsecured Debt and Over-investment

We begin our analysis by asking whether B can implement the efficient strategy by borrowing via (only) unsecured debt at Date 0. Thus, unlike in Lemma 2 above, B's contract is neither state-contingent nor exclusive. Hence, the question is whether B can commit to follow the efficient investment strategy. I.e. can B satisfy the following two necessary conditions for efficiency?

(i) *B undertakes Project 1 if $\tau = H$.* Since existing debt is unsecured, B can secure all of his pledgeable cash flow to Date-1 creditors. Hence, B is able to finance Project 1 if and only if this total pledgeable cash flow exceeds the cost of Project 1, i.e.

$$q\big(X_0 + X_1^H\big) \geq I_1. \tag{10}$$

By Assumption 3, this condition holds and B is able to fund Project 1 if $\tau = H$. B is also willing to fund Project 1 if his payoff from doing so exceeds that without Project 1, i.e.

$$q\left(Y_0 + Y_1^H + \max\left\{0\ ,\ X_0 + X_1^H - F_0 - \frac{I_1}{q}\right\}\right) \geq q\Big(Y_0 + \max\big\{0\ ,\ X_0 - F_0\big\}\Big), \tag{11}$$

where I_1/q is the face value of secured debt needed to fund Project 1. This can be simplified as

$$Y_1^H + \max\left\{0\ ,\ X_0 + X_1^H - F_0 - \frac{I_1}{q}\right\} \geq \max\big\{0\ ,\ X_0 - F_0\big\}, \tag{12}$$

which is satisfied (by Assumption 1). This simply reflects that Project 1 has positive NPV in state H—B captures at least the NPV, and may also benefit from dilution.

(ii) *B does not undertake Project 1 if $\tau = L$.* Again, since the existing debt is unsecured, B can secure all of his pledgeable cash flow to Date-1 creditors to fund Project 1. Hence B is able to finance Project 1 if and only if

$$q\big(X_0 + X_1^L\big) \geq I_1, \tag{13}$$

which is satisfied (by Assumption 3). Thus, he chooses not to invest in Project 1 only if financing Project 1 with secured debt would (weakly) decrease his payoff, or

$$Y_1^L + \max\left\{0\ ,\ X_0 + X_1^L - F_0 - \frac{I_1}{q}\right\} \leq \max\big\{0\ ,\ X_0 - F_0\big\}. \tag{14}$$

This says that as long as Project 1 is sufficiently pledgeable (i.e. Y_1^L is low), then B does not invest in it. Otherwise, he over-invests, since dilution is effectively a tax on old debt and a subsidy to new financing/investment—in this case, old creditors bear the cost of investment, but B captures (at least) the entire non-pledgeable part of it, Y_1^L.

After solving for the equilibrium face value F_0, we find that the two conditions can be satisfied together whenever Y_1^L is sufficiently small:

Proposition 1. (Unsecured debt) *B can implement the efficient outcome borrowing unsecured (without covenants) at Date 0 if and only if*

$$Y_1^L \leq \min\left\{X_0 - \frac{I_0}{q}\,,\, X_0 - \frac{I_0}{q} + \frac{p\left(X_0 + X_1^H - \frac{I_0}{q} - \frac{I_1}{q}\right)}{1-p}\right\} \tag{15}$$

When pledgeability is low if Project 1's quality is low, B can implement first best borrowing unsecured at Date 0. However, when pledgeability is high if Project 1's quality is low, B can take advantage of dilution, leading to over-investment in state L. He may wish to use secured debt to commit not to, as we turn to next.

4.2 Secured Debt and Under-investment

Here we ask whether B can implement efficiency by borrowing via secured debt at Date 0. Since secured debt is time prioritized (cf. Subsection 2.2.3), B cannot dilute it at all. This can prevent the dilution that could be necessary to implement efficiency in state H. But if only a fraction σ_0 of B's debt is secured at Date 0, he can still dilute the fraction $(1-\sigma_0)$ that is unsecured: $(1-\sigma_0)$ is a cap on dilution. If B can keep this cap loose enough to allow dilution in state H, while still keeping it tight enough to prevent it in state L, he can satisfy the two necessary conditions for efficiency:

(i) *B undertakes Project 1 if $\tau = H$.* B can invest in state H as long as he has enough financial flexibility, or the cost of investment is less than what he can promise to repay Date-1 creditors, i.e. less than his "unencumbered pledgeable cash flow":

$$q\Big((1-\sigma_0)X_0 + X_1^H\Big) \geq I_1. \tag{16}$$

(ii) *B does not undertake Project 1 if $\tau = L$.* B will not invest in the low quality project as long as he does not have too much financial flexibility:[7]

$$q\Big((1-\sigma_0)X_0 + X_1^L\Big) < I_1. \tag{17}$$

[7]He also will not invest if Y_1^L is small enough that he has no incentive to invest; we do not focus on this case here, since we already showed that B can implement the first best with unsecured debt in that case anyway (Proposition 1).

There is a fraction of secured debt σ_0 such that these conditions are satisfied together whenever X_1^L is sufficiently small:

Proposition 2. (Secured debt) *B can implement the efficient outcome via a mix of secured and unsecured debt at Date 0 if*

$$X_1^L < X_1^H. \tag{18}$$

X_1^τ is the pledgeable cash flow created by funding Project 1. If it is low, then B can fund Project 1 only by diluting existing debt, and the lower it is, the more dilution is needed to fund the project. Hence, the condition $X_1^L < X_1^H$ in the proposition says that we can implement the first best if financing the low-quality project dilutes existing debt more than financing the high-quality project. Otherwise, B cannot use collateral to constrain financing of the low-quality project without constraining financing of the high-quality one too:

Corollary 1. (Collateral overhang) *Suppose*

$$X_1^L \geq X_1^H \tag{19}$$

and B secures a fraction σ_0 of its cash flow to his Date-0 creditors so that he cannot finance Project 1 in the low-quality project at Date 1. He cannot finance the high-quality project either (even if his Date-0 debt can be renegotiated).

This is a manifestation of the "collateral overhang problem" in Donaldson, Gromb, and Piacentino (2018): whereas secured debt prevents B from diluting Date-0 creditors to fund an inefficient investment, it also prevents him from diluting them to fund an efficient investment—collateralization encumbers B's assets. Perhaps, then, negative pledge covenants, which do not necessarily prevent dilution, can help? This is what we turn to next.

5 Negative Pledge Covenants

In this section, we ask whether negative pledge covenants can help B to implement efficiency, considering first Date-0 financing entirely via debt with covenants, and then with a mix of debt with and without covenants.

5.1 Financing Entirely via Unsecured Debt with Covenants

Here we suppose that B finances his first project entirely via unsecured debt with negative pledge covenants to a single creditor. Given the debt is unsecured, B can always dilute it with new secured debt, even in violation of its explicit covenants. However, the creditor has the right to accelerate her debt following the violation. Since acceleration forces liquidation, the threat of acceleration could deter dilution, and potentially even lead B to invest efficiently. The acceleration threat must be credible, however. And it is only credible if the creditor has something to gain from accelerating her debt.

But what can she gain by accelerating once a violation has already taken place, given the violation itself entailed prioritizing new debt? In this case, nothing.

Lemma 3. *Suppose B finances his first project entirely via unsecured debt with negative pledge covenants to a single creditor. The creditor never accelerates. Hence, the outcome coincides with that in which B finances the project entirely via unsecured debt (Proposition 1).*

To understand the result, suppose that B cannot fully repay his creditor even in success (this is necessary for dilution, hence it is without loss). If the creditor accelerates, she has a junior claim on B's assets in liquidation, and gets $qX_{\text{tot.}} - F_1$, where F_1 is the face value of the secured debt that B took on to finance Project 1. If she does not accelerate, she has a junior claim on B's assets at maturity, and gets $q(X_{\text{tot.}} - F_1)$. Since

$$qX_{\text{tot.}} - F_1 < q\big(X_{\text{tot.}} - F_1\big), \tag{20}$$

the creditor never accelerates, and B is not deterred from taking on new debt. The reason is that liquidation subsidizes secured debt, since it makes it less risky: it is repaid F_1 for sure, not just with probability q. This subsidy is a tax on unsecured debt. To avoid it, the unsecured creditor does not accelerate.

5.2 Financing with a Mix of Unsecured Debt with and without Covenants

We now suppose that B finances his first project via a fraction ϕ of unsecured debt with negative pledge covenants to one creditor and a fraction $(1-\phi)$ of unsecured debt without covenants to other creditors. Ironically, having less debt with negative pledge covenants can make the covenants more effective.

The reason is that the creditor with covenants now has more to gain from acceleration, so her acceleration threat could be credible. Although acceleration does nothing to reverse the dilution imposed on her via the new secured debt, it now has a benefit: it allows her to dilute the fraction $(1-\phi)$ of unsecured debt without covenants, getting paid before B defaults on his other unsecured debt at maturity. Here is yet another side of dilution: the option to dilute other unsecured debt (through acceleration) creates a credible threat to deter dilution with secured debt (through priority).

The fraction ϕ of debt with covenants determines the strength of the acceleration threat—the smaller ϕ is, the more other debt there is to dilute, and the more there is to gain from accelerating. Thus, B may be able to choose ϕ to make the threat credible at the right time, deterring investment in the low-quality project, but not the high-quality project at Date 1, i.e. satisfying the two necessary conditions for efficiency:

(i) *B undertakes Project 1 if* $\tau = H$. B finances the high-quality project, borrowing secured in violation of covenants, only if he anticipates that the creditor with covenants will not accelerate afterward, i.e. if she prefers to get paid given success at maturity, behind the secured debt F_1 but pari passu with other unsecured debt, than to accelerate and force liquidation to get paid for sure today, still behind secured debt but now ahead of other unsecured debt:

$$q\big(X_0 + X_1^H\big) - F_1 \leq q\phi\big(X_0 + X_1^H - F_1\big). \tag{21}$$

(We have supposed for simplicity that dilution is severe enough that B cannot repay the debt with negative pledge covenants in full given acceleration. This turns out to be without loss of generality; see the proof of Proposition 3.)

(ii) *B does not undertake Project 1 if* $\tau = L$. B does not finance the low-quality project if he anticipates that creditors will accelerate afterward, or, analogously to the previous case,

$$q\big(X_0 + X_1^L\big) - F_1 \geq q\phi\big(X_0 + X_1^L - F_1\big). \tag{22}$$

There is a fraction of debt ϕ with negative pledge covenants such that these conditions are satisfied together whenever X_1^L is sufficiently large:

Proposition 3. (Covenants) *B can implement the efficient outcome via a mix of unsecured debt with and without negative pledge covenants at Date 0 if*

$$X_1^L \geq X_1^H \tag{23}$$

(even if his Date-0 debt can be renegotiated).

Recall that X_1 reflects how much B must dilute existing debt to finance Project 1 (see Subsection 4.2). Hence, condition (23) in the proposition says that we can implement the first best with covenants exactly when we might not be able to with secured debt (Proposition 2): if financing the low-quality project dilutes less than financing the high-quality project. Otherwise, there is no mix of way to make use the acceleration threat to deter financing the low-quality project without deterring financing the high-quality project too.

The reason is that there is more to gain from acceleration when dilution is less severe, making the threat is credible when dilution it is relatively small, but not when it is large. To see why, observe that if B violates a covenant by financing a new project with secured debt, the existing unsecured debt is ipso facto junior. It is paid after new secured debt, but ahead of equity. Hence, it is both debt-like and equity-like. And the more it is diluted, the closer it is to a residual claim—the more it resembles equity, a call option on B's assets that creditors are reluctant to exercise early—and the less credible the acceleration threat is. When dilution is large, it is better not to accelerate, but to "gamble for resurrection" as in the prototypical problem of firm in distress.

Unlike in the prototypical problem, however, this risk-shifting incentive distortion is exactly what leads to the efficient action: it makes the acceleration threat credible at the right time, and hence covenants allow for some dilution—good dilution—despite their stated intention not to.

It is worth stressing that although liquidation is inefficient, B cannot renegotiate with his creditors to bribe them not to accelerate, which would undermine the liquidation threat. The reason is that continuation only produces extra non-pledgeable cash flow, and B cannot credibly promise to give it to his creditors. Hence, creditors (weakly) prefer just to liquidate and seize B's assets at Date 1.

6 Equilibrium Characterization and Discussion

6.1 Characterization

Our analysis above implies that B can always find a debt structure to implement the first best, but how the structure looks depends on parameters.

Proposition 4. (Characterization) *The equilibrium is (first-best) efficient and can be implemented as follows. At Date 0, B finances Project 0 by borrowing I_0 via debt with total face value*

$$F_0 = \frac{I_0}{q} + \max\left\{0\,,\frac{p}{1-p}\left(\frac{I_0}{q}+\frac{I_1^H}{q}-X_0-X_1^H\right)\,,\,\frac{1-p}{p}\left(\frac{I_0}{q}-X_0\right)\right\}, \tag{24}$$

where the proportions of this debt that are unsecured without covenants, secured, and unsecured with covenants depend on parameters as follows:

- *If* $Y_1^L \leq \min\left\{X_0-\frac{I_0}{q}\,,\,X_0-\frac{I_0}{q}+\frac{p}{1-p}\left(X_0+X_1^H-\frac{I_0}{q}-\frac{I_1}{q}\right)\right\}$*, the debt is all unsecured without covenants.*

- *Otherwise, if* $X_1^H > X_1^L$*, a fraction* $\sigma_0 \in \left[1 - \frac{I_1 - qX_1^L}{qX_0}, 1 - \frac{I_1 - qX_1^H}{X_0}\right)$ *is secured.*
- *Otherwise, the debt is unsecured, and a fraction* $\phi \in \left[\frac{q\left(X_0 + X_1^H\right) - I_1/q}{q\left(X_0 + X_1^H - I_1/q\right)}, \frac{q\left(X_0 + X_1^L\right) - I_1/q}{q\left(X_0 + X_1^L - I_1/q\right)}\right]$.

At Date 1 in state H*, B finances Project 1 by borrowing via* I_1^H *via secured debt with face value* $F_1^H = I_1^H$*; at Date 1 in state* L*, B does not finance Project 1.*

This result rationalizes the real-world priority structure, in the sense that it allows B to use the instruments at his disposal to implement the first-best outcome. The way he uses the instruments also reflects practice, as we discuss in the next section.

6.2 Discussion

Covenants vs. collateral. The literature stresses the substitutability of covenants and collateral; for example, Schwartz (1989) says that

> Secured debt and covenants are substitutes (both are issued to protect against dilution (p. 1418).

Indeed, this is true in our model. But we show that there is also complementarity between covenants and collateral: covenants can implement efficiency only in conjunction with collateral. Although you need covenants to promise not to use collateral—not to dilute unsecured debt inefficiently—you also need to collateral break that promise—to dilute efficiently.

Maturity vs. collateral. Folk wisdom suggests that maturity and collateral are substitutes.[8] Indeed, shortening maturity and pledging collateral are two ways to establish priority in our model. But they can still be complements: shortening maturity via acceleration is not only a way for unsecured creditors to get priority, it is also a way for them to prevent secured creditors from getting priority, since the acceleration threat makes it unattractive for the borrower to pledge collateral to new creditors.

Debt vs. debt. The literature stresses how covenants address conflicts between debt and equity. Notably, Smith and Warner (1979) say

> In this paper, we examine how debt contracts are written to control the bondholder-stockholder conflict. We investigate the various kinds of bond covenants which are included in actual debt contracts (p. 117).

[8]This folk wisdom seems to come from a combination of theories; for example, Hertzberg, Liberman, and Paravisini (2018) say "In theory, lenders can partially mitigate these inefficiencies by using contract terms...such as high collateral (Bester (1985), short maturity (Flannery (1986)), or strict covenants (Levine and Hughes (2005))."

Our analysis suggests that conflicts among different debts could be as important as conflicts between debt and equity—indeed, negative pledge covenants need not exist at all in our model if creditors did not have conflicting priorities.

Creditors vs. creditors. In our model, the threat of acceleration helps to mitigate conflicts among debts. But to make the acceleration threat credible, creditors are pitted against each other—one creditor has the incentive to accelerate only to dilute another's debt. Thus, efficiency relies on how some conflicts among multiple creditors mitigate others. One creditor, with negative pledge covenants, must act strategically, deciding whether to accelerate its debt or waive a covenant violation. Such a large, strategic creditor could represent a bank. Other creditors, without negative pledge covenants, are passive by comparison. Whether they are diluted or not depends on what the borrower and the bank do. These creditors could represent bondholders. Indeed, in practice, bank debt is concentrated and relatively covenant heavy, whereas bonds are dispersed and relatively covenant lite.

Dilution vs. dilution. Debt dilution is largely viewed as a "serious danger" for firms (Schwartz (1997)) and, likewise, a "major problem" for countries (Eyigungor (2013)). Indeed, dilution can be bad in our model: dilution via collateral can lead to over-investment and dilution via acceleration can lead to inefficient liquidation. But it can also be good: dilution via collateral can prevent under-investment and dilution via acceleration creates a threat that deters other, inefficient dilution. The optimal debt structure—the fractions σ of secured debt and ϕ of unsecured debt with negative pledge covenants—allows for good dilution while preventing bad dilution.

Contingent outcomes vs. non-contingent contracts (and contingent debt structure). The literature has paid a lot of attention to contingent contracting. In corporate finance, it has also focused a lot on the debt vs. equity decision, and explored how contingent contracts can be implemented via a mix of debt and equity, as well as some other instruments, such as credit lines. Our model is about implementing a contingent contract too; for the equilibrium to be efficient, B should invest if $\tau = H$ but not if $\tau = L$. But we focus on the debt vs. debt decision, and show that the efficient strategy can be implemented with a variety of debt contracts that are not contingent at all. Rather, contingencies are implemented via contingent dilution, which itself is implemented by mixing debts with different covenants and priorities. The mix of debt contracts B uses resembles firms' real-world funding structure: it is almost all debt, but debt is heterogeneous.

Absolute vs. partial priority. The absolute priority rule dictates secured debt is paid in full before anyone else is paid anything. Bebchuk and Fried (1996) argue that such absolute priority of secured debt can create inefficiencies, because it gives secured debt the power to defeat other claims. We argue that this is not always a bad thing, because dilution can be good, helping to overcome limited pledgeability. Moreover, we show how borrowers can use a mix of different types of (non-contingent) debt to allow for contingent dilution, allowing efficient dilution, but still preventing inefficient dilution.

The price of debt with vs. without covenants. How do covenants affect debt pricing? Their being prevalent in contracts suggests they matter a lot. But their being enforced seldom could suggest they might not. Some papers have taken up this question empirically, and found significant value of covenants (see Bradley and Roberts (2015) and Matvos (2013)). These papers estimate the value of covenants by the price of debt with covenants to debt without. But in our model, debt with covenants has the same price as debt without, even when covenants are effective (cf. Lemma 4 in the Appendix). Indeed, covenants are effective exactly because there is debt without covenants that can be diluted—it is this option to dilute that makes the acceleration threat credible. However, all debt, not just that with covenants, is more valuable because some of it has covenants (which discipline the borrower through the acceleration threat) and some of it does not (which makes this threat credible). Hence, our model suggests that this empirical work could underestimate the value of covenants, since the value of covenants spills over from the "treatment group" to the "control group."

Flexibility vs. rigidity. In many models, covenants are hard restrictions, and hence impose the cost of limited flexibility. In ours, in contrast, covenants can be violated, and indeed bring the benefit of increased flexibility with respect to secured debt.

7 Conclusion

We present model of financial contracting in which contracts are non-exclusive, and hence can conflict: contracts may contain covenants putting restrictions on other contracts, but these covenants can be violated. In this case, a priority rule is needed to resolve conflicts among contracts. Hence, contracts are meaningful only with respect to the priority rule.

In practice, secured debt has absolute priority. This creates the risk of dilution: new secured debt overrides existing unsecured debt. Given this priority rule, negative pledge covenants restricting new secured debt might seem futile—they can be overridden by the very dilution they are supposedly there to prevent. But we show that this can be a good thing. The reason is that in addition to the usual bad side of dilution (it leads to over-investment), there are good sides as well. First, it can loosen borrowing constraints that could be too tight due to limited pledgeability, and hence prevent over-investment. Second, it subsidizes accelerating creditors, hence making their threat credible and preventing bad dilution. In our environment, a borrower who understands the existing priority structure can choose his debt structure to get the good sides of dilution without the bad, and hence implement the efficient investment strategy. Hence, our model rationalizes the existing priority rules.

A Proofs

A.1 Proof of Lemma 1

The result follows immediately from Assumption 1. □

A.2 Proof of Lemma 2

Assumption 2 and Assumption 3 imply these inequalities can be satisfied.

One easy way to see this is to make the first two bind, so $F_0^H = X_0 + X_1^H$ and $F_0^L = X_0$. In this case, the third (inequality (9)) reduces to Assumption 2. □

A.3 Proof of Proposition 1

To prove the proposition, we consider face value F_0 if B follows the efficient strategy and determine when B has no incentive to deviate and invest if $\tau = L$. (We know B will invest if $\tau = H$ irrespective of F_0.) If $\tau = H$, B can borrow I_1 with secured debt with face value F_1 such that $q \min\{X_0 + X_1^H\ ,\ F_1\} = I_1$. Thus, by Assumption 3, $F_1 = I_1/q$.

Case 1: $X_0 + X_1^H \geq I_0/q + I_1/q$ **and** $X_0 \geq I_0/q$.

In this case, if the projects succeed, B is able to pay I_0/q to Date-0 creditors irrespective of τ and so

$$F_0 = I_0/q. \tag{25}$$

Condition (14) becomes

$$Y_1^L + \max\{0\ ,\ X_0 + X_1^L - I_0/q - I_1/q\} \leq X_0 - I_0/q. \tag{26}$$

There are two subcases, depending on whether B defaults on Date-0 creditors if he invests when $\tau = L$ and the projects succeed.

Subcase 1.1 $X_0 + X_1^L > I_0/q + I_1/q$.

In this case, B does not default. As a result, he would bear the full negative value of Project 1 when $\tau = L$ and so does not undertake it in that case.

Subcase 1.2 $X_0 + X_1^L < I_0/q + I_1/q$.

In this case, if B invests in Project 1 when $\tau = L$ and the projects succeed, he defaults on Date-0 creditors. Hence, condition (14) becomes

$$Y_1^L \leq X_0 - I_0/q. \tag{27}$$

Summing up, B will undertake Project 1 when $\tau = L$ if

$$X_0 + X_1^L < I_0/q + I_1/q \;\text{ and }\; Y_1^L > X_0 - I_0/q. \tag{28}$$

By Assumption 1, $Y_1^L < I_1/q - X_1^L$, so one condition implies the other: in this case, there is over-investment if and only if $Y_1^L > X_0 - I_0/q$, and conversely, B will not undertake Project 1 when $\tau = L$ if and only if

$$Y_1^L \leq X_0 - I_0/q. \tag{29}$$

Case 2: $X_0 + X_1^H < I_0/q + I_1/q$ **and** $X_0 \geq I_0/q$**.**

In this case, if B undertakes Project 1 and the projects succeed, he defaults on his Date-0 debt for $\tau = H$ but not for $\tau = L$. Hence, F_0 is given by the following break-even condition for Date-0 creditors:

$$I_0 = q\big(p\left(X_0 + X_1^H - I_1/q\right) + (1-p)F_0\big) \tag{30}$$

so

$$F_0 = \frac{I_0/q - p\left(X_0 + X_1^H - I_1/q\right)}{1-p}. \tag{31}$$

Note that given $X_0 + X_1^H < I_0/q + I_1/q$, Assumption 2 implies $F_0 \leq X_0$, so B does not default if $\tau = L$. Thus, condition (14) becomes

$$Y_1^L + \max\{\ 0\ ,\ X_0 + X_1^L - F_0 - I_1/q\ \} \leq X_0 - F_0 \tag{32}$$

There are two subcases, depending on whether B defaults if B undertakes Project 1 when $\tau = L$ and the projects succeed.

Subcase 2.1: $X_0 + X_1^L \geq F_0 + I_1/q$.

In that case, B would not default and so would bear the full negative value of Project 1. Hence, he does not undertake Project 1 if $\tau = L$.

Subcase 2.2: $X_0 + X_1^L < F_0 + I_1/q$. In that case, B would default and condition (14) becomes

$$Y_1^L \leq X_0 - F_0, \tag{33}$$

which, by Assumption 1, implies the subcase's condition, i.e.

$$X_0 + X_1^L < F_0 + I_1/q. \tag{34}$$

Hence, B does not undertake Project 1 when $\tau = L$ if and only if condition (33) holds which, plugging in for F_0, can be rewritten as

$$(1-p)Y_1^L \le X_0 - I_0/q + p(X_1^H - I_1/q). \tag{35}$$

Case 3: $X_0 < I_0/q$. In this case, B defaults if $\tau = L$ but not if $\tau = H$ and the projects succeed. Thus, Date-0 creditors' break-even condition is

$$I_0 = q\big(pF_0 + (1-p)X_0\big) \tag{36}$$

so

$$F_0 = \frac{I_0/q - (1-p)X_0}{p}. \tag{37}$$

Note that given $X_0 < I_0/q$ in this case, Assumption 2 implies that $F_0 + I_1/q \le X_0 + X_1^H$, so B does not default if $\tau = H$ and the projects succeed. In this case B always defaults. Hence, inequality (14) reduces to $Y_1^L \le 0$, which is never satisfied.

Efficiency conditions. In summary, efficient investment requires that $X_0 - I_0/q \ge 0$ (from Case 3) and that (from Case 1)

$$Y_1^L \le X_0 - I_0/q \quad \text{if} \quad X_0 + X_1^H - I_0/q - I_1/q \ge 0 \tag{38}$$

and (from Case 2)

$$Y_1^L \le X_0 - I_0/q + \frac{p}{1-p}(X_0 + X_1^H - I_0/q - I_1/q) \quad \text{if} \quad X_0 + X_1^H - I_0/q - I_1/q < 0. \tag{39}$$

Taken together, equations (38) and (39) can be written as condition 15 in the proposition. Finally, note that we can omit the condition that $X_0 \ge I_0/$q, since it is implied by the condition that $Y_0 \le X_0 - I_0/q$.

□

A.4 Proof of Proposition 2

Immediately from equations (16) and (17), efficiency is implementable whenever there is σ_0 such that

$$\frac{I_1 - qX_1^H}{qX_0} \le 1 - \sigma_0 < \frac{I_1 - qX_1^L}{qX_0}. \tag{40}$$

Since the LHS is less than one by Assumption 3 and the RHS is greater than zero by Assumption 1, such a σ_0 exists whenever the LHS is less than the RHS, or $X_1^H > X_1^L$, which is the condition in the proposition. □

A.5 Proof of Lemma 3

The argument for why the single creditor never accelerates is in the text. Without the acceleration threat, unsecured debt with negative pledge covenants is equivalent to unsecured debt. Hence, the outcome is that described in Proposition 1.

A.6 Proof of Proposition 3

Before starting the proof, we write down creditors' payoffs from accelerating or not. First, observe that if B borrows at Date 1, he always borrows fully secured, to maximize the benefit of dilution. Hence, from Date-1 creditors' break-even condition, the face value of Date-1 debt is

$$F_1 = \frac{I_1}{q} \tag{41}$$

Now, we denote the total face value of Date-0 debt with and without covenants F_0^{c} and F_0^{nc} respectively, with $F_0^{\text{c}} + F_0^{\text{c}} \equiv F_0$, and, likewise, amount borrowed with and without covenants by I_0^{c} and I_0^{nc} respectively, with $I_0^{\text{c}} + I_0^{\text{nc}} \equiv I_0$, and, by the definition of ϕ, $I_0^{\text{c}} \equiv \phi I_0$. There are three relevant cases:[9]

1. **B does not borrow at Date 1.** In this case, B repays in full at Date 2 if $X_{\text{tot.}} \geq F_0$ and defaults otherwise, in which case creditors are paid pro rata:
   - Unsecured creditors with covenants get $q \min\{ F_0^{\text{c}}\,,\, \phi X_{\text{tot.}} \}$.
   - Unsecured creditors without covenants get $q \min\{ F_0^{\text{nc}}\,,\, (1-\phi) X_{\text{tot.}} \}$.
2. **B borrows secured at Date 1, but debt is not accelerated.** In this case, B repays in full at Date 2 if $X_{\text{tot.}} \geq F_0 + F_1$ and defaults otherwise, in which case he repays the secured debt first and the unsecured debt pro rata:
   - Secured creditors break even, getting F_1 with probability q (recall that $F_1 = I_1/q$ from equation (41)).
   - Unsecured creditors with covenants get $q \min\big\{ F_0^{\text{c}}\,,\, \phi(X_{\text{tot.}} - F_1) \big\}$.
   - Unsecured creditors without covenants get $q \min\big\{ F_0^{\text{nc}}\,,\, (1-\phi)(X_{\text{tot.}} - F_1) \big\}$.

[9] We omit cases in which B takes on new unsecured debt at Date 1, because it is easy to show that doing so is dominated by taking on new secured debt at Date 1.

3. **B borrows secured at Date 1, and debt is accelerated.** In this case, B repays in full at Date 1 if $qX_{\text{tot.}} \geq F_0 + F_1$ and defaults otherwise, in which case he repays secured debt first, the accelerating unsecured creditors (those with covenants) next, and other unsecured creditors last:

   - Secured creditors get F_1 (given $qX_{\text{tot.}} \geq F_1$ by Assumption 3).
   - Unsecured creditors with covenants get $\min\{\, F_0^{\text{c}}\,,\, qX_{\text{tot.}} - F_1\}$.
   - Unsecured creditors without covenants get either the smaller of their face value and the assets remaining after all other creditors have been repayed: $\min\Big\{\, F_0^{\text{nc}}\,,\, qX_{\text{tot.}} - F_1 - \min\{\, F_0^{\text{c}}\,,\, qX_{\text{tot.}} - F_1\}\Big\}$.

Before moving on the main argument, wex prove a lemma that said that the interest rates on debt with and without covenants are the same under the efficient strategy.

Lemma 4. *If B does the first-best strategy, then the interest rates on debt with and without covenants coincide: $F^c/I_0^c = F_0^{nc}/I_0^{nc}$. Hence $I_0^c = \phi I_0$ and $I_0^{nc} = (1-\phi)I_0$.*

Proof. There are three cases.

Case 1: $q\big(X_0 + X_1^H\big) > I_0 + I_1^H$ and $qX_0 \geq I_0$. In this case, all debt is repaid in full in the event of success and repaid nothing otherwise, $1/q$. Thus, $F_0^{\text{c}} = I_0^{\text{c}}/q$ and $F_0^{\text{nc}} = I_0^{\text{nc}}/q$. Hence $F_0 = I_0/q$ which implies that $F_0^{\text{c}} = \phi I_0$ and $F_0^{\text{nc}} = (1-\phi)F_0 = (1-\phi)I_0$.

Case 2: $q(X_0 + X_1^H) < I_0 + I_1$ and $qX_0 \geq I_0$. In this case, B following success if $\tau = H$, but not if $\tau = L$. Using $I_0^{\text{c}} = \phi I_0$ and $I_0^{\text{nc}} = (1-\phi)I_0$, creditors' break-even conditions are

$$\phi I_0 = q\left(p\phi\left(X_0 + X_1^H - \frac{I_1}{q}\right) + (1-p)F_0^{\text{nc}}\right), \tag{42}$$

$$(1-\phi)I_0^{\text{nc}} = q\left(p(1-\phi)\left(X_0 + X_1^H - \frac{I_1}{q}\right) + (1-p)F_0^{\text{nc}}\right), \tag{43}$$

having used $F_1 = I_1/q$. Solving for F_1^{c} and F_1^{nc} above gives the result.

Case 3: $qX_0 < I_0$. In this case, B defaults given success if $\tau = L$ but not if $\tau = H$. Again, we use $I_0^{\text{c}} = \phi I_0$ and $I_0^{\text{nc}} = (1-\phi)I_0$ to write creditors' break-even conditions:

$$\phi I_0 = q\Big(pF_0^{\text{c}} + (1-p)\phi X_0\Big), \tag{44}$$

$$(1-\phi)I_0 = q\Big(p(1-\phi)F_0^{\text{nc}} + (1-p)(1-\phi)X_0\Big). \tag{45}$$

Again, solving for F_0^{c} and F_0^{nc} gives the result.

□

We now turn to the proof of the proposition. As we argued in the text, acceleration must be incentive compatible following a covenant violation if $\tau = L$ but not if $\tau = H$. Rather than the conditions (21) and (22), in which we assumed that B never repaid in full, we now have

$$\min\left\{\phi F_0\,,\, q\big(X_0 + X_1^H\big) - \frac{I_1}{q}\right\} \leq q \min\left\{\phi F_0\,,\, \phi\left(X_0 + X_1^H - \frac{I_1}{q}\right)\right\}, \tag{46}$$

$$\min\left\{\phi F_0\,,\, q\big(X_0 + X_1^L\big) - \frac{I_1}{q}\right\} \geq q \min\left\{\phi F_0\,,\, \phi\left(X_0 + X_1^L - \frac{I_1}{q}\right)\right\}. \tag{47}$$

Before launching into the main argument, we can dispense with a few cases relatively easily:

1. We can focus on cases in which there is dilution if $\tau =^L$, or $F_0 > X_0 + X_1^L - I_1/q$. Otherwise, B will not finance Project 1 in this case, since it has negative NPV.

2. By implication, we can focus on cases in which there is dilution if $\tau = H$ as well, since $X_1^H < X_1^L$ by hypothesis.

3. We can focus on cases in which the accelerated debt is not paid in full if $\tau = L$, since we are looking only for a sufficient condition (and otherwise acceleration is always IC).

Now, the ICs can be simplified to read

$$\min\left\{\phi F_0\,,\, q\big(X_0 + X_1^H\big) - \frac{I_1}{q}\right\} \leq q\phi\left(X_0 + X_1^H - \frac{I_1}{q}\right), \tag{48}$$

$$q\big(X_0 + X_1^L\big) - \frac{I_1}{q} \geq q\phi\left(X_0 + X_1^L - \frac{I_1}{q}\right). \tag{49}$$

To get sufficient conditions, we can split (48) in two, and write

$$q\big(X_0 + X_1^H\big) - \frac{I_1}{q} \leq q\phi\left(X_0 + X_1^H - \frac{I_1}{q}\right), \tag{50}$$

$$q\big(X_0 + X_1^H\big) - \frac{I_1}{q} \leq \phi F_0, \tag{51}$$

$$q\big(X_0 + X_1^L\big) - \frac{I_1}{q} \geq q\phi\left(X_0 + X_1^L - \frac{I_1}{q}\right). \tag{52}$$

Combining the above, we have

$$\max\left\{\frac{q\big(X_0+X_1^H\big)-I_1/q}{F_0},\frac{q\big(X_0+X_1^H\big)-I_1/q}{q\big(X_0+X_1^H-I_1/q\big)}\right\}\leq\phi\leq\frac{q\big(X_0+X_1^L\big)-I_1/q}{q\big(X_0+X_1^L-I_1/q\big)}. \tag{53}$$

Now, we can do away with the max above. Recall that we are focused on a case in which there is dilution (hence default) given success if $\tau=H$. It follows that $F_0>X_0+X_1^H-I_1^q>q\big(X_0+X_1^H-I_1^q\big)$ and hence

$$\max\left\{\frac{q\big(X_0+X_1^H\big)-I_1/q}{F_0},\frac{q\big(X_0+X_1^H\big)-I_1/q}{q\big(X_0+X_1^H-I_1/q\big)}\right\}=\frac{q\big(X_0+X_1^H\big)-I_1/q}{q\big(X_0+X_1^H-I_1/q\big)}. \tag{54}$$

So we can implement the first best if we can find ϕ satisfying

$$\frac{q\big(X_0+X_1^H\big)-I_1/q}{q\big(X_0+X_1^H-I_1/q\big)}\leq\phi\leq\frac{q\big(X_0+X_1^L\big)-I_1/q}{q\big(X_0+X_1^L-I_1/q\big)}. \tag{55}$$

Since the LHS is always less than one and the RHS is greater than zero by Assumption 3, such a ϕ exists whenever the LHS is less than the RHS, or $X_1^H\leq X_1^L$, which is the condition in the proposition.

Renegotiation proofness. This argument hinges on acceleration being a credible threat when $\tau=L$, even though liquidation is inefficient. To complete the proof, we show that this is robust to the possibility of renegotiation. For renegotiation to be feasible, all parties, i.e. (i) B, (ii) Date-1 secured creditors, (iii) Date-0 creditors, both protected by covenants and not, must be strictly better off. However, if B avoids liquidation and continues, the most he can promise his creditors is $q(X_0+X_1^L)$. But this is only equal to the liquidation value that creditors are already dividing up among themselves. Hence, there is no way to make them collectively better off.

A.7 Proof of Proposition 4

The expression face value F_0 follows from equations (25), (31), and (37) in the proof of Proposition 1. The regions in which B uses secured debt or covenants and the ranges of σ_0 and ϕ follow from Proposition 2 and Proposition 3 (and their proofs).

References

Aghion, P. and B. Hermalin (1990). Legal restrictions on private contracts can enhance efficiency. *Journal of Law, Economics, & Organization 6*(2), 381–409.

Attar, A., C. Casamatta, A. Chassagnon, and J.-P. Décamps (2015). Multiple lenders, strategic default, and the role of debt covenants. Working paper, Toulouse School of Economics.

Ayotte, K. and P. Bolton (2011). Optimal property rights in financial contracting. *Review of Financial Studies 24*(10), 3401–3433.

Badoer, D., E. Dudley, and C. James (2018). Priority spreading of corporate debt. Working paper, University of Missouri.

Barclay, M. J. and C. W. Smith (1995). The priority structure of corporate liabilities. *The Journal of Finance 50*(3), 899–916.

Bebchuk, L. A. and J. M. Fried (1996). The uneasy case for the priority of secured claims in bankruptcy. *The Yale Law Journal 105*(4), 857–934.

Beneish, M. D. and E. Press (1993). Costs of technical violation of accounting-based debt covenants. *The Accounting Review 68*(2), 233–257.

Beneish, M. D. and E. Press (1995). The resolution of technical default. *The Accounting Review 70*(2), 337–353.

Bester, H. (1985). The level of investment in credit markets with imperfect information. *Zeitschrift fÃijr die gesamte Staatswissenschaft / Journal of Institutional and Theoretical Economics 141*(4), 503–515.

Billett, M., T.-H. D. King, and D. Mauer (2007). Growth opportunities and the choice of leverage, debt maturity, and covenants. *The Journal of Finance 62*(2), 697–730.

Bjerre, C. S. (1999). Secured transactions inside out: Negative pledge covenants, property and perfection. *Cornell Law Review*.

Bolton, P. and M. Oehmke (2015). Should derivatives be privileged in bankruptcy? *Journal of Finance 70*(6), 2352–2394.

Bradley, M. and M. R. Roberts (2015). The structure and pricing of corporate debt covenants. *Quarterly Journal of Finance 05*(02), 1550001.

Chava, S. and M. Roberts (2008). How does financing impact investment? the role of debt covenants. *Journal of Finance 63*(5), 2085–2121.

Colla, P., F. Ippolito, and K. Li (2013). Debt specialization. *The Journal of Finance 68*(5), 2117–2141.

Deloitte Blogs (2014). Asset encumbrance: The elephant in the room?

Dichev, I. D. and D. J. Skinner (2002). Largeï£¡ï£¡ sample evidence on the debt covenant hypothesis. *Journal of Accounting Research 40*(4), 1091–1123.

Donaldson, J. R., D. Gromb, and G. Piacentino (2018). The paradox of pledgeability. Technical report, Washington University in St. Louis.

Erel, I., B. Julio, W. Kim, and M. S. Weisbach (2012). Macroeconomic conditions and capital raising. *The Review of Financial Studies 25*(2), 341–376.

Eyigungor, B. (2013). Debt dilution: when it is a major problem and how to deal with it. *Business Review* (Q4), 1–8.

Flannery, M. (1986). Asymmetric information and risky debt maturity choice. *The Journal of Finance 41*(1), 19–37.

Gârleanu, N. and J. Zwiebel (2009). Design and renegotiation of debt covenants. *Review of Financial Studies 22*(2), 749–781.

Gennaioli, N. (2006). Contracting in the shadow of the law. Working paper, Institute for International Economic Studies, Stockholm University.

Gopalakrishnan, V. and M. Prakash (1995). Borrower and lender perceptions of accounting information in corporate lending agreements. *9*(1).

Hahn, D. (2010). The roles of acceleration. *DePaul Business and Commerce Law Journal 8*.

Hansmann, H. and R. Kraakman (2002). Property, contract, and verification: The *Numerus Clausus* problem and the divisibility of rights. *The Journal of Legal Studies 31*(S2), S373–S420.

Hansmann, H. and M. Santilli (1997). Authors' and artists' moral rights: A comparative legal and economic analysis. *The Journal of Legal Studies 26*(1), 95–143.

Hertzberg, A., A. Liberman, and D. Paravisini (2018). Screening on loan terms: Evidence from maturity choice in consumer credit. *The Review of Financial Studies 31*(9), 3532–3567.

Ivashina, V. and B. Vallée (2018). Weak credit covenants. Working paper, Harvard Business School.

Kronman, A. T. and T. H. Jackson (1979). Secured financing and priorities among creditors. *Yale Law Journal 88*.

Levine, C. B. and J. S. Hughes (2005). Management compensation and earnings-based covenants as signaling devices in credit markets. *Journal of Corporate Finance 11*(5), 832 – 850.

Matvos, G. (2013). Estimating the benefits of contractual completeness. *The Review of Financial Studies 26*(11), 2798–2844.

Nini, G., D. C. Smith, and A. Sufi (2012). Creditor control rights, corporate governance, and firm value. *The Review of Financial Studies 25*(6), 1713–1761.

Park, C. (2002). Monitoring and structure of debt contracts. *The Journal of Finance 55*(5), 2157–2195.

Rajan, R. and A. Winton (1995). Covenants and collateral as incentives to monitor. *Journal of Finance 50*(4), 1113–46.

Rauh, J. and A. Sufi (2010). Capital structure and debt structure. *The Review of Financial Studies 23*(12), 4242–4280.

Roberts, M. and A. Sufi (2009). Control rights and capital structure: An empirical investigation. *The Journal of Finance 64*(4), 1657–1695.

Schwarcz, S. L. (1997). The Easy case for the priority of secured claims in bankruptcy. *Duke Law Journal*.

Schwartz, A. (1984). The continuing puzzle of secured debt. *Vanderbilt Law Review 37*(5).

Schwartz, A. (1989). A theory of loan priorities. *The Journal of Legal Studies 18*(2).

Schwartz, A. (1994). The absolute priority rule and the firm' s investment policy. *Washington University Law Review 72*(3), 1213–1229.

Schwartz, A. (1997). Priority contracts and priority in bankruptcy. *Cornell Law Review 82*, 1396–1419.

Smith, C. W. (1993). A perspective on accounting-based debt covenant violations. *The Accounting Review 68*(2), 289–303.

Smith, C. W. and J. B. Warner (1979). On financial contracting: An analysis of bond covenants. *Journal of Financial Economics 7*(2), 117 – 161.

Sweeney, A. P. (1994). Debt-covenant violations and managers' accounting responses. *Journal of Accounting and Economics 17*(3), 281 – 308.

Tirole, J. (2006). *The Theory of Corporate Finance*. Princeton University Press.