Disclosures, Rollover Risk, and Debt Runs

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Abstract

Procedures such as stress tests increase the transparency of the financial system by dampening information asymmetries between banks and the regulator. Is this transparency desirable? How should the regulator disclose information to the public? I construct a dynamic model of runs where debt and beliefs evolve endogenously. This allows to capture a funding cost channel and a beliefs channel which are absent from models with a single rollover date but matter for equilibrium outcomes. I find that opacity reduces run likelihood and inefficiency if and only if fundamentals are strong; that transparency can be more efficient even when it entails more runs; and that the regulator should commit to disclosure except at large levels of opacity.

Keywords: dynamic debt runs, opacity, disclosure policy.
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1 Introduction

Financial institutions issuing short-term debt collateralized by long-term assets are exposed to bank run phenomena: creditors may demand to withdraw their funds and trigger costly liquidations. Debt runs are prominent features of financial crises: during the turmoil of 2007-2008, runs hit the asset-backed commercial paper market, the repo market, money market mutual funds and banks such as Northern Rock and Bear Stearns.[1]

Many institutions managing opaque assets struggled during the crisis (Gorton (2008)). This ignited a debate among both academics and policy makers about the impact of opacity on financial fragility. One line of thinking, represented by Gorton and Ordoñez (2014) and Dang, Gorton, Holmström, and Ordoñez (2017), advocates that opacity is actually a desirable characteristic of the financial sector and should be fostered. On the other hand, the policy responses to the crisis seemed to go in a different direction: regulators engaged in a considerable effort to both gather and disclose more information about banks. This was evidenced by the start of the Supervisory Capital Assessment Program (SCAP) in February 2009, a massive effort to submit all major banking institutions in the United States to thorough stress tests. Gathering and disclosing information are two distinct decisions: the regulator may collect information to have the option to reduce opacity by releasing it to the public, but prefer ex post not to do so. In fact, there were concerns that fully releasing the results of the SCAP stress tests might have a destabilizing effect.[2]

The present paper aims at answering the following questions: how does the accessibility of information impact the resilience of financial institutions to debt runs? under which circumstances should the regulator strive to collect information regularly? if the regulator has incentives to withhold information in some states, should he commit ex ante to a policy of full disclosure?

To do so, I modify the discrete time dynamic debt runs model of Acharya, Gale, and Yorulmazer (2011) by allowing the bank's assets to be opaque and information release to be strategic. My model features an uninsured financial institution ("bank") trying to roll over its short-term debt until its assets mature. The bank cannot communicate credibly. Instead, creditors have to rely on regulatory disclosures when deciding whether to renew their credit to the bank. Because the bank assets can be complex and investigation is costly, the regulator may not be able to constantly assess the soundness of the bank: the frequency at which he can and wishes to obtain bank-specific information defines the degree of transparency in the model. Opacity is defined as the opposite of transparency. In a regime

[1] Gorton and Metrick (2012) document the run on the repo market, and Covitz, Liang, and Suarez (2013) investigate the run on the ABCP market.

[2] Bernanke (2010) mentions these concerns; see also Goldstein and Sapra (2014).

of commitment (*mandatory* disclosure), the regulator conveys truthfully any information he has to the bank's creditors. Absent commitment (*voluntary* disclosure), the regulator finds it optimal to only release good news.

Because of the simple structure chosen for the bank's asset process, I am able to characterize analytically the interest rates demanded by creditors to roll over the bank's debt, and the states in which they instead decide to run. Runs are assumed to entail deadweight liquidation costs proportional to the fundamental value of the asset at the run time. Inefficiency is then defined as the expected liquidation costs.

Constructing a dynamic framework where the cost of debt is endogenous allows to uncover two channels that would not be apparent in a model with a single rollover date. First, I capture a *funding cost channel*: a signal provided to a creditor has a contemporaneous effect (it will trigger a run with some probability today), but also impacts the required interest rate. Hence, it affects future debt levels and thereby future incentives to run. The efficiency of an opacity level and a disclosure policy depend on both the direct and the indirect effect. Second, the model recognizes that the disclosure policy of the regulator impacts the beliefs dynamics, which in turn impact future rollover decisions. When the regulator does not commit to disclosure, short-term funding costs are lower in good times. However, the lack of commitment generates systematically depressed beliefs, potentially leading to a larger probability of bank failure at longer horizons. A model with a single rollover date would obliterate this *beliefs channel* and the costs it entails; when in fact, all the costs associated with non-commitment are due to the fact that it generates worse beliefs.

The interaction between the information structure, debt dynamics and beliefs dynamics is rich. Short-term debt yields are determined by the number of default states tomorrow under the given information structure, not by the expected value of the collateral computed under the beliefs generated by this structure: yields do not primarily reflect the current expected collateral quality. Two results of the paper relate to this intuition. First, there need not necessarily be a warning sign of a run in the time-series of short-term returns: yields may remain low while risk builds in the background. Second, there are situations in which the expected collateral value is always larger under one disclosure regime, but the bank nevertheless faces larger financing costs under this regime, and therefore fails only in the seemingly better scenario.

At the policy level, the main results are the following. First, I find that opacity reduces run probability and inefficiency only when fundamentals are strong enough: in situations where the regulator believes that the economy is healthy and likely to remain so for a long time, collecting and releasing information about banks is detrimental; in other situations, the regulator wants to implement transparency. Second, opacity may decrease run probability but increase inefficiency: the objective of the regulator is not to minimize the probability of a bank failure, but rather the expected costs associated with liquidation. Under transparency, runs may occur more frequently but they are concentrated on bad banks, for which liquidation is less inefficient. Third, voluntary disclosure is more efficient than mandatory disclosure except at large levels of opacity: this implies that the regulator should commit to disclose stress test results as soon as his access to information is relatively easy. Thus, my model shows that whether stress test results should be systematically disclosed depends on the degree of asset opacity.

Relation to the literature. The game-theoretic study of bank runs traces back to the seminal paper of Diamond and Dybvig (1983): in the bad equilibrium, agents "panic" about the run decision of others, leading to an outcome where all creditors run on the bank and force an inefficient liquidation. Building on the global games literature pioneered by Carlsson and van Damme (1993) and Morris and Shin (1998), Rochet and Vives (2004) and Goldstein and Pauzner (2005) provide bank run models where the equilibrium is unique and runs arise as the result of both a coordination failure and concerns about the fundamentals. In these models, the coordination problem comes from the fact that creditors are dispersed and must decide simultaneously whether to withdraw their funds. Models of dynamic debt runs provide a related but distinct approach. There, the coordination problem is intertemporal in the sense that an agent may withdraw his funds because of concerns about future rollover decisions of other creditors. He and Xiong (2012) and Schroth, Suarez, and Taylor (2014) provide such models and use them to quantify the impact of factors such as maturity mismatch, leverage and liquidation costs on run likelihood, with a focus on the 2007 run on ABCP.

As Acharya, Gale, and Yorulmazer (2011), my paper highlights the importance of the specific nature of the information structure to the outcome of the rollover problem. In a broader framework, Kamenica and Gentzkow (2011) show how one can optimally design information structures (*i.e.* select signals[3]) to maximize non-linear functions of some agent's beliefs, what they call Bayesian persuasion. Finding the optimal opacity level and disclosure policy in the present model can be seen as a Bayesian persuasion problem, because it means choosing ex ante which signals about the fundamental to show to investors, and the non-linear function of their belief is the rollover decision. Papers linking explicitly the Bayesian persuasion approach to the research on stress tests include Goldstein and Leitner (2017), Inostraza and Pavan (2017) and Quigley and Walther (2017).

While the models of dynamic debt runs mentioned above assume full information, there is also a significant body of literature on banking under opacity. Alvarez and Barlevy (2014)

[3] By "selecting signals" one means of course selecting *ex ante* a random variable, rather than being able to show or conceal the realization of a given signal.

develop a network model of banking where imposing mandatory disclosure of losses can only improve welfare when contagion concerns are strong. de Faria e Castro, Martinez, and Philippon (2016) study the interaction between the fiscal capacity of the government and optimal disclosure policies. When deposit insurance can be provided at a low social cost, a disclosure policy that would be suboptimal absent insurance because of the run risk it implies may become desirable. In a model of coordination failures *à la* Goldstein and Pauzner (2005), Bouvard, Chaigneau, and de Motta (2015) investigate how a regulator endowed with perfect information about aggregate and idiosyncratic shocks on the banking sector should communicate with the public. My model does not distinguish between these shocks, but introduces the possibility that the regulator herself has no information: this generates a different commitment problem. Additionally, their model features a single rollover date and threfore does not capture the funding cost channel and the beliefs channel described above. Finally, Monnet and Quintin (2017) map the need for transparency to the degree of a bank's asset liquidity and show that opacity is preferable when secondary markets are shallow.

My paper also bears a connection with the series of papers by Gorton and Pennacchi (1990), Dang, Gorton, and Holmström (2013), Dang, Gorton, and Holmström (2015), Gorton and Ordoñez (2014) and Dang, Gorton, Holmström, and Ordoñez (2017). These authors focus on the notion of *information sensitivity*. A security is information insensitive when agents have no incentive to acquire costly signals about it. Because of their capped payoff, debt contracts are natural candidates for information insensitivity, and more so if collateral is opaque. If, in addition, the expected value of collateral is large enough, debt is risk-free and of constant value: it can be used as money. Therefore bank should be "secret keepers" (Dang, Gorton, Holmström, and Ordoñez (2017)). Deterring information acquisition with opaque collateral also ensures that information is always symmetrical. This prevents market freezes due to adverse selection issues (Dang, Gorton, and Holmström (2015)). One can similarly define the information sensitivity status of debt in my model and map this status to the current state of the world, the degree of opacity, and the disclosure regime.

2 The Model

Time is discrete: $t = 0, 1, 2, \ldots$. The model features an uninsured financial institution ("bank") whose short-term debt must be refinanced by successive creditors until its asset reaches maturity.[4]

[4] A significant part of the short-term debt of financial institutions is not insured, and even bank deposits are typically insured only up to some limit. Moreover, ex-post liquidity assistance may not be systematical but contingent to some criteria (see for instance Santos and Suarez (forthcoming)). For simplicity, I consider uninsured debt, but it is straightforward to amend the model solution to the case where the institution is

2.1 The Bank

2.1.1 Asset side

The bank holds a long-term asset. For tractability purposes, its maturity is modelled as a random time ζ_ϕ. ζ_ϕ is assumed to be independent of all other variables and geometrically distributed with parameter $\phi \in (0,1)$.[5] At time ζ_ϕ, the asset delivers its payoff, agents receive their payments, and the world ends. The asset does not pay anything before maturity.

The asset side of the bank is modelled by a Markov chain $(y_t)_{t\geq 0}$ with two states: $y^G > y^B$. The meaning of y_t is the following: if maturity occurs at time t ($\zeta_\phi = t$), the asset payoff is y_{ζ_ϕ}. Assume that the asset is initially in the good state: $y_0 = y^G$. The transition matrix of (y_t) is

$$\Lambda = \begin{pmatrix} \lambda^{GG} & 1-\lambda^{GG} \\ \lambda^{BG} & 1-\lambda^{BG} \end{pmatrix}. \tag{1}$$

λ^{GG} represents the probability to stay in the good state from one period to the next, while λ^{BG} can be interpreted as a recovery probability. Under the conditions $\lambda^{GG} > \frac{1}{2}$ and $\lambda^{BG} < \frac{1}{2}$, we have

$$V^G \equiv \mathbb{E}[y_{\zeta_\phi} | y_t = y^G, t < \zeta_\phi] > \mathbb{E}[y_{\zeta_\phi} | y_t = y^B, t < \zeta_\phi] \equiv V^B. \tag{2}$$

(2) means that being in the state y^G before maturity signals a high expected payoff at maturity, so y^G is indeed the "good state".

2.1.2 Liability side

The initial capital structure of the bank is taken as given. The bank has raised an amount D_0 of short-term (*i.e.* one-period) debt D_0.[6] Equity is the residual claim and is owned by the banker. Since the asset does not pay anything before maturity, short-term debt must be refinanced: to do so, the bank has access to a pool of potential short-term creditors (see section 2.2). No other sources of financing are available.

Short-term debt can stop being rolled over in two cases. (i) (*strategic default*) The bank can decide to default on the debt, in which case its asset is liquidated at a fraction of its current expected value. The strategic default time is denoted ζ_s. (ii) (*rollover freeze*) If debt is too high, there is no short-term debt contract that compensates adequately for default

bailed out with some exogenous probability when a run occurs.

[5] The expected maturity is $\mathbb{E}[\zeta_\phi] = \frac{1}{\phi}$.

[6] Explicit motivations for short-term debt include Calomiris and Kahn (1991) and Diamond and Rajan (2001). Brunnermeier and Oehmke (2013) show how debt maturities can endogenously shorten in response to dilution concerns. Carré and Klossner (2018) provide a global games model for the short-term leverage choice of a bank whose debt provides liquidity but creates rollover risk.

risk. No creditor accepts to roll over the debt, forcing the bank into liquidation. The time at which this happens is denoted ζ_z.

Important details on the liquidation procedure are given in section 2.4.3. Let $\zeta_\ell = \min\{\zeta_s, \zeta_z\}$ be the liquidation time. I will use the convention $\zeta_l = \infty$ when liquidation does not occur prior to maturity. Finally, define the end date as

$$\zeta_f = \min\{\zeta_\ell, \zeta_\phi\}. \tag{3}$$

It is convenient to introduce the following assumption.

Assumption 1. $D_0 > V^B$.

This condition ensures that the bank is insolvent when the bad state is revealed, which triggers liquidation.

2.2 Creditors

The bank has access to an unlimited pool of risk-neutral and competitive creditors.

I assume that all the short-term debt is held by a single investor at each period, and that the investor entering the debt contract at date t exits forever the pool of creditors after receiving his payment at $t + 1$.

Given an amount of debt to roll over at time t, the bank offers a contract with a promised repayment at time $t + 1$, the face value F. The risk-free rate is normalized to zero. Hence, since creditors compete to obtain the debt contract, the equilibrium face value is such that a creditor makes zero profit on average. If no face value satisfies the zero profit condition, liquidation occurs (*i.e.* ζ_z is reached). I use the convention $F = \emptyset$ in that case, since the bank cannot offer any acceptable face value.

2.3 The Regulator

There is a regulator who may obtain information about the bank's asset and can disclose them to creditors. When the regulator does not commit to reveal all its information, he selects his disclosure policy to minimize inefficiency. Note that since creditors make zero expected profit in equilibrium, the regulator's objective is in fact to maximize the banker's equity value: see the equilibrium definition in section 2.5.3. The next section describes the information structure and provides details about the constraints under which the regulator operates.

2.4 Information Structure

2.4.1 Asset Opacity

I make the following assumptions. First, the bank observes (y_t) but cannot credibly communicate any information to investors. Second, at each time t, the regulator observes the current state of the chain, y_t, with probability p, independently of everything else.

It will be convenient to define the dummy variables

$$\omega_t = \begin{cases} 1 & \text{if the regulator observes } y_t \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

By assumption, $(\omega_t)_{t\geq 0}$ is an i.i.d. sequence of Bernoulli variables with parameter p. p characterizes the degree of opacity of the asset. When $p = 1$, there is full information, while $p = 0$ corresponds to the extreme case of a fully opaque asset.

Agents in the pool of creditors cannot make any direct observation and rely on the regulator's disclosures.

The motivation for this particular modelling of opacity is the following. One wants to capture the fact that it is not feasible for the regulator to monitor the bank at all times, because of the excessive costs this would imply. As Bernanke (2010) noted, "The SCAP represented an extraordinary effort on the part of the Federal Reserve staff and the staff of other banking agencies. In a relatively short time, the supervisors had to gather and evaluate an enormous amount of information".

Considering an exogenous ω_t allows to maintain tractability; and letting $p < 1$ incorporates the regulator's constraints into the model as desired.

2.4.2 Disclosure Regimes

At each time t, the regulator has the opportunity to disclose information to the pool of creditors after the realization of ω_t. Disclosure takes the form of an announcement δ_t:

$$\delta_t = \begin{cases} \emptyset & \text{“I did not observe the asset value ”} \\ y^G & \text{“I observed the asset value and } y_t = y^G \text{”} \\ y^B & \text{“I observed the asset value and } y_t = y^B \text{”.} \end{cases} \tag{5}$$

I compare two disclosure regimes: *voluntary* and *mandatory.*

Under mandatory disclosure, the regulator is compelled by law to announce the truth. That is, he has been able to credibly commit to communicate any information he has. In that case, disclosure is mechanical:

$$\delta_t = \begin{cases} y_t & \text{if } \omega_t = 1 \\ \emptyset & \text{if } \omega_t = 0. \end{cases} \tag{6}$$

Under voluntary disclosure, the regulator can conceal news. That is, he can claim to be uninformed while he is. Formally, it means he can play $\delta_t = \emptyset$ when $\omega_t = 1$. However, if a state is announced, is must be accompanied with evidence. Hence, it is impossible to announce that a state has been observed when it is not the case. Formally, it means that $\delta_t = y^i$ implies $\omega_t = 1$ and $y_t = y^i$ for $i = G, B$. These assumptions on the voluntary disclosure regime are borrowed from Dye (1985).

The equilibrium under voluntary disclosure will feature a *sanitization strategy*:[7] the regulator discloses the good state and conceals the bad state. That is, he plays $\delta = \delta^S$, where

$$\delta_t^S \equiv \begin{cases} y^G & \text{if } \omega_t = 1 \text{ and } y_t = y^G \\ \emptyset & \text{otherwise} \end{cases} \tag{7}$$

is the sanitization strategy.

Denote $(\mathcal{F}_t^I)_{t\geq 0}$ the filtration of the investors:

$$\mathcal{F}_t^I = \sigma\left((\delta_s)_{s\leq t}, \zeta_\ell \mathbb{I}_{\{\zeta_\ell \leq t\}}, \zeta_\phi \mathbb{I}_{\{\zeta_\phi \leq t\}}\right), \tag{8}$$

$(\mathcal{F}_t^R)_{t\geq 0}$ the filtration of the regulator:

$$\mathcal{F}_t^R = \sigma\left((\omega_s, y_s \mathbb{I}_{\{\omega_s = 1\}})_{s\leq t}, \zeta_\ell \mathbb{I}_{\{\zeta_\ell \leq t\}}, \zeta_\phi \mathbb{I}_{\{\zeta_\phi \leq t\}}\right), \tag{9}$$

and $(\mathcal{F}_t^B)_{t\geq 0}$ the filtration of the bank, which observes everything but cannot communicate information credibly.

2.4.3 Liquidation

If liquidation occurs at time t ($t < \zeta_\phi$), the value αV is recovered, where $\alpha \in [0,1]$ and $V = \mathbb{E}[y_{\zeta_\phi} | \mathcal{F}_t^I]$ is the fundamental value of the asset computed under the outsiders' information set at time t.

In case of a strategic liquidation under asymmetric information, the liquidation decision has a signalling content and we need to specify the beliefs of outsiders. For simplicity, I focus on equilibria where the bank's decision to liquidate at t when $\delta_t = \emptyset$ is interpreted as

[7]See Shin (2003).

the fact that the bank has observed the bad state ($y_t = y^B$). When $\delta_t \neq \emptyset$, (payoff-relevant) information is symmetric because announcements of states are trustworthy. Hence, there is no signalling problem in that case.

$1-\alpha$ is a measure of illiquidity as it represents the fraction of asset value destroyed due to premature liquidation.[8] Because of the deadweight cost $(1-\alpha)V$, liquidation is never efficient in this model; the inefficiency is large for good banks (*i.e.* V high) and small for bad banks (*i.e.* V low).[9]

2.5 Equilibrium

2.5.1 Debt Dynamics

Assume we are at time $t < \zeta_f$ with a level of debt D_t. The bank has promised the face value D_{t+1} to the current creditor. The actual payment, $\tilde{D}_{t+1}$, satisfies:

$$\tilde{D}_{t+1} = \begin{cases} \min\{y_{t+1}, D_{t+1}\} & \text{if } \zeta_\phi = t+1 \\ \min\{\alpha V_{t+1}, D_{t+1}\} & \text{if } \zeta_\phi > t+1 \text{ and } \zeta_\ell = t+1 \\ D_{t+1} & \text{otherwise.} \end{cases} \tag{10}$$

In the first case, maturity occurs at time $t+1$ and the asset delivers the payoff y_{t+1}. In the second case, the bank is liquidated at the value αV_{t+1} where $V_{t+1} = \mathbb{E}[y_{\zeta_\phi} | \mathcal{F}^I_{t+1}]$. In the third case, the banker is able to roll its debt over. That is, she obtains the financing necessary to repay D_{t+1} in full.

Lemma 1. *The break-even condition of lenders is equivalent to the property that* $(\tilde{D}_{t\wedge\zeta_f})_{t\geq 0}$ *is a* $(\mathcal{F}^I_t)$*-martingale.*

(All proofs are relegated to the Appendix).

2.5.2 Quantities of Interest

My goal is to understand how asset opacity and disclosure regimes impact the likelihood of debt runs and their inefficiency. In this section, I explain how these quantities are measured in the model. I then define formally the equilibrium.

[8] If the bank is the first-best user of the asset, transferring its control rights to another party reduces its value (Shleifer and Vishny (1992)).

[9] The typical situation in 3-dates models of runs is that premature liquidations are efficient when the fundamental is very low and inefficient otherwise. The common conclusion is that the deadweight cost of liquidating good banks is larger.

The probability of a run is simply

$$\mathcal{P} \equiv \mathbb{P}(\zeta_\ell < \zeta_\phi). \tag{11}$$

Now note that because lenders make zero profit on average, the bank bears the costs of inefficient runs. Optimality for the bank coincides with a social planner's optimality in the model, that is, maximizing the expectation of the payoff U of the asset. This quantity is given by

$$U \equiv \alpha V_{\zeta_l} \mathbb{I}_{\{\zeta_\ell < \zeta_\phi\}} + y_{\zeta_\phi} \mathbb{I}_{\{\zeta_\phi \leq \zeta_\ell\}}. \tag{12}$$

Equivalently, the measure of *inefficiency* is the expected deadweight cost

$$\mathcal{I} \equiv \mathbb{E}[(1-\alpha) y_{\zeta_\ell} \mathbb{I}_{\zeta_\ell < \zeta_\phi}] = V^G - \mathbb{E}[U]. \tag{13}$$

Saying that the banker maximizes equity value is equivalent to saying that she maximizes $\mathbb{E}[U]$ or minimizes $\mathcal{I}$.

We are now ready to define the equilibrium.

2.5.3 Equilibrium concept

Definition 1. *Given a disclosure policy δ, a consistent bank policy is a promised face value schedule F and a time of strategic liquidation ζ_s such that*

i) F_t is Markov in (D_t, q_t) where $q_t \equiv \mathbb{E}[y_t | \mathcal{F}^I_t]$; $D_{t+1} = F_t$ and the process $(\tilde{D}_t)$ associated with (D_t) is a $(\mathcal{F}^I_t)$-martingale. F is required to satisfy

(M) F_t is non-decreasing in D_t and non-increasing in q_t[10]*,*

(NP) $F \leq K$ for some constant $K > y^G$.

ii) ζ_s is $\mathcal{F}^B_t$-adapted, and, given F, it minimizes $\mathcal{I}$.

An equilibrium is (δ, F, ζ_s) such that

i) given δ, (F, ζ_s) is a consistent bank policy that minimizes $\mathcal{I}$.

ii) δ is $\mathcal{F}^R_t$-adapted and given (F, ζ_s) it minimizes $\mathcal{I}$.[11]

[10] Recall the convention $F = \emptyset$ when there is no acceptable face value. The meaning of the monotonicity condition is then that if $F(D_1) = \emptyset$ and $F(D_2) \in \mathbb{R}$, $D_1 > D_2$.

[11] Under mandatory disclosure, δ is mechanical and is *de facto* not an equilibrium object.

The implicit assumption here is that the banker commits to an interest rate schedule at date 0. Otherwise, the banker would convey signalling information when offering a face value to creditors. In particular due to the specification of out-of-equilibrium beliefs, this would complicate significantly the formalization of the game without bringing additional insights. With the formulation of the text, all the signalling is contained in the disclosure decision.

Requiring that F_t is Markov in (D_t, q_t) is to simplify the exposition. We could just demand that F_t is $(\mathcal{F}_t^I)$-adapted; but since q_t encapsulates all the relevant information about the asset payoff, the bank has nothing to gain to condition its face value to other $\mathcal{F}_t^I$-measurable variables.

Condition (NP) rules out Ponzi schemes, and, as usual, the constraint $F \leq K$ is never binding in equilibrium.[12] This is a consequence of the following useful lemma:

Lemma 2. *In a consistent bank policy, an insolvent bank is necessarily forced into liquidation.*

This is the standard result that insolvency implies illiquidity (of course, the converse is not true). Hence, since $K > y^G$, the bank would be ran upon before debt can reach K, so the constraint $F \geq K$ does not bind.

3 Model Solution

The first step towards solving the model is to establish that the bank never wishes to force liquidation:

Lemma 3. *The bank never liquidates strategically in a consistent bank policy for $\alpha \in [0, 1)$: $\zeta_s = \infty$.*

(In the extreme case $\alpha = 1$, there is no cost associated with liquidation. Thus, when $y_t = y^G$ is observed, the bank is indifferent between holding the asset or liquidating it.) The intuition behind this result is the following. Since debt comes at a zero expected cost for the bank, the banker has no incentive to incur the deadweight liquidation cost today: she is always better off waiting.

[12] In the sense that the banker never actually sets F at K. But of course the constraint binds in a dynamic sense since it rules out Ponzi schemes. Also note that absent requirement (NP), there is a Ponzi equilibrium where each lender is simply betting against maturity, *i.e.* hoping he is not the last in line (this is made possible by the random maturity assumption). Of course, the actual asset value is irrelevant in that case. See *e.g.* Blanchard and Watson (1982).

3.1 Voluntary Disclosure

We now characterize the policy of the regulator in the voluntary disclosure case.

Lemma 4. *The regulator follows the sanitization strategy δ^S (defined in (7)) in equilibrium.*

This result is very intuitive. When the regulator observes the good state, it is clearly in his best interest to communicate it to investors. When the regulator observes the bad state, it is always best to conceal it. Even if creditors understand that the regulator may be hiding information, their updated belief about the probability of the good state cannot be worse than if the regulator had revealed the bad state.[13]

3.1.1 State Variables

Suppose we are at time $t < \zeta_\phi$ and current debt is D. From Lemma 4, we know that the regulator discloses only $\emptyset$ or y^G in equilibrium. Let τ be the time elapsed since the last disclosure of y^G:

$$\delta_{t-\tau} = y^G, \delta_{t-\tau+1} = \emptyset, \delta_{t-\tau+2} = \emptyset, \dots, \delta_t = \emptyset. \tag{14}$$

Given the stationarity of the problem, the data of (D, τ) contains all the relevant information for decision making and we can select (D, τ) as the state variable:

Remark 1. *Any face value schedule F in a consistent bank policy is Markov in (D, τ). Due to Lemmas 3 and 4, what remains to be determined in order to find the equilibrium is which $F(D, \tau)$ are compatible with a consistent bank policy, and which one maximizes the banker's equity value.*

The full characterization of the equilibrium is in section 3.2.5. The next sections explain how to get there.

3.1.2 Beliefs Dynamics

The banker offers a face value to rollover its debt, and investors play second by either accepting or rejecting the offer. Hence, what matters is the outsiders' beliefs about the asset. The probability to be in state y^G, under $\mathcal{F}_t^I$, sums up the outsiders' beliefs: denote it q. Initially we have $q = 1$, and immediately after any disclosure $q = 1$ as well, because disclosure only occurs when the regulator observes y^G. Now assume no disclosure at $t = 1$. Either the state was bad and observed (probability $p(1-\lambda^{GG})$) or the state was not observed (probability $1 - p$). So non-disclosure happens with probability $1 - p + p(1 - \lambda^{GG})$. And

[13] I simplify the strategic disclosure problem to the maximum in order to focus on the comparison between voluntary and mandatory disclosure regimes.

non-disclosure in the good state happens with probability $(1-p)\lambda^{GG}$. Hence, the probability to be in state y^G after one non-disclosure period is

$$q_1 = \frac{(1-p)\lambda^{GG}}{1-p+p(1-\lambda^{GG})}. \tag{15}$$

And the probability to be in state y^G at $t = 2$ is

$$\gamma_1 = q_1\lambda^{GG} + (1-q_1)\lambda^{BG}. \tag{16}$$

Recall that τ is the time elapsed since the last disclosure. Let

$$q_k(t) = \mathbb{P}(y_t = y^G | \tau = k, \zeta_\phi > t) \tag{17}$$

be the value of q after k periods of non-disclosure and

$$\gamma_k(t) = \mathbb{P}(y_{t+1} = y^G | \tau = k, \zeta_\phi > t) \tag{18}$$

be the probability to be in state y^G tomorrow after k periods of non-disclosure. These quantities only depend on t to the extent that t must be smaller than the maturity time. Hence, we can drop the dependency in t. Also for notational simplicity, the subscript k will be denoted τ. Using Bayesian updating, as in the case $k = 1$ detailed above, we obtain recursively:

$$q_{\tau+1} = \frac{(1-p)\gamma_\tau}{1-p+p(1-\gamma_\tau)}, \tag{19}$$

$$\gamma_\tau = q_\tau\lambda^{GG} + (1-q_\tau)\lambda^{BG}. \tag{20}$$

To each τ corresponds one q_τ; Figure 1 provides a graphical representation. Note that q_τ decreases to a limit weight q_V^*, which bears an economic interpretation, discussed in section 3.2.2.

3.1.3 Fundamental Value

Let $V(q)$ be the fundamental value of the asset when the probability to be in state y^G is q. Let $\mathbf{y} = (y^G \; y^B)$ be the vector of states, and $\mathbf{q} = (q \; 1-q)^{\mathbf{T}}$ be the vector of weights on the two states. By assumption the asset has not matured at time $t = 0$, and the probability of the maturity being $\zeta_\phi = t+1$ for $t \geq 0$ is $(1-\phi)^t\phi$. At time $t+1$, the weights on the 2 states are given by the vector $\Lambda^{t+1}\mathbf{q}$, so the expected asset value conditional on $t+1 = \zeta_\phi$ is

$\mathbf{y}\Lambda^{t+1}\mathbf{q}$. Therefore

$$\begin{aligned} V(q) &= \sum_{t\geq 0} \mathbb{E}[y_t|\zeta_\phi = t+1]\mathbb{P}(t+1=\zeta_\phi) \\ &= \sum_{t\geq 0}(1-\phi)^t\phi\mathbf{y}\Lambda^{t+1}\mathbf{q} \\ &= \phi\mathbf{y}\Lambda(\mathrm{Id}_2-(1-\phi)\Lambda)^{-1}\mathbf{q}. \end{aligned} \tag{21}$$

Note that V is affine in q:

$$V(q) = qV^G + (1-q)V^B. \tag{22}$$

V can also be expressed as a function of τ, the time since last disclosure:

$$V_\tau \equiv V(q_\tau). \tag{23}$$

3.1.4 Debt Capacity

Definition 2. *The debt capacity is the maximal amount of debt financing that can be obtained by pledging the assets under management as collateral. Under a consistent bank policy, it depends on the state τ and is defined by*

$$C(\tau) = \inf\{D \geq 0, F(D,\tau) = \emptyset\}. \tag{24}$$

By definition, if debt exceeds debt capacity during the lifespan of the asset, it is no longer possible to find investors to roll debt over. In my model, this forces a premature liquidation, because no other sources of financing are available: a run occurs. Hence, debt capacity coincides here with a *run threshold*.

Definition 3. *The fair pricing function in state τ, m_τ, is the mapping that associates to any promise F the expectation of the actual payment, under the creditors' information. In a consistent bank policy,*

$$m_\tau(F(D,\tau)) = D \tag{25}$$

holds in state (D,τ).

Of course, m_τ, an inverse of F, is also an equilibrium object and remains to be determined, jointly with the debt capacities. In general, we have the following relationship:

$$C(\tau) = \sup_{F\geq 0} m_\tau(F). \tag{26}$$

That is, today's debt capacity is the maximum amount of financing that a promise of F tomorrow can buy.

The first key observation towards the analytical characterization of debt capacities is the following:

Lemma 5. *Assume we are in state τ and let $\chi_1, \ldots, \chi_k$ be the possible states of the world tomorrow, and $C(\chi_i)$ the maximum available financing in state χ_i. Then today's debt capacity satisfies*

$$C(\tau) = \max\{m_\tau(C(\chi_1)), \ldots, m_\tau(C(\chi_k))\}. \tag{27}$$

This means that we do not need to consider *all* promises, as suggested by equation (26), but only the maximal viable promises in tomorrow's states of the world. The intuition is the following. When the banker increases the face value from F to $F + dF$, two cases are possible. If the states χ_i in which there is default are unchanged, then the expected repayment under $F + dF$ must be larger: $m_\tau(F + dF) > m_\tau(F)$. By contrast, if the increase in face value creates an additional default state, the expected repayment *decreases* because of the deadweight liquidation cost. Hence, as F increases, $m_\tau(F)$ increases, except when a new default state is created, in which case it jumps downwards. When is χ_i a default state? It is precisely when the face value is larger than $C(\chi_i)$. Hence Lemma 5.

I now proceed and describe tomorrow's states of the world in my model (from the point of view of outsiders). There are always four:

- χ_1: the asset has just matured ($\zeta_\phi = t + 1$), in the good state y^G.
- χ_2: the asset has just matured ($\zeta_\phi = t + 1$), in the bad state y^B.
- χ_3: the asset has not matured ($\zeta_\phi > t + 1$), and a disclosure was made ($\tau = 0$).
- χ_4: the asset has not matured ($\zeta_\phi > t + 1$), no disclosure was made ($\tau \to \tau + 1$).

From Lemma 5, we obtain

$$C(\tau) = \max\{m_\tau(C(0)), m_\tau(C(\tau + 1)), m_\tau(y^G), m_\tau(y^B)\}. \tag{28}$$

The second key observation in determining the debt capacities is that there is only "one kind of good news": the observation of y^G. In order to sustain today's debt capacity, one must promise a face value that will be paid in better states of the world, because in worst states, less financing is available than today. Hence, we can directly map the $C(\tau)$ to $C(0)$, and $C(0)$ to y^G: through a simple choice of asset process, we have been able to obtain an analytically tractable functional equation for debt capacity. The following derivations make these intuitions formal.

From condition (M), $C(\tau+1) \leq C(\tau)$, and since $m_\tau(F) \leq F$ always holds, equation (28) reduces to:

$$C(\tau) = \max\{m_\tau(C(0)), m_\tau(y^G), m_\tau(y^B)\}, \tag{29}$$

for $\tau \geq 1$ and

$$C(0) = \max\{m_0(y^G), m_0(y^B)\}. \tag{30}$$

y^B is the worst state of the world, so in equilibrium the banker can make a risk-free promise:

$$m_\tau(y^B) = y^B. \tag{31}$$

Let us now deal with the pricing of bonds with face value y^G and $C(0)$, respectively.

- In case the asset matures tomorrow, there will be full payment in the good state (state χ_1) and payment of y^B in state χ_2. Otherwise, there will be liquidation, since $C(\tau) < y^G$. The liquidation value will be either V_0 (in state χ_3) or $V_{\tau+1}$ (in state χ_4). So

$$m_\tau(y^G) = \phi(\gamma_\tau y^G + (1-\gamma_\tau)y^B) + \alpha(1-\phi)(p\gamma_\tau V_0 + (1-p\gamma_\tau)V_{\tau+1}). \tag{32}$$

- Payments in states χ_1 to χ_4 are respectively $C(0)$, y^B, $C(0)$, $\alpha V_{\tau+1}$. So

$$m_\tau(C(0)) = \phi(\gamma_\tau C(0) + (1-\gamma_\tau)y^B) + (1-\phi)(p\gamma_\tau C(0) + \alpha(1-p\gamma_\tau)V_{\tau+1}). \tag{33}$$

Notice that m_τ was *a priori* unknown. But equation (32) provides a necessary expression for $m_\tau(y^G)$, which determines $C(0)$ thanks to (30) and (31). In turn, $m_\tau(C(0))$ is determined by equation (33). (29) concludes the characterization of the debt capacities in all states. We have thus proven the following.

Proposition 1. *The equilibrium debt capacities in the voluntary disclosure case are characterized analytically by equations* (29) *to* (33)*.*

Figure 2 provides a graphical representation.

3.1.5 Endogenous Bond Yields

Proposition 1 says that we know the debt capacities in all states, and so we also know the default states, which means that the equilibrium fair pricing functions m_τ are determined. We are then in a position to obtain the following.

Proposition 2. *The equilibrium face value schedule is characterized by*

$$F(D, \tau) = \min\{F \geq 0, m_\tau(F) = D\}. \tag{34}$$

(Details and the explicit expression for m_τ can be found in the Appendix). Notice that the gross bond yield in state τ is $R(D, \tau) = F(D, \tau)/D$.

3.2 Mandatory Disclosure

The model solution under mandatory disclosure is both similar and simpler: there is no disclosure policy and information is symmetric. I quickly repeat the analysis above in order to obtain the debt capacities and bond yields under mandatory disclosure.

3.2.1 Beliefs Dynamics

Let again denote q the probability of the asset being in state y^G (now the same for the bank and outsiders) and τ be the time since the last disclosure of y^G. As in the voluntary disclosure case, there is a correspondence between τ and q. The updating rule is modified. Here, $q(\tau = 0) = 1$ and

$$q_{\tau+1} = q_\tau \lambda^{GG} + (1 - q_\tau)\lambda^{BG}. \tag{35}$$

Figure 1 provides a graphical representation.

Since asset observability is now independent from asset value, the weights on states after τ periods without observation are simply given by the iterated transition matrix, $\Lambda^{\tau+1}$. The qualitative behaviour of q_τ is the same as in the voluntary disclosure case. Here, it decreases to the stationary weight

$$q^*_M = \frac{\lambda^{BG}}{1 + \lambda^{BG} - \lambda^{GG}}, \tag{36}$$

which is above the limit q^*_V of q_τ in the voluntary disclosure case. The intuition is that with mandatory disclosure, no information does not mean a higher chance of bad news being concealed. Under voluntary disclosure, a protracted lack of disclosure seriously hints at the state being y^B.

I again define γ_τ as the probability to be in state y^G tomorrow given τ periods of non-disclosure. Here, we simply have

$$\gamma_\tau = q_{\tau+1}. \tag{37}$$

3.2.2 The Stationary Weights

There is an economic intuition behind q^*_M, which represents (up to an affine transformation) the asymptotic expected value of collateral, as the economy becomes information-less. If even in the information-less economy, agents accept to roll over debt because the expected value of collateral is high enough —q^*_M large enough—, it is pointless to gather information. It would even be inefficient, since the (rare) bad banks would be inefficiently closed. This is one message of Gorton and Ordoñez (2014). But even if q^*_M is large enough, the expected value of the bond collateral in the information-less economy can be insufficient to ensure information insensitivity when disclosure is strategic. Indeed, the absence of information under voluntary disclosure is worse news than under mandatory disclosure. Formally, we have the following:

Lemma 6. *For any opacity parameter $p \in (0,1)$, the stationary weights (the probability to be in the good state when the time since the last disclosure becomes large) satisfy $q^*_V < q^*_M$. As a consequence, the expected value of the bond collateral in the information-less economy is smaller in the voluntary disclosure case.*

3.2.3 Fundamental Value

The formula for $V(q)$ established above is still valid. We now have that the fundamental value after τ periods without disclosure is $V_\tau \equiv V(q = q_\tau)$, where the probability q_τ is computed assuming mandatory disclosure.

3.2.4 Debt Capacity and Endogenous Bond Yields

Using the same method as before, we obtain the parallel of Propositions 1 and 2:

Proposition 3. *The equilibrium debt capacities in the mandatory disclosure case are characterized analytically by equations (29) to (31), where the operator m_τ is modified (see the Appendix for its explicit expression). Again, the equilibrium face value schedule satisfies*

$$F(D, \tau) = \min\{F \geq 0, m_\tau(F) = D\}. \tag{38}$$

3.2.5 Equilibrium characterization

Collecting the results obtained so far, we can exhibit the equilibrium:

In the voluntary disclosure case, the equilibrium is $(\delta = \delta^S, F, \zeta_s = \infty)$ where F is given *in Proposition 2. In the mandatory disclosure case, the equilibrium is* $(F, \zeta_s = \infty)$ *where* F *is given in Proposition 3.*

4 Results

When necessary, I denote by Θ the set of parameters, and Θ_{-x} this set without the variable x. The notation is useful when we want to study comparative statics with respect to x.

Table 1. Baseline model parameters.

Variable	Description	Value
y^G	Good state	100
y^B	Bad state	0
p	Opacity parameter	0.5
λ^{GG}	Prob. of staying in the good state	97%
λ^{BG}	Prob. of recovery	3%
ϕ	Intensity of maturity	15%
α	Asset liquidity parameter	85%

4.1 Opacity, Information Sensitivity and Rollover Risk

4.1.1 Notions of Information Sensitivity

The notion of *information sensitivity* is at the heart of a series of papers: Gorton and Pennacchi (1990), Dang, Gorton, and Holmström (2013), Dang, Gorton, and Holmström (2015), and Gorton and Ordoñez (2014). A security is information-insensitive when agents accept to trade it without paying to obtain a costly signal about it, and has a high information sensitivity when agents are ready to spend a lot to obtain a signal. Debt is a natural candidate to information insensitivity because its payoff is constant over all the range of non-default states.

Adverse selection. In the papers of Dang et al., this property is desirable mainly because it allows to sidestep adverse selection issues. Debt is liquid because agents are not concerned that the next buyer knows more about the collateral than they do. In this context, opacity is efficient since it makes debt information-insensitive in more states of the world.

Pooling. In Gorton and Ordoñez (2014), opacity permits the pooling of firms with good collateral with firms with bad collateral. If the average quality of collateral is high enough, firms obtain credit from lenders who do not verify firm-specific collateral quality. This financing is invested in positive NPV projects, and opacity is therefore desirable: it provides insurance to banks in terms of their access to financing. To the contrary, when information about a firm's collateral is cheap, debt becomes information-sensitive: lenders verify collateral quality and lend only conditional on good news. Firms with bad collateral are deprived of credit and welfare is lower.

One can also define the notion of information sensitivity in my model:

Definition 4 Let (D, τ) be the state today, and $F(D, \tau)$ the promised face value due tomorrow. I say that debt is *information-insensitive* if the full repayment of $F(D, \tau)$ does not imply disclosure tomorrow. To the contrary, debt is *information-sensitive* if the absence of disclosure tomorrow entails a run.

Endowed with this definition, it will be easier to understand how the information structure – the degree of transparency and the disclosure policy – impact rollover risk and the price of debt.

4.1.2 Rollover Risk, Funding Costs and the Information Structure

In this section, we back up formally the following claims.

- Transparency increases funding costs in good times; the reverse holds in bad times. As long as debt remains information-insensitive, there are less default states under opacity. This can backfire as conditions deteriorate: when debt becomes information-sensitive, the release of good news is required to avoid a bank failure, but this release is unlikely under opacity.

- Voluntary disclosure implies lower funding costs than mandatory disclosure as long as debt remains information-insensitive. However, we will see (Lemma 7) that voluntary disclosure also systematically induces more pessimistic beliefs that mandatory disclosure.

Figure 3 plots the gross yields $R(D, \tau) = F(D, \tau)/D$ after $\tau = 1$ period of non-disclosure in the voluntary disclosure case. The plot is qualitatively similar for other values of τ. $R(., \tau)$ exhibits upwards jumps, which correspond the the creation of an additional default state, as explained in section 3.1.4. The jump points define regions, labelled II, IS, P and L in the Figure, with the following economic interpretation.

In the information-insensitive region (II), debt is safe: the face value satisfies $F(D, \tau) \leq C(\tau + 1)$: it is below tomorrow's debt capacity if there is no disclosure. Hence, unless the asset matures tomorrow in the bad state, debt will necessarily be rolled over. In the II region, debt is money-like.

The information-sensitive (IS) region corresponds to face values $F(D,\tau)$ between $C(\tau+1)$ and $C(0)$: those are higher than tomorrow's debt capacity if there is no disclosure. Hence, a run will occur tomorrow in the case no disclosure is made. Since $F(D,\tau) \leq C(0)$, however, a run will not occur if the bank discloses y^G tomorrow. Avoiding liquidation is contingent on the disclosure of good news.

The region P (for "pre-liquidation") corresponds to face values $F(D,\tau)$ above $C(0)$: liquidation will happen tomorrow unless the project matures in state y^G. This means that the bank can survive for one more period but not more. In order to incentivize lenders to stay in the game, the bank has to offer very high yields.

Finally, the liquidation region L corresponds to levels of debt where a run occurs today, for lack of an admissible face value to roll debt over: $D > C(\tau)$.

As Figure 4 shows, the behaviour of bond yields in the mandatory disclosure case is qualitatively similar. Note that now, the bank can survive long periods of non-disclosure (here $\tau = 7$) because investors know that the regulator is genuinely uninformed. When the probability to fall into the bad state is low, the asset still has a good chance to be in state y^G after several non-disclosure periods.

I now present a series of analytical results implied by the expression of yields found in Section 3. In turn, a first set of economic conclusions are derived from these results.

Proposition 4. *Let the superscript $[p]$ designate a variable relative to the model solution for opacity parameter p. The following holds:*

(a) (safer information-insensitive debt) *If regions II and IS both exist, short-term debt is less risky in the II region.*

(b) (opacity and information sensitivity) *For high opacity, i.e. small values of p, debt cannot be information-sensitive. In the voluntary disclosure case, when $p \to 1$, the information-insensitive region shrinks: for any τ, (D,τ) can not be in the II zone for p close enough to 1.*

(c) (bond yield discontinuity) *Bond yields are discontinuous in the value of debt for a given τ. As debt reaches the information-sensitivity threshold, yields jump upward.*

(d) (opacity component of short-term spreads) For a given (D,q), short-term spreads can vary with the opacity level:

(d1) *If (D,q) is at the right of the information-sensitive region for opacity parameters p_1*

and p_2 with $p_1 < p_2$ then $F^{[p_1]}(D,q) > F^{[p_2]}(D,q)$.

(d2) *If (D,q) is in the information-insensitive region for opacity parameters p_1 and p_2 with $p_1 < p_2$ then $F^{[p_1]}(D,q) \leq F^{[p_2]}(D,q)$, with equality if and only if disclosure is voluntary.*

(e) (disclosure component of short-term spreads) *For a given (D,q), voluntary disclosure provides lower yields in the V-II region: $F^V(D,q) < F^M(D,q)$.*

Points **(a)** and **(b)** confirm that information-insensitive debt is safer, and that opacity can increase the size of the information-insensitive region. This does not mean, however, that opacity is always desirable. Indeed, when $p = \mathbb{P}(\omega_t = 1)$ is small, disclosures are rare, and τ is large on average, meaning that q_τ often takes low values, and the debt level may escape the II zone. By contrast, under transparency (p close to 1), the II region is tiny and a single period of non-disclosure can trigger default; but non-disclosure is very rare.

Point **(d)** contains the prediction that there is an opacity component in short-term spreads. Spreads are primarily linked to future rollover decisions, not to the asset fundamental value. But rollover decisions occur at each node of the asset tree, whose structure depends on disclosures. Therefore opacity matters: the model predicts that in good times (D low, or q high) transparency increase spreads, while in crisis (at the right of the IS zone) transparency decrease spreads.

Point **(e)** contains the prediction that there is a disclosure component in short-term spreads and states that for a given belief about the current state of the world, voluntary disclosure allows the bank to borrow at better terms as long as debt is information-insensitive under this disclosure regime. This does not mean, however, that voluntary disclosure is always desirable. Indeed, voluntary disclosure systematically produces more pessimistic beliefs than mandatory disclosure: see Lemma 7 in the next section, where we formalize the comparison between the two disclosure regimes.

As Figures 3 and 4 show, the IS zone is in general tiny, and can also not exist. In that case, the debt directly switches from being information-insensitive to being defaulted upon, making the trade off between short-term protection and long-term exposure even clearer. This occurs typically when the maturity of short-term debt is small compared to the expected time before the next observation of the asset. In this situation, disclosure is unlikely. An information-sensitive debt would therefore be defaulted upon with such a high probability that no information-sensitive contract is feasible.

So far, the analysis was *local*, since I focused on the behaviour of short-term yields. The model suggests that while opacity indeed makes debt safer and money-like in the short-run, it may induce a high exposure to runs in the longer run, when q becomes too low, or D too high. The next sections attempt to quantify *globally* this trade-off, *i.e.* to study the impact of opacity and disclosures on the run probability $\mathcal{P}$ and our inefficiency measure $\mathcal{I}$.

4.2 Impact of the Disclosure Regime on Equilibrium Outcomes

4.2.1 Methodology

In order to understand how the nature of disclosure affects the dynamics of debt and our quantities of interest — run probability and our measure of inefficiency — we need to compare them all other things being equal. This is achieved in the following way. Consider some

$$\psi \equiv \left(\zeta_\phi, (y_t)_{t\leq\zeta_\phi}, (\omega_t)_{t\leq\zeta_\phi}\right). \tag{39}$$

ψ is the data of a maturity date ζ_ϕ, all the positions of the asset in the Markov chain before ζ_ϕ, and all the observability shocks before ζ_ϕ. We can now compute, for the same ψ, the equilibrium paths of debt $(D_t^V(\psi))$, $(D_t^M(\psi))$ and liquidation times (if any) $\zeta_\ell^V(\psi)$, $\zeta_\ell^M(\psi)$ in the cases of voluntary and mandatory disclosure, respectively.

The fundamental value of the asset is identical at all times across both scenarios. The same holds true for the information collected by the regulator. Moreover, if along ψ, $y_t = y^G$ for all t, the signals received by the creditors are also the same at all times across both scenarios (at any t, they received either $\delta_t = y^G$, announcement of the good state, or $\delta_t = \emptyset$, announcement that the asset has not been observed by the regulator). Even in that case, the debt and beliefs dynamics will be different across the two disclosure regimes considered. This can lead to dramatically different outcomes, as shown in the next section.

Hence, we are able to isolate effects due the disclosure policy by fixing a history ψ and computing the debt and beliefs dynamics along ψ in both disclosure regimes. Having defined formally the comparison between regimes, the following result, announced in section 4.1.2, is now clear:

Lemma 7. *Along any ψ, $q_t^V \leq q_t^M$ for any $t \leq \min\{\tau_f^V, \tau_f^M\}$ with equality only when $\omega_t = 1$ and $y_t = y^G$.*

The lemma simply states that voluntary disclosure consistently produces depressed beliefs, because investors anticipate the possibility that the bank may conceal bad news. The only case where the beliefs are the same under both disclosure regimes is when the bank has just announced the good state.

We are also in a position to define the following:

Definition 4. *A voluntary disclosure-induced run (or credibility run) is a fundamental path ψ that produces a run when disclosure is voluntary but no run under mandatory disclosure. A mandatory disclosure-induced run is defined similarly.*

The alternative name "credibility run" for a voluntary disclosure-induced run comes from the fact that under voluntary disclosure, the regulator lacks credibility when she announces no observation of the asset, even when this is actually the case. Because she has not taken any commitment, the absence of news release is interpreted as very bad news by the creditors. In situations where no observations are made for a protracted period of time ($\omega_t = 0$ for several consecutive t), creditors rationally downgrade a lot their beliefs about the asset quality. This potentially leads to a run that would have been avoided under mandatory disclosure. Indeed, under mandatory disclosure, creditors are safe in the knowledge that the regulator is genuinely uninformed, and not trying to conceal bad news.

4.2.2 Voluntary disclosure-induced and Mandatory disclosure-induced Runs

Figures 5 and 6 plot two sample paths of debt for both disclosure regimes, and the associated beliefs dynamics: $q_t = \mathbb{P}(y_t = y^G | \mathcal{F}_t^I)$ is the probability to be in the good state under the creditors' information set. A black dot at time t indicates that the regulator has observed the asset at time t: $\omega_t = 1$. Along both sample paths, the asset actually always was in the good state: $y_t = y^G$ for all t. As mentioned in the previous section, this means that the fundamentals, the regulator's information, and the signals received by the creditors are identical in each example across the two disclosure regimes. All differences in outcomes are explained by the difference in the commitment decision of the regulator, which leads to different information structures and therefore different beliefs and debt dynamics.

Figure 5 depicts a credibility run. In the beginning, interest rates are lower under voluntary disclosure. This is because the bad state, should it occur, will not be revealed under voluntary disclosure, but will be revealed under mandatory disclosure. Hence, voluntary disclosure produces less default states and reduces the bank's cost of financing, leading to a slower growth of the stock of debt. But a run suddenly occurs: this is because news have not been released for a protracted period of time, leading to a sharp decline in the creditors' beliefs, as illustrated by the bottom panel: observe the plunge of q_t between periods $t = 10$ and $t = 14$. In turn, this strong decline in beliefs leads to a strong decline in debt capacities.

By contrast, under mandatory disclosure, the bank is resilient to long non-disclosure periods because creditors know that the regulator would be forced to reveal the bad state, had it been observed.

Prior to $\tau_\ell^V - 1$, the yields $\frac{D_{t+1}}{D_t}$ are lower under voluntary disclosure, but *it is under this* *disclosure regime that the bank undergoes a run.* This means that one cannot unconditionally map the current value of the short-term yield to the health of a financial institution: yields are to a large extent determined by the opacity of the collateral and the disclosure policy; and because they only reflect next period's rollover risk, they can remain low even when the probability of a run at a small horizon is very large.

Figure 6 shows a mandatory disclosure-induced run.

At $\tau_\ell^M = 52$, the bank undergoes a run under mandatory disclosure. Prior to that date, good news were regularly released, producing consistently large values of q_t and maintaining the information-insensitive status of debt under voluntary disclosure. Similar to point **(e)** in Proposition 4, the bank was therefore able to borrow at better terms under voluntary disclosure.

Across both disclosure regimes, the fundamentals and the signals are identical at all times, and the probability to be in the good state is in fact *always weakly larger under mandatory disclosure*, but it is nevertheless under this disclosure regime that the bank undergoes a run. *The critical channel here is the endogenous refinancing cost:* the funding cost channel. Mandatory disclosure produces an information structure that generates more default states in good times (even though it produces better *average* beliefs). This implies larger financing costs, and the stock of debt grows faster. This can lead to a run that only occurs under mandatory disclosure.[14]

As is apparent from these examples, two opposite forces are at play and it is not clear a priori which one dominates, *i.e.* whether mandatory disclosure dominates voluntary disclosure in terms of efficiency. This question is investigated in section 4.3.2.

4.3 Global Results

4.3.1 Impact of opacity on run probability and efficiency

In this section only, we abstract from the disclosure regime and ask whether opacity is efficient. We compare the polar cases $p = 0$ and $p = 1$, where disclosure regimes are equivalent. To obtain more compact expressions, assume symmetrical transition probabilities: $\lambda \equiv \lambda^{BG} = 1 - \lambda^{GG}$.

When $p = 0$, the only random variable actually observed is maturity. Hence, before maturity, the paths of debt and beliefs about the current state are deterministic. Given an

[14] Note that debt became information-sensitive at $t = 79$ under voluntary disclosure, consistent with a sharp decrease in q_t (see bottom panel of Figure 6). This corresponds to an upward "jump" in the stock of debt. A run was nevertheless avoided, because the required good news were indeed announced: $\omega_{80} = 1$.

initial debt D_0, there is a deterministic $t_0(D_0)$ such that liquidation always occurs at t_0 if maturity is not reached yet. t_0 is obtained by computing the path of debt using the formulas for bond yields derived above. t_0, as a function of D_0, is a non-increasing, piecewise constant function.

t_0 can be interpreted as a "time-to-crisis". Until time $t_0 - 1$, Effect 1 is at play and the bond is money-like. The key point here is that the quasi-absence of risk in the beginning is only due to the possibility to liquidate the asset in the future. The bond is not risky because it will always be possible to run when the liquidation value approaches the debt level. Short-term spreads are by no means informative about the longer-term risk of the project and are low precisely because of the option to run.

Denote $\mathbf{e}_1$ the column vector $(1\ 0)^{\mathbf{T}}$, $q_0 = \mathbf{e}_1^{\mathbf{T}} \Lambda^{t_0} \mathbf{e}_1$ the probability to be in state y^G at time t_0. Recall that $V(q)$ stands for the fundamental value when the probability to be in state y^G is q.

Proposition 5. *When $p = 0$, run probability and expected output are respectively given by*

$$\begin{aligned} \mathcal{P}(p=0) &= (1-\phi)^{t_0}, && (40) \\ \mathbb{E}[U](p=0) &= \boldsymbol{y}\phi\Lambda(Id_2 - (1-\phi)^{t_0}\Lambda^{t_0})(Id_2 - (1-\phi)\Lambda)^{-1}\boldsymbol{e}_1 \\ &+ (1-\phi)^{t_0} \underbrace{\alpha V(q_0)}_{\textit{Liq. value of average bank.}} . && (41) \end{aligned}$$

Conditional on $y_t = y^G$ for all t, there is a deterministic time $t_1(D)$ such that liquidation occurs at t_1 as soon as $\zeta_\phi > t_1$. This is because debt grows while the states remain the same.

Proposition 6. *When $p = 1$, run probability and expected output are respectively given by*

$$\begin{aligned} \mathcal{P}(p=1) &= 1 - \phi\frac{1-(1-\lambda)^{t_1}(1-\phi)^{t_1}}{1-(1-\lambda)(1-\phi)}, && (42) \\ \mathbb{E}[U](p=1) &= \alpha(1-\phi)^{t_1}((1-\lambda)^{t_1}V^G + (1-(1-\lambda)^{t_1})V^B) + \alpha(1-(1-\phi)^{t_1})V^B \\ &+ \phi\left((1-\lambda)y^G - \underbrace{\alpha V^B}_{\textit{Liq. value of bad bank.}} + y^B\lambda\right)\frac{1-(1-\phi)^{t_1}(1-\lambda)^{t_1}}{1-(1-\phi)(1-\lambda)}. && (43) \end{aligned}$$

Endowed with these analytical expressions, we can now efficiently explore the parameter space and compare efficiency and run likelihood. We obtain the following result.[15]

[15]The result is only numerical in the sense that we can only compute the quantities of interest on a discretization of the parameter space. But for each parameter set, the computation is exact, due to Propositions 5 and 6.

Numerical Result 1. *For any parameter set* Θ*, there are* $\lambda^*(\Theta_{-\lambda})$*,* $\phi^*(\Theta_{-\phi})$*,* $\alpha^*(\Theta_{-\alpha})$ *and* $D_0^*(\Theta_{-D_0})$ *such that* $\mathcal{I}(p=0) < \mathcal{I}(p=1)$ *if and only if*

. $\lambda < \lambda^*$

. $\phi > \phi^*$

. $\alpha > \alpha^*$

. $D_0 < D_0^*$.

The same result holds for the comparison of the run probabilities $\mathcal{P}(p=0)$ *and* $\mathcal{P}(p=1)$.

The intuition is the following. A small λ, a large ϕ, a large α or a small D_0 all correspond to situation with good fundamentals: the probability that the asset matures in the good state is large, liquidation costs are low, or the initial stock of debt is a lot below debt capacity. In those cases, debt is more likely to be information-insensitive, so the drawbacks associated with opacity matter less.

Figure 7 provides a graphical illustration.

At this stage, it is important to note that our measure of inefficiency is not aligned with the run probability. In other words, maximizing efficiency does not imply minimizing the likelihood of runs: one can have $\mathcal{I}(p=0) > \mathcal{I}(p=1)$ even if $\mathcal{P}(p=0) < \mathcal{P}(p=1)$. This is well illustrated by the following analytical result obtained in a special case:

Proposition 7. *Consider the continuous-time limit of a short-term debt with vanishing maturity and assume* $V^B = 0$.

When $\phi > 1 - \lambda^{GG}$*, there exists* α^* *such that for* $\alpha_{min} := \frac{D_0}{V^G} < \alpha < \alpha^*$*,* $\mathcal{P}(\alpha; p=1) < \mathcal{P}(\alpha; p=0)$ *and for* $\alpha > \alpha^*$*,* $\mathcal{P}(\alpha; p=1) > \mathcal{P}(\alpha; p=0)$.

When $\phi < 1 - \lambda^{GG}$*, we always have* $\mathcal{P}(\alpha; p=1) > \mathcal{P}(\alpha; p=0)$.

However, $\mathcal{I}(p=1; \alpha) < \mathcal{I}(p=0; \alpha)$ *for any interior* α.

According to Proposition 7, there are parameters such that a bank undergoes more runs on average under transparency, but where the expected costs of premature liquidation are nevertheless larger under opacity. The intuition is that under opacity, a bank can be hit by a runs and nevertheless be healthy. With our assumption that runs on good banks are more costly, it follows that the average cost of a run under opacity is larger.

It is interesting to link this result to the discussion of section 4.1.1, where we discussed the argument of Gorton and Ordoñez (2014) that opacity provides insurance to banks in their access to funding and may therefore be desirable. Proposition 7 says that opacity may improve access to financing in the sense that it lowers the probability that creditors refuse to refinance the debt, but be less efficient. Intuitively, the pooling of good and bad banks "backfires", as some healthy institutions, for which credit is the most valuable, can be denied credit.

4.3.2 Comparison between mandatory and voluntary disclosure

We now return to the general case of interior levels of opacity, $p \in (0, 1)$. In that case, the disclosure policy matters. When $p \notin \{0; 1\}$, no closed-form expression of the quantities of interest are available and I use Monte-Carlo simulations of the model. The main result of this section is the following.

Numerical Result 2. *Mandatory disclosure is more efficient than voluntary disclosure for large values of p; the reverse holds for small values of p ("small" and "large" being relative to the set of other parameters Θ_{-p}).*

This result can be made formal in the limit $p \to 1$:

Proposition 8. *For any parameter set Θ, there exists $p^*(\Theta_{-p}) < 1$ such that for any $p \in (p^*, 1)$ mandatory disclosure dominates voluntary disclosure in terms of efficiency and run probability: $\mathcal{I}^M < \mathcal{I}^V$ and $\mathcal{P}^M < \mathcal{P}^V$.*

The intuition is the following. At low levels of opacity, non-disclosure ($\delta_t = \emptyset$) is a very negative signal on the asset quality, because it is probable that the regulator is concealing bad news. Since the regulator does not have a way to credibly communicate that he is genuinely uninformed when this is the case, runs become likely then. Credibility runs, as described in section 4.2.2, are a significant possibility; since they can hit good banks, they are also particularly inefficient. By contrast, at high levels of opacity, non-disclosure only marginally downgrades the belief of creditors, and it allows the bank to borrow at better terms. In turn, debt grows at a lower rate and it is less likely to reach the bank's debt capacity, reducing the probability of a premature liquidation.

We now look at two particular examples in order to make the economic discussion more precise. For $p = 1$ (last column of Table 2) the two disclosure regimes are equivalent, and the results can also be obtained with the closed-form formulas obtained in Section 4.3.1.

Table 2. **Opacity, Runs and Efficiency.** $D_0 = 40$.

p	0.2	0.5	0.95	1
$\mathbb{E}$[Residual Claim] - Voluntary	43.8	43.8	44.2	44.7
$\mathbb{E}$[Residual Claim] - Mandatory	44.3	44.6	44.7	44.7
$\mathbb{P}$(Run under V and M)	0.096	0.103	0.130	0.151
$\mathbb{P}$(Run only under V)	0.040	0.059	0.034	0
$\mathbb{P}$(Run only under M)	0.033	0.037	0.020	0
$\mathbb{P}$(Run under V)	0.136	0.162	0.164	0.151
$\mathbb{P}$(Run under M)	0.129	0.141	0.150	0.151

Table 3. **Liquidity, Runs and Efficiency.** $D_0 = 40$

α	0.7	0.8	0.9	0.95
$\mathbb{E}$[Residual Claim] - Voluntary	41.3	43.1	44.4	44.8
$\mathbb{E}$[Residual Claim] - Mandatory	43.2	44.3	44.8	45.0
$\mathbb{P}$(Run under V and M)	0.142	0.118	0.096	0.091
$\mathbb{P}$(Run only under V)	0.074	0.068	0.050	0.043
$\mathbb{P}$(Run only under M)	0.029	0.030	0.040	0.042
$\mathbb{P}$(Run under V)	0.216	0.186	0.146	0.135
$\mathbb{P}$(Run under M)	0.172	0.147	0.137	0.133

Rows 1 and 2 give the expected residual claim under both voluntary and mandatory disclosure. Given that debt payment is equal to D_0 in expectation, this quantity is equal to $\mathbb{E}[U] - D_0$.

Row 3 gives the probabilities that a run occurs under both disclosure regimes.

Row 4 gives the probabilities that a run occurs only under mandatory disclosure. This can happen in a situation where the bank can successfully weather a crisis under voluntary disclosure: $\omega_t = 1, y_t = y^B, \delta_t = \emptyset$ for some t and $\omega_{t'} = 1, y_{t'} = y^G, \delta_{t'} = y^G$ for some $\zeta_\phi \geq t' > t$. By contrast, a run occurs at t under mandatory disclosure. This can also happen if $\omega_t = 1$ sufficiently often so that debt grows at a slower rate under voluntary disclosure, as illustrated in section 4.2.2.

Row 5 gives the probabilities of credibility runs, *i.e.* runs that occur only under voluntary disclosure.

Rows 6 and 7 are the sum of rows 3 and 4, and 3 and 5 respectively, and show the total run probability under each regime.

Impact of varying the degree of opacity. Table 2 shows that the efficiency under full transparency (column $p = 1$) is higher, although this regime may feature more runs. The intuition is as in Effect 3 and Section 4.3.1: under full transparency, the bank is exposed to the revelation of a bad shock. However, in the long term, the likely absence of information will prevent the bank from keeping rolling debt over, while a transparent bank with the asset in the good state could do it.

Another conclusion that can be drawn from Table 2 is that at high levels of opacity (columns $p \in \{0.2, 0.5\}$), the disclosure regime matters significantly to the realized outcome of the model: a significant fraction of runs are regime-specific, *i.e.* do not happen under the same scenario for the other disclosure regimes. This sharp contrast at the path-wise level gets dampened to some extent at the aggregate level: the differences in the expected efficiency of both regimes are sizeable but remain quantitatively moderate. The whole of Table 3 also supports these conclusions (recall that the baseline value of p is 0.5).

Impact of the liquidity parameter. A larger α increases debt capacities and decreases run likelihood. Additionally, it reduces the deadweight loss upon liquidation for any given fundamental value of the asset at the liquidation time. Hence, as illustrated by Table 3, low liquidation costs imply both less runs and less inefficiency.

It is a somewhat woeful consequence of the intertemporal coordination problem that *runs are the most likely precisely when they are the most harmful.*

Conclusion

Opacity and disclosure regimes matter to the outcome of the rollover game because they shape the information tree, and therefore the short-term yields and the beliefs dynamics. Starting from the good state, opacity provides protection in the short run, but is likely to increase exposure at longer horizons — a tension which is amplified under voluntary disclosure. At the aggregate level, the model predicts that opacity reduces run probability and inefficiency only in situations where the fundamentals are strong anyways; that opacity may decrease run probability but increase inefficiency; and indicates that mandatory disclosure is more efficient than voluntary disclosure except at large levels of opacity.

Several extensions of the model appear interesting. First, relaxing the rigid structure of the bank's balance sheet should provide valuable additional insights; for instance, the bank may also have long-term debt, or use cash reserves to manage its risk of run. Second, one could introduce state-contingent regulation rather than fixing ex ante the disclosure regime.

Third, the information structure could be refined by considering a richer set of signals about the current asset value, in order to hone the modelling of the regulator's strategy set. One could then reformulate the model as an explicit Bayesian persuasion problem and compare it to the existing Bayesian persuasion literature on stress tests. Finally, the bank could have access to several investment opportunities, and may have moral hazard incentives to engage into inefficient projects. Clearly, the bank's portfolio decision and the regulator's opacity and disclosure choices would affect each other, giving rise to a potentially rich interaction.

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A Proofs

A.1 Lemma 1

By definition of $(\tilde{D}_t)$, $\mathbb{E}\left[\tilde{D}_{(t+1)\wedge\zeta_f}|\mathcal{F}_t^I\right] = \mathbb{E}\left[\tilde{D}_{t+1}|\mathcal{F}_t^I\right] = \tilde{D}_t = \tilde{D}_{t\wedge\zeta_f}$ over $\{t < \zeta_f\}$. And $\tilde{D}_{(t+1)\wedge\zeta_f} = \tilde{D}_{t\wedge\zeta_f} = D_{\zeta_f}$ over $\{t \geq \zeta_f\}$. Therefore $\mathbb{E}[\tilde{D}_{(t+1)\wedge\zeta_f}|\mathcal{F}_t^I] = \tilde{D}_{t\wedge\zeta_f}$.

A.2 Lemma 2

From Lemma 1, $(\tilde{D}_{t\wedge\zeta_f})$ is a martingale, and it is bounded due to condition (NP). Therefore it is a closed martingale. Now consider a bank at t which is not forced into liquidation. This means that $\zeta_f > t$, and since $(\tilde{D}_{t\wedge\zeta_f})$ is a closed $(\mathcal{F}_t^I)$-martingale, $\mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^I] = D_t$. Now, note that $\zeta_f = \zeta_\ell \wedge \zeta_\phi$ and that

$$\tilde{D}_{\zeta_\ell}\mathbb{I}_{\{\zeta_\ell<\zeta_\phi\}} + \tilde{D}_{\zeta_\phi}\mathbb{I}_{\{\zeta_\ell\geq\zeta_\phi\}} \leq V_{\zeta_\ell}\mathbb{I}_{\{\zeta_\ell<\zeta_\phi\}} + y_{\zeta_\phi}\mathbb{I}_{\{\zeta_\ell\geq\zeta_\phi\}}. \tag{44}$$

Taking expectations and noting that the expectation of the right-hand side is the same average of maturity values y_{ζ_ϕ} as $V_t \equiv \mathbb{E}[y_{\zeta_\phi}|\mathcal{F}_t^I]$, we obtain

$$D_t = \mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^I] \leq V_t. \tag{45}$$

The bank is solvent.

A.3 Lemma 3

Assume voluntary disclosure (the proof in the mandatory disclosure case is included in this one). Consider a time $t < \zeta_f$. If $\delta_t = \emptyset$ and the banker decides to liquidate, she obtains the value $\alpha V^B < D_0 < D_t$ and her equity is worth zero. Now, if $\delta_t \neq \emptyset$, payoff-relevant information is symmetric:

$$\begin{aligned} V_t \equiv \mathbb{E}[y_{\zeta_\phi}|\mathcal{F}_t^I] &= \mathbb{E}[y_{\zeta_\phi}|\mathcal{F}_t^B] \\ \mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^I] &= \mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^B]. \end{aligned} \tag{46}$$

Since $(\tilde{D}_{t\wedge\zeta_f})$ is a closed martingale,

$$D_t = \tilde{D}_t = \mathbb{E}\left[\lim_{s\to\infty} \tilde{D}_{s\wedge\zeta_f}|\mathcal{F}_t^I\right] = \mathbb{E}[\tilde{D}_{\zeta_f}|\mathcal{F}_t^I]. \tag{47}$$

Combining (46) and (47), we obtain

$$\begin{aligned}\alpha V_t - D_t &= \mathbb{E}[\alpha y_{\zeta_\phi}\mathbb{I}_{\{\zeta_\phi \le \zeta_\ell\}} + \alpha y_{\zeta_\ell}\mathbb{I}_{\{\zeta_\phi > \zeta_\ell\}} - \tilde{D}_{\zeta_f}|\mathcal{F}_t^I] \\ &= \mathbb{E}[\alpha y_{\zeta_\phi}\mathbb{I}_{\{\zeta_\phi \le \zeta_\ell\}} + \alpha y_{\zeta_\ell}\mathbb{I}_{\{\zeta_\phi > \zeta_\ell\}} - \tilde{D}_{\zeta_f}|\mathcal{F}_t^B] \\ &< \mathbb{E}[y_{\zeta_\phi}\mathbb{I}_{\{\zeta_\phi \le \zeta_\ell\}} + \alpha y_{\zeta_\ell}\mathbb{I}_{\{\zeta_\phi > \zeta_\ell\}} - \tilde{D}_{\zeta_f}|\mathcal{F}_t^B]. \end{aligned} \tag{48}$$

The first line is the payoff from liquidating today. The last line is the $\mathcal{F}_t^B$-expected payoff of never liquidating strategically.

A.4 Lemma 4

First note that for $p \in (0,1)$, in a consistent belief system, $q_t = 0$ after disclosure of the bad state, $q_t = 1$ after disclosure of the good state, and $q_t \in (0,1)$ absent disclosure. Fix such a belief system and show that the sanitization strategy is optimal. Due to discounting (Blackwell (1965)) we can focus on one-shot deviations. Note that under F and for a given D_0, the event tree is discrete. This is because there are always at most 4 possible states tomorrow, given the state today. Let us consider the choice between playing the sanitization strategy δ^S at some node or something else, leaving the rest of the strategies unchanged. Let $\mathcal{O}$ be the event tree following playing δ^S at this node and $\mathcal{D}$ the event tree following the other move (the "deviation"). The deviation is either the regulator switching from concealing the bad state to disclosing it, or concealing the good state instead of disclosing it. The former case is equivalent to a strategic default enforced by the regulator, which is never optimal, similarly to Lemma 3. Thus, focus on the latter case and relabel $t = 0$ the deviation time (at which $y_0 = y^G$), and let $y_1, \ldots, y_n, \ldots$ and $J = \zeta_\phi$ be a possible realization of future asset states and maturity. By condition (M) and induction, the face values $\tilde{F}_0, \ldots, \tilde{F}_n, \ldots$ associated with y_0 undisclosed and the realizations $y_1, \ldots, y_n, \ldots$ disclosed according to the sanitization strategy satisfy $\tilde{F}_i \ge F_i$, where F_i are the face values in $\mathcal{O}$. Let j be the liquidation time in $\mathcal{D}$. Three cases are possible. (i) $j \le J - 1$ and there is liquidation at time j in $\mathcal{O}$: then there is liquidation at time j in both $\mathcal{O}$ and $\mathcal{D}$. Since due debt is higher in $\mathcal{D}$ ($\tilde{F}_{j-1} \ge F_{j-1}$), the residual claim is lower in $\mathcal{D}$. (ii) $j \le J - 1$ and there is no liquidation at time j in $\mathcal{O}$: there, debt is lower, and the arguments of the proof of Lemma 3 allow to conclude that the expected residual claim at time j conditional on $\zeta_\phi = J$ is higher in the original tree. (iii) $j \ge J$: the asset matures before liquidation both in $\mathcal{O}$ and $\mathcal{D}$. Since debt is lower in $\mathcal{O}$, the expected residual claim is higher in $\mathcal{O}$. Finally, note that the expected profit at date $t = 0$ is an average of expectations of the residual claim conditional on $y_0 = y^G, y_1, \ldots, y_j, \zeta_\phi = J$. Cases (i), (ii) and (iii) above show that these quantities are higher in $\mathcal{O}$ for all J, j, $y_1, \ldots, y_j$. Hence, it is optimal for the regulator to play the sanitization strategy. The belief system consistent with this strategy is then the one given in section 3.1.2.

A.5 Lemma 6

Voluntary disclosure case. In equations (19) and (20), we obtained the recursive relationship

$$q_{\tau+1} = \frac{(1-p)(q_\tau \lambda^{GG} + (1-q_\tau)\lambda^{BG})}{1-p+p(1-q_\tau\lambda^{GG} - (1-q_\tau)\lambda^{BG})}. \tag{49}$$

A standard sequence analysis reveals that (q_τ) decreases to the root of G that lies in $[0,1]$, with $G \equiv G_1 - G_2$ and

$$G_1(q) \equiv q(1-p+p(1-q\lambda^{GG} - (1-q)\lambda^{BG})) \tag{50}$$
$$G_2(q) \equiv (1-p)\lambda^{BG} + q(1-p)(\lambda^{GG} - \lambda^{BG}). \tag{51}$$

This root is the stationary weight in case of voluntary disclosure, q_V^*, and satisfies

$$q_V^* = \frac{1-(1-p)\lambda^{GG} - (2p-1)\lambda^{BG} - \sqrt{(\lambda^{BG})^2 + 2\lambda^{BG}(\lambda^{GG}(p-1) - 2p+1) + (1-(1-p)\lambda^{GG})^2}}{2p(\lambda^{GG} - \lambda^{BG})}. \tag{52}$$

Mandatory disclosure case. The expression of q_M^* in (36) results directly from considering the fixed point of (35). We now set out to obtain the inequality $q_V^* < q_M^*$. Since G can only be non-negative for $q \geq q_V^*$, it is sufficient to show that $G(q_M^*; p) \geq 0$ (with obvious notation) for any value of p. Direct calculation shows that

$$\frac{\partial}{\partial p} G(q_M^*; p) \tag{53}$$

has the same sign as $1 + \lambda^{BG} - 2\lambda^{GG}$. In particular, it is of constant sign and we obtain

$$G(q_M^*; p) \geq \min\{G(q_M^*; 0), G(q_M^*; 1)\}. \tag{54}$$

Since $G(q_M^*; 0) = 0$ and $G(q_M^*; 1) > 0$, we obtain $G(q_M^*; p) \geq 0$ indeed.

A.6 Proposition 2

We know that in a consistent bank policy, $m_\tau(F(D,\tau)) = D$. The banker picks the lowest F that satisfies this equation, because expected liquidation costs are increasing in F. Hence, in equilibrium,

$$F(D,\tau) = \min\{F \geq 0, m_\tau(F) = D\}. \tag{55}$$

In order to find F, we need to make m_τ explicit. If $F \leq y^B$, the promise of F is never defaulted upon: $m_\tau(F) = F$. If $F \leq C(\tau+1)$, there is one default state (state χ_2, see section 3.1.4) and

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)F. \tag{56}$$

If $F \in (C(\tau+1), C(0)]$, there are two default states (χ_2 and χ_3) and

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)(p\gamma_\tau F + \alpha(1-p\gamma_\tau)V_{\tau+1}). \tag{57}$$

If F belongs to $(C(0), y^G]$, there are three default states (χ_2, χ_3 and χ_4) and

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + \alpha(1-\phi)(p\gamma_\tau V_0 + (1-p\gamma_\tau)V_{\tau+1}). \tag{58}$$

A.7 Proposition 3

The probability of an announcement tomorrow is $p\gamma_\tau$. The probability of no announcement is $1-p$. Otherwise, state y^B is disclosed (probability $p(1-\gamma_\tau)$). First, if $F \leq y^B$, $m_\tau(F) = F$. If $y^B < F \leq C(\tau+1)$, then

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)((1-p(1-\gamma_\tau))F + p(1-\gamma_\tau)\alpha V^B). \tag{59}$$

If $C(\tau+1) < F \leq C(0)$, then

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)(p\gamma_\tau F + (1-p)\alpha V(\tau+1) + p(1-\gamma_\tau)\alpha V^B). \tag{60}$$

If $C(0) < F \leq y^G$, then

$$m_\tau(F) = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + \alpha(1-\phi)(p\gamma_\tau V^G + (1-p)V(\tau+1) + p(1-\gamma_\tau)V^B). \tag{61}$$

A.8 Proposition 4

Proofs are presented in the voluntary disclosure case, and work identically in the mandatory disclosure case. I first need to introduce the

Lemma 8. *Let τ be a fixed integer and $0 < p^* < 1$. If $\alpha < 1$, there is $K_\tau > 0$ such that for all $p \leq p^*$,*

$$C^{[p]}(\tau) \geq \alpha V_\tau^{[p]} + K_\tau, \tag{62}$$

where the superscript $[p]$ designates a variable relative to the model solution under the opacity parameter p.

Proof. Promising y^G entails costly liquidation tomorrow unless the asset matures. Hence,

$$m_\tau^{[p]}(y^G) = \phi(1-\alpha)\left(\gamma_\tau^{[p]} y^G + (1-\gamma_\tau^{[p]})y^B\right) + \alpha V_\tau^{[p]}. \tag{63}$$

We obtain the result by setting $K_\tau = \phi(1-\alpha)\gamma_\tau^{[p^*]}$, noting that $C^{[p]}(\tau) \geq m_\tau^{[p]}(y^G)$. ■

We now come back to the proof of Proposition 4.

(a) Debt in the II region satisfies

$$D = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)F, \tag{64}$$

with $F \leq C(\tau+1)$. Thus, the inverse yield verifies

$$\frac{D}{F} \geq \phi\gamma_\tau + \phi(1-\gamma_\tau)\frac{y^B}{C(\tau+1)} + 1 - \phi. \tag{65}$$

Debt in the IS region satisfies

$$D = \phi(\gamma_\tau F + (1-\gamma_\tau)y^B) + (1-\phi)(p\gamma_\tau F + \alpha(1-p\gamma_\tau)V_{\tau+1}), \tag{66}$$

with $C(\tau+1) < F \leq C(0)$. From there,

$$\begin{aligned} \frac{D}{F} &\leq \phi\gamma_\tau + \phi(1-\gamma_\tau)\frac{y^B}{C(\tau+1)} + (1-\phi)\left(p\gamma_\tau + \alpha\frac{(1-p\gamma_\tau)V_{\tau+1}}{F}\right) \\ &\leq \phi\gamma_\tau + \phi(1-\gamma_\tau)\frac{y^B}{C(\tau+1)} + 1 - \phi, \end{aligned} \tag{67}$$

where the last inequality holds because of Lemma 8. We conclude by comparison with Equation (65).

(b) We first need to show that for p small, $m_\tau(C(0)) < m_\tau(C(\tau+1))$. This implies that promising face values between $C(\tau+1)$ and $C(0)$ does not allow to roll over other debt levels than the ones in the II zone: there is no IS zone. Given the expressions of $m_\tau(C(0))$ and $m_\tau(C(\tau+1))$, the desired inequality is equivalent to

$$p\gamma_\tau^{[p]}C(0) + \alpha(1-p\gamma_\tau^{[p]})V_{\tau+1}^{[p]} \leq C^{[p]}(\tau+1). \tag{68}$$

We conclude by letting $p \to 0$ and using Lemma 8. For the case $p \to 1$, recall that debt capacity is always below the fundamental value from Lemma 2. In the voluntary disclosure case, as p goes to 1, $q_{\tau+1}$ goes to 0, so the fundamental value goes to V^B. Now let $D > V^B$. We have

$$m_\tau^{[p]}(C^{[p]}(\tau+1)) < C^{[p]}(\tau+1) \leq V_{\tau+1}^{[p]} \to y^B, \tag{69}$$

hence D can not be in the II zone for p close enough to 1.

(c) is a consequence of the fact that $m(C(\tau+1)+\varepsilon) < m(C(\tau+1))$ for ε close to 0 and $\alpha < 1$. Recall that this is because the face value is only infinitesimally higher, but there will be default in one more state of the world (the non-disclosure state), meaning that the proportional cost $1-\alpha$ now applies to an additional, non-zero probability, state of the world.

(d) **(d1)** Let $p_1 < p_2$, τ_1, τ_2 such that

$$q_{\tau_1}^{[p_1]} = q_{\tau_2}^{[p_2]} = q. \tag{70}$$

The probability to be in state y^G tomorrow is $q' = \lambda^{GG}q + \lambda^{BG}(1-q) = \gamma_{\tau_1}^{[p_1]} = \gamma_{\tau_2}^{[p_2]}$. Then, the probability to be in state y^G tomorrow conditional on no disclosure under parameter p_1 is $\frac{(1-p_1)q'}{1-p_1q'}$. Using the expression of the yield in the IS region, we find

$$\begin{aligned} m^{[p_1]}(F) &= \phi(q'y^G + (1-q')y^B) \\ &+ (1-\phi)\left(q'p_1F + \alpha(1-p_1q')\left[\frac{(1-p_1)q'}{1-p_1q'}V^G + \frac{1-q'}{1-p_1q'}V^B\right]\right). \end{aligned} \tag{71}$$

From there,

$$m^{[p_1]}(F) - m^{[p_2]}(F) = (p_2 - p_1)q'(\alpha V^G - F), \tag{72}$$

which is negative for F close to $C(0)$ by Lemma 8. Given that $D = m^{[p_1]}(F^{[p_1]})$, we have $D < m^{[p_2]}(F^{[p_1]})$, from which we deduce that $F^{[p_1]} > F^{[p_2]}$. Indeed, (D, τ_2) belongs to the IS region under p_2, and $m^{[p_2]}(.)$ is increasing over this region, and must satisfy $D = m^{[p_2]}(F^{[p_2]})$.

(d2) This part of the proposition is clear from the expression of yields. There is equality in the voluntary disclosure case, and strict inequality in the mandatory disclosure, because increasing p increases the probability of having to disclose bad news.

(e) Let $F^V \equiv F^V(D, q)$ and q' be defined as above. We have

$$\phi\left(q'F^V + (1-q')y^B\right) + (1-\phi)F^V = D. \tag{73}$$

For $F \leq F^V$,

$$\begin{aligned} m^M(F) &\leq \phi\left(q'F^V + (1-q')y^B\right) + (1-\phi)\left((1-q')py^B + (1-(1-q')p)F\right) \\ &< \phi\left(q'F + (1-q')y^B\right) + (1-\phi)F \\ &\leq \phi\left(q'F^V + (1-q')y^B\right) + (1-\phi)F^V \\ &= D. \end{aligned} \tag{74}$$

Hence, $m^M(F) < D$ for $F \leq F^V$, implying that $F^M > F^V$.

A.9 Proposition 5

The first equality is because a run never happens before t_0 and always happens at t_0 if maturity is not reached yet (ζ_l is either t_0 or $+\infty$). Thus $\mathcal{P} = \mathbb{P}(\zeta_\phi > t_0)$. To compute expected output (see equation (12)) $\mathbb{E}[U]$, write

$$\mathbb{E}[U] = \sum_{t=0}^{t_0-1} \phi(1-\phi)^t \mathbb{E}[U|\zeta_\phi = t+1] + \mathbb{E}[U|\zeta_\phi > t_0]\mathbb{P}(\phi > t_0). \tag{75}$$

Note that

$$\mathbb{P}(\zeta_\phi > t_0) = (1-\phi)^{t_0}, \tag{76}$$

and

$$\mathbb{E}[U|\zeta_\phi > t_0] = V(q_0) \tag{77}$$

by the Markov property and given that $\mathbb{P}(\zeta_\phi = t+k|\zeta_\phi \geq t) = \mathbb{P}(\zeta_\phi = k)$. Then

$$\mathbb{E}[U|\zeta_\phi = t+1] = \mathbf{y}\Lambda^{t+1}\mathbf{e}_1, \tag{78}$$

and the result obtains by computing the geometric sum.

A.10 Proposition 6

Note that liquidation occurs in two cases: either maturity is not reached at t_1 or the state switches to y^B before t_1. Therefore the probability that no run occurs, $1-\mathcal{P}$, satisfies

$$\begin{aligned}
1-\mathcal{P} &= \mathbb{P}(\zeta_\phi \leq t_1,\ \zeta_l > \zeta_\phi) \\
&= \sum_{t=0}^{t_1-1} \mathbb{P}(\zeta_\phi = t+1)\mathbb{P}(\zeta_l > t+1) \\
&= \sum_{t=0}^{t_1-1} \phi(1-\phi)^t(1-\lambda)^t \\
&= \phi\frac{1-(1-\lambda)^{t_1}(1-\phi)^{t_1}}{1-(1-\lambda)(1-\phi)}. \qquad (79)
\end{aligned}$$

And

$$\mathbb{E}[U] = \sum_{t=0}^{t_1-1} \phi(1-\phi)^t \mathbb{E}[U|\zeta_\phi = t+1] + \mathbb{E}[U|\zeta_\phi > t_1]\mathbb{P}(\phi > t_1). \qquad (80)$$

Now write

$$\begin{aligned}
& \mathbb{E}[U|\zeta_\phi > t_1] \\
=\ & \mathbb{E}[U|\zeta_\phi > t_1, y_0 = \ldots = y_{t_1} = y^G]\mathbb{P}(y_0 = \ldots = y_{t_1} = y^G) \\
+\ & \mathbb{E}[U|\zeta_\phi > t_1, \exists k \leq t_1,\ y_k = y^B]\mathbb{P}(\exists k \leq t_1,\ y_k = y^B) \\
=\ & (1-\lambda)^{t_1}\alpha V^G + (1-(1-\lambda)^{t_1})\alpha V^B. \qquad (81)
\end{aligned}$$

Similarly,

$$\begin{aligned}
& \mathbb{E}[U|\zeta_\phi = t+1] \\
=\ & \mathbb{E}[U|\zeta_\phi = t+1, y_0 = \ldots = y_{t+1} = y^G]\mathbb{P}(y_0 = \ldots = y_{t+1} = y^G) \\
+\ & \mathbb{E}[U|\zeta_\phi = t+1, \exists k \leq t,\ y_k = y^B]\mathbb{P}(\exists k \leq t,\ y_k = y^B) \\
+\ & \mathbb{E}[U|\zeta_\phi = t+1, y_0 = \ldots = y_t = y^G, y_{t+1} = y^B]\mathbb{P}(y_0 = \ldots = y_t = y^G, y_{t+1} = y^B) \\
=\ & y^G(1-\lambda)^{t+1} + (1-(1-\lambda)^t)\alpha V^B + \lambda(1-\lambda)^t\alpha V^B. \qquad (82)
\end{aligned}$$

The result finally obtains by computing the geometric sums.

A.11 Proposition 7

The dynamics of the model in continuous time are provided in the next section of the Appendix. First, we need to show that for any $\alpha \in \left(\frac{D_0}{V^G}, 1\right)$, $\mathcal{I}_1(\alpha) < \mathcal{I}_0(\alpha)$ (with obvious notation). Under our assumptions, we have

$$t_1(\alpha) = \frac{1}{p_c} \ln \frac{\alpha V^G}{D_0} \tag{83}$$

and

$$\mathcal{I}_1(\alpha) = (1-\alpha)e^{-(p_c+\phi)t_1(\alpha)}V^G. \tag{84}$$

Moreover, we know that $t_0(\alpha)$ is the unique solution to $f_\alpha(t) = g_\alpha(t)$, where

$$f_\alpha(t) \equiv \alpha V^G e^{-p_c t} \tag{85}$$

$$g_\alpha(t) \equiv D_0 e^{\phi t + \frac{\phi}{p_c}\left(e^{-p_c t}-1\right)}, \tag{86}$$

and

$$\mathcal{I}_0(\alpha) = (1-\alpha)V^G e^{-(p_c+\phi)t_0(\alpha)}. \tag{87}$$

Therefore, we need to show that $t_1 > t_0$ over $\left(\frac{D_0}{V^G}, 1\right)$. Since f_α decreases and g_α increases, it is sufficient to show that $f_\alpha(t_1(\alpha)) < g_\alpha(t_1(\alpha))$. This boils down to show

$$D_0 < D_0 \exp\left(\frac{\phi}{p_c}\ln\frac{\alpha V^G}{D_0} + \frac{\phi}{p_c}\left(\frac{D_0}{\alpha V^G} - 1\right)\right), \tag{88}$$

or

$$-\ln\frac{D_0}{\alpha V^G} + \frac{D_0}{\alpha V^G} - 1 > 0 \tag{89}$$

which holds true because $\ln x < x - 1$ for $x \in (0,1)$.

Now, $\mathcal{P}_1 < \mathcal{P}_0$ is equivalent to saying that $t_0(\alpha) < t(\alpha) \equiv -\frac{1}{\phi}\ln \mathcal{P}_1(\alpha)$. As before, this is equivalent to $f_\alpha\left(t(\alpha)\right) < g_\alpha\left(t(\alpha)\right))$ for α small, or

$$\frac{\alpha V^G}{D_0} < \exp\left((\phi + p_c)t(\alpha) + \frac{\phi}{p_c}\left(e^{-p_c t(\alpha)} - 1\right)\right). \tag{90}$$

Noting that the expressions only depend on the ratio ϕ/p_c and $x = \frac{\alpha V^G}{D_0}$, we can assume w.l.o.g. that $p_c = 1$ and it is sufficient to study when the inequality

$$x < h(j(x)) \tag{91}$$

holds, with

$$j(x) = -\frac{1}{\phi}\log\left(\frac{1}{1+\phi}+\frac{\phi}{1+\phi}e^{-(1+\phi)\log x}\right) \quad (92)$$

$$h(x) = \exp\left((1+\phi)x+\phi\left(e^{-x}-1\right)\right). \quad (93)$$

j is increasing and

$$j^{-1}(y) = \exp\left(-\frac{1}{1+\phi}\log\left(\frac{\phi+1}{\phi}e^{-\phi y}-\frac{1}{\phi}\right)\right). \quad (94)$$

(91) is equivalent to $j^{-1}(y) < h(y)$, or, taking logs:

$$r(y) \equiv (1+\phi)y+\phi\left(e^{-y}-1\right) > s(y) \equiv -\frac{1}{1+\phi}\log\left(\frac{\phi+1}{\phi}e^{-\phi y}-\frac{1}{\phi}\right), \quad (95)$$

with $0 \leq y \leq y_M \equiv \frac{1}{\phi}\log(1+\phi)$. Now note that $r(0) = s(0) = 0$, $r'(0) = s'(0) = 1$, $r''(0) = \phi$, $s''(0) = 1$, $r(y_M^-) < s(y_M^-) = +\infty$. Given these variations, it is now sufficient to show that $(r-s)''$ can only switch sign at most once. But

$$(r-s)'''(y) = -\phi e^{-y} - \phi^3 e^{\phi y}\frac{1+\phi+e^{\phi y}}{(1+\phi-e^{\phi y})^3} < 0 \quad (96)$$

for $0 \leq y < y_M$. If $\phi < p_c$, $r-s < 0$ over $(0, y_M)$. When $\phi > p_c$, $r-s$ is positive in the neighborhood of 0 and negative close to y_M, so there exists y_0 with $(r-s)(y_0) = 0$ and given the variations of $r-s$ given above, we have $r-s > 0$ over $(0, y_0)$ and $r-s < 0$ over (y_0, y_M). This concludes the proof.

A.12 Proposition 8

From Proposition 4, point **(b)**, we know that for p close enough to 1, the II zone disappears in the voluntary disclosure case. This means that disclosure of y^G is necessary to avoid liquidation; the regulator cannot conceal the bad state, which is a default state under both mandatory and voluntary disclosure. But there is a default state that exists only under voluntary disclosure and has positive probability $1-p$: the event that the regulator actually did not observe the asset ($\omega_t = 0$). It is then immediate from the expressions of the debt capacities and the bond yields in Propositions 1,2 and 3 that the voluntary debt capacity is necessarily attained first, *i.e.* liquidation always occurs weakly after under mandatory disclosure. Since it occurs strictly after with positive probability (the cases where $\omega_t = 0$), we obtain that mandatory disclosure is both strictly more efficient and produces strictly less runs.

B Additional Material

B.1 Details on the Numerical Approach

The procedure to compare mandatory and voluntary disclosure is given in section 4.2.1.

The standard deviation of a random variable with support $[a, b]$ is smaller than $\frac{b-a}{2}$. The variable $\mathbb{I}_{\{\zeta_\ell < \zeta_\phi\}}$ takes values in $\{0; 1\})$ and the final payoff takes values in $[0, y^G]$. Hence the asymptotic standard deviation of the Monte-Carlo error is smaller than $\frac{1}{2\sqrt{N}}$ for the run probabilities and smaller than $\frac{y^G}{2\sqrt{N}}$ for expected output. This allows to select the suitable value for N given a desired level of confidence. I select 1% for a precision at the third significative digit.

To obtain the numerical result 2, I run Monte-Carlo simulations of the model for parameter sets in a 10×64-point discretization of the $p \times (\lambda^{GG} \times \lambda^{BG} \times \alpha)$ space, with a fixed value of D_0.

To obtain the numerical result 1, no Monte-Carlo simulation is required. I use a 10000-point discretization of the $\alpha \times \phi \times \lambda \times D_0$ space and apply the closed-form formulas of Propositions 5 and 6 at each point of the grid.

B.2 Debt Dynamics in the Continuous-Time Limit

In the limit of vanishing debt maturity, the Markov chain with transition matrix Λ becomes a continuous-time Markov chain with infinitesimal generator

$$A = \begin{pmatrix} -p_c & p_c \\ p_r & -p_r \end{pmatrix}. \tag{97}$$

$p_c dt$ is the instantaneous probability to move from the good to the bad state. Let y be the 2×1 vector of states: $y = (y^G, y^B)^T$. To simplify, set $y^B = 0$.

Taking the limit in the analytical expression of debt capacities, we obtain that $C(\tau) = \alpha V_\tau$ in both regimes.

Full transparency. Away from $C = \alpha V^G$, the only risk is the observation of the bad state, which happens with probability $p_c dt$. Hence we have

$$D_t = \alpha V^B + (D_0 - \alpha V^B) e^{p_c t}. \tag{98}$$

Let

$$t^1(\alpha) = \frac{1}{p_c} \ln \frac{\alpha(V^G - V^B)}{D_0 - \alpha V^B}. \tag{99}$$

t^1 is the maximal time the bank can survive (even in the best scenario). Let ζ_c be the time of jump to the bad state.

Case 1: t^1 realizes before ζ_ϕ and ζ_c. Probability $e^{-(p_c+\phi)t^1}$. Liquidation happens at αV^G (repaying exactly creditors, leaving 0 to banker).

Case 2: ζ_ϕ realizes before t^1 and ζ_c. Probability

$$\frac{\phi}{p_c + \phi}\left(1 - e^{-(p_c+\phi)t^1}\right). \tag{100}$$

Full repayment, y^G realizes.

Case 3: ζ_c realizes first, probability

$$\frac{p_c}{p_c + \phi}\left(1 - e^{-(p_c+\phi)t^1}\right). \tag{101}$$

Liquidation at αV^B.

Hence

$$\mathcal{P} = 1 - \frac{\phi}{p_c + \phi}\left(1 - e^{-(p_c+\phi)t^1(\alpha)}\right) \tag{102}$$

$$\mathcal{I} = (1-\alpha)\left(e^{-(p_c+\phi)t^1(\alpha)} V^G + \frac{p_c}{p_c+\phi}\left(1 - e^{-(p_c+\phi)t^1(\alpha)}\right) V^B\right). \tag{103}$$

Full opacity. Away from C_t, the only risk is that maturity occurs, in the bad state, so

$$dD_t = \phi(1-\pi_t) D_t dt, \tag{104}$$

with $1 - \pi_t = \mathbb{P}(y_t = y^B | y_0 = y^G) = P_{GB}(t) = \frac{p_c}{p_r+p_c} - \frac{p_c}{p_r+p_c} e^{-(p_c+p_r)t}$. So the stock of debt evolves according to

$$D_t = D_0 \exp\left(\phi a t + \frac{\phi a}{b}\left(e^{-bt} - 1\right)\right) \tag{105}$$

with $a = \frac{p_c}{p_c+p_r}$ and $b = p_c + p_r$. And

$$\alpha V_t = \alpha(a - a e^{-bt})(V^B - V^G) + \alpha V^G. \tag{106}$$

Let $t^0(\alpha)$ be the unique solution to $\alpha V_t = D_t$.

Case 1: ζ_ϕ realizes before t^0. Then with probability π_{ζ_ϕ}, payoff realizes at y^G (full repayment) and with probability $1 - \pi_{\zeta_\phi}$, payoff realizes at $y^B = 0$.

Case 2: ζ_ϕ realizes after t_0. Then there is liquidation at αV_{t^0}.

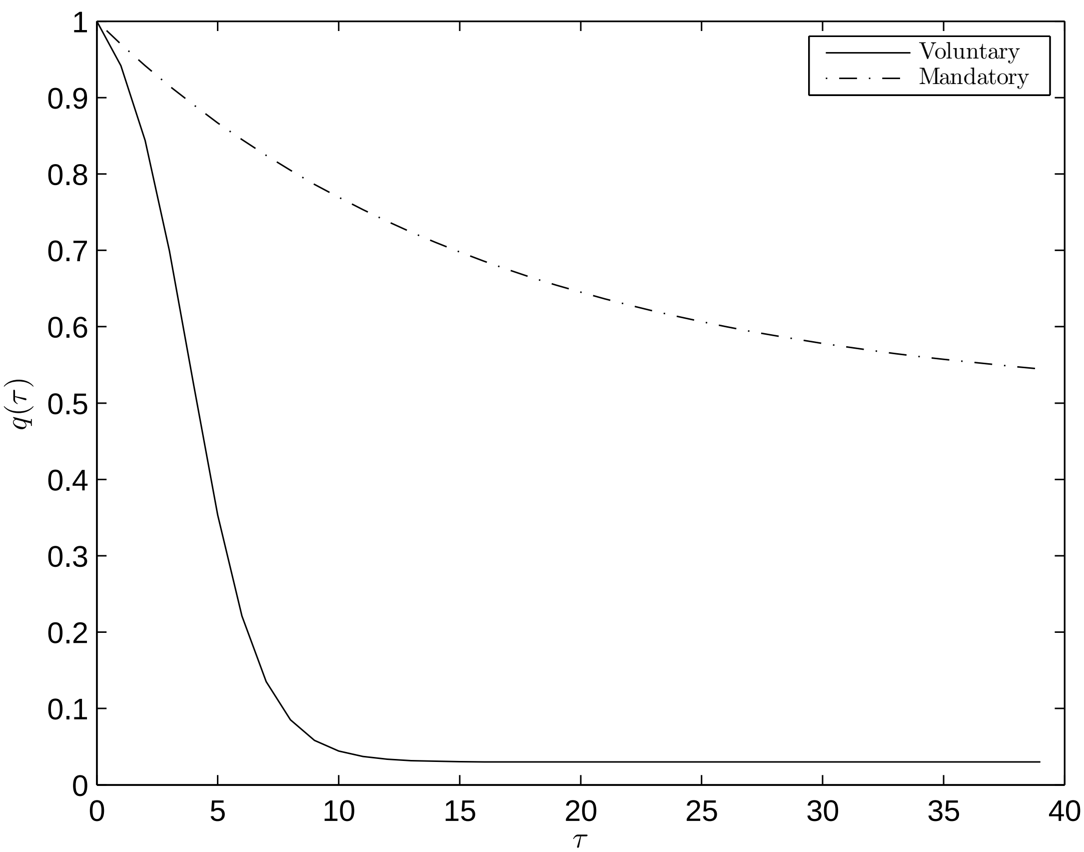
Hence

$$\mathcal{P} = e^{-\phi t^0(\alpha)} \tag{107}$$

$$\mathcal{I} = (1-\alpha) V_{t_0} e^{-\phi t^0(\alpha)}. \tag{108}$$

C Figures

Figure 1: Probability q_τ to be in state y^G after τ periods of non-disclosure.



When no information arrives, outsiders' perceived probability to be in the good state decreases and goes to a limit weight. When disclosure is voluntary, the downgrade is much faster because the regulator is increasingly likely to be concealing bad news. The limit weight on state y^G is lower in that case.

Figure 2: Debt capacities under both regimes.

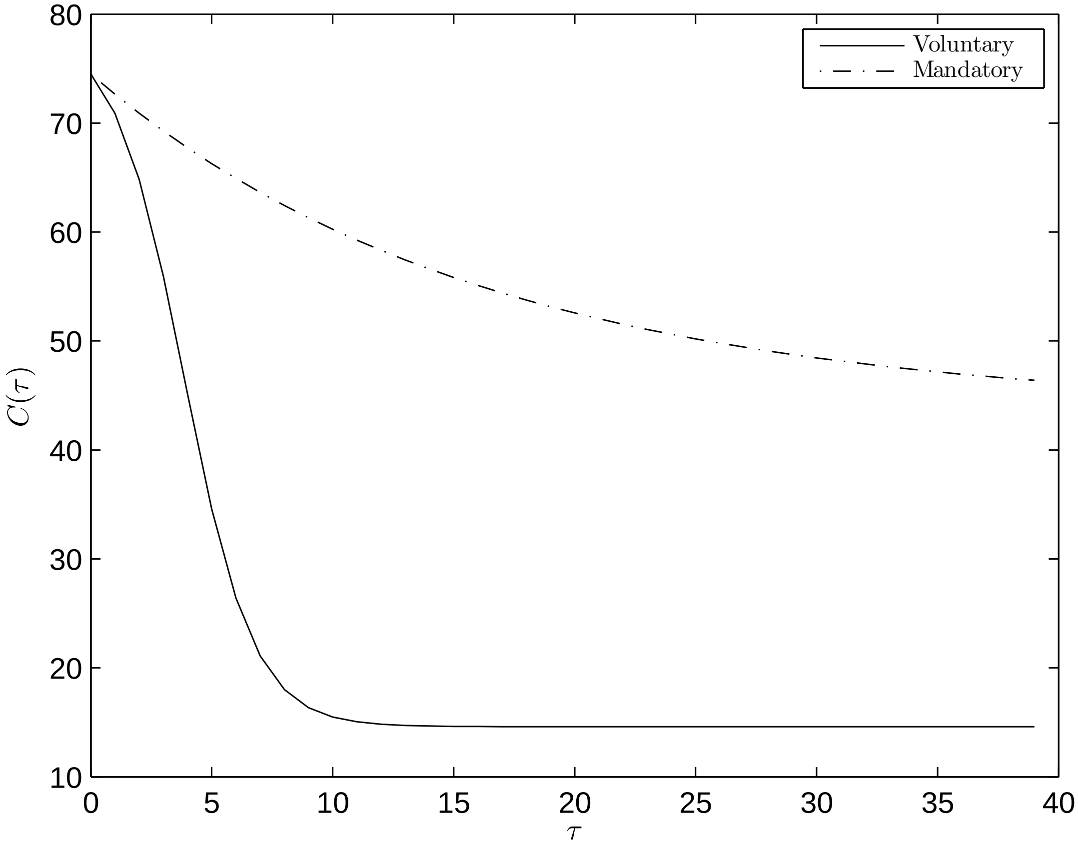


Figure 3: Bond yields as a function of debt for $\tau = 1$ under voluntary disclosure.

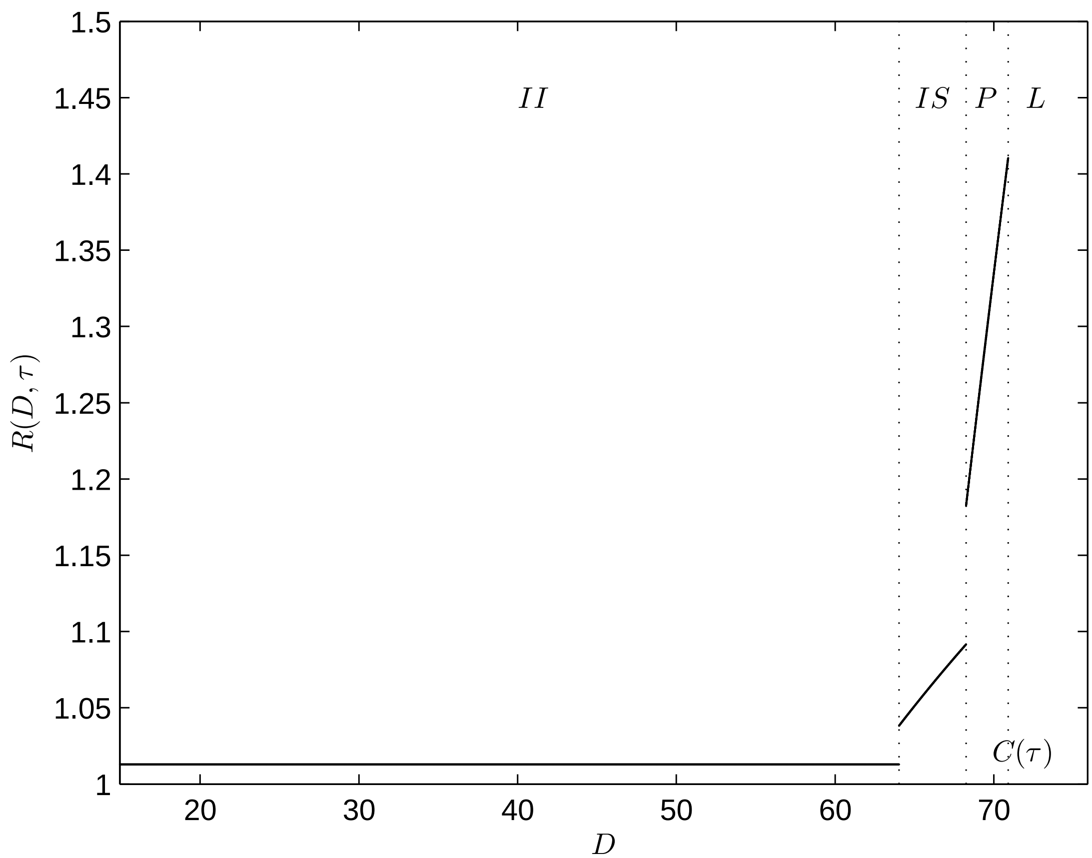


Figure 4: Bond yields as a function of debt for $\tau = 7$ under mandatory disclosure.

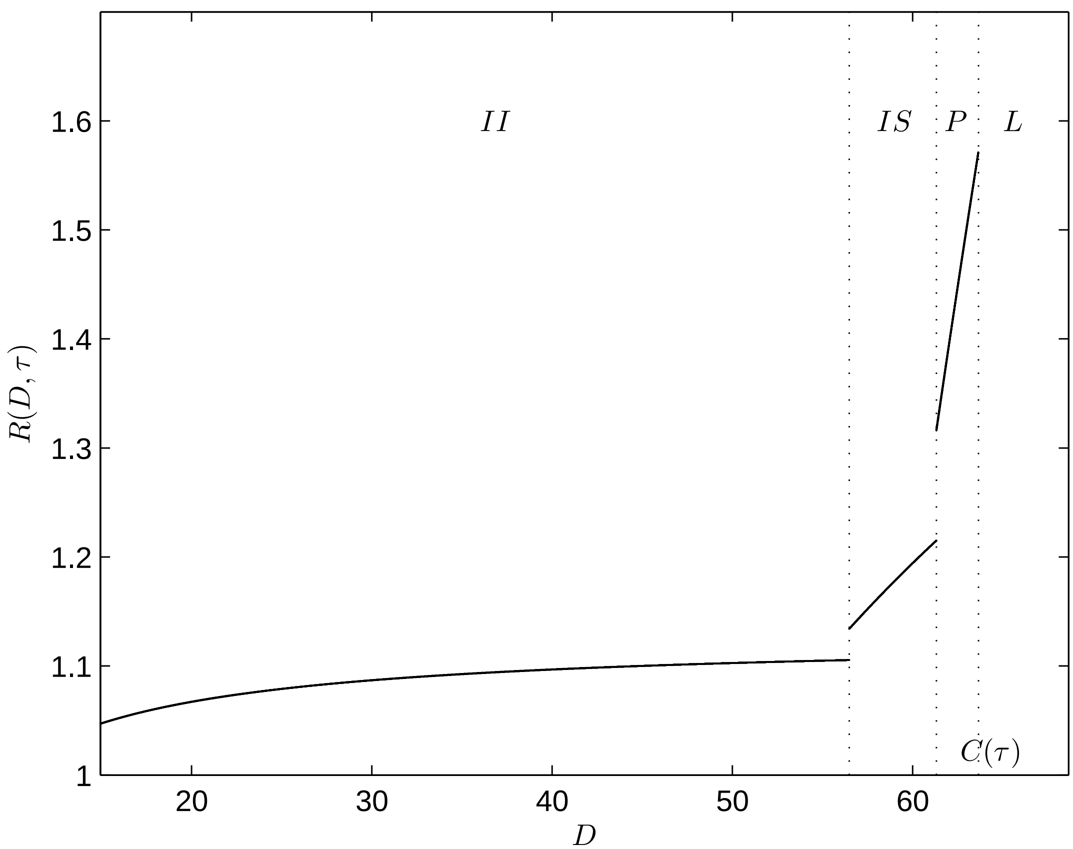
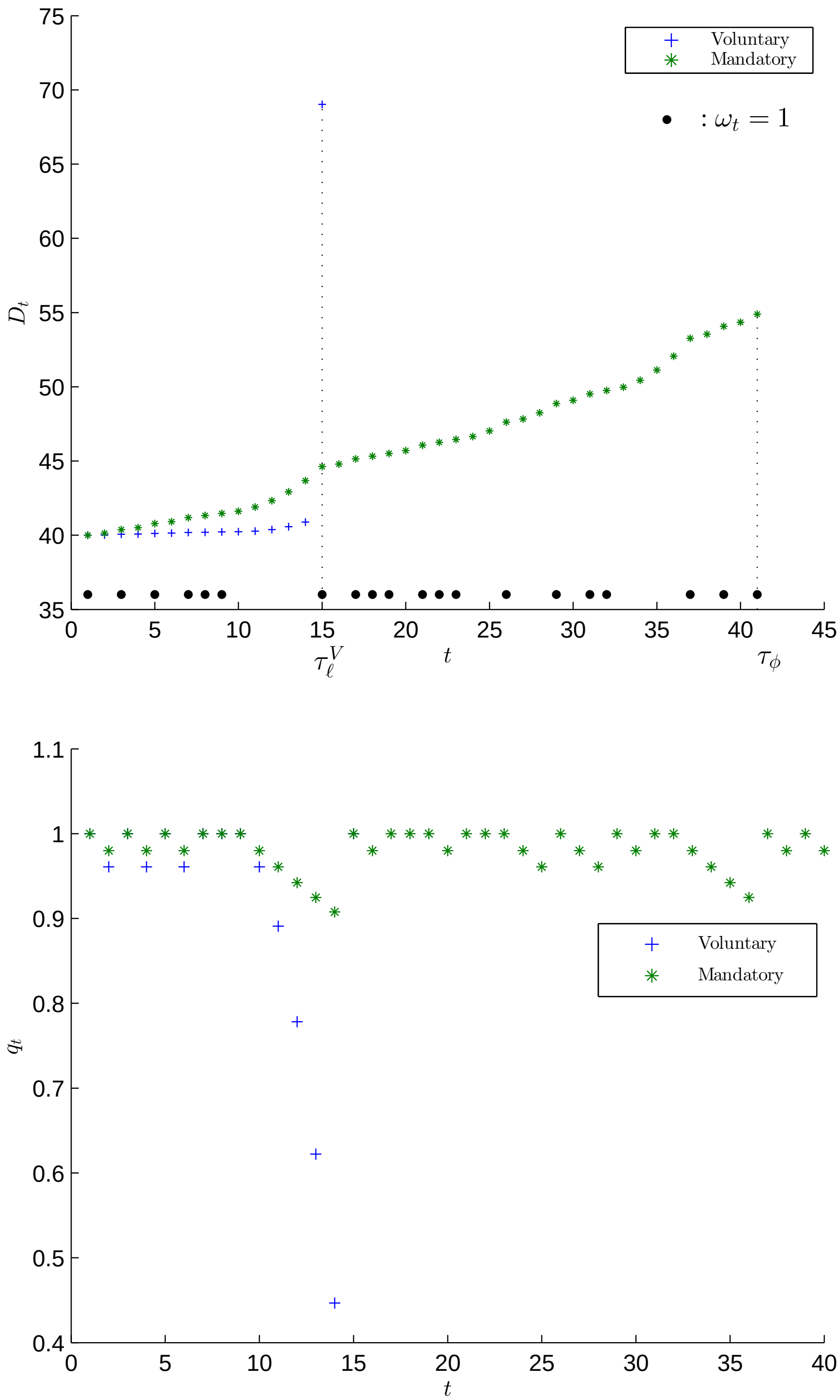
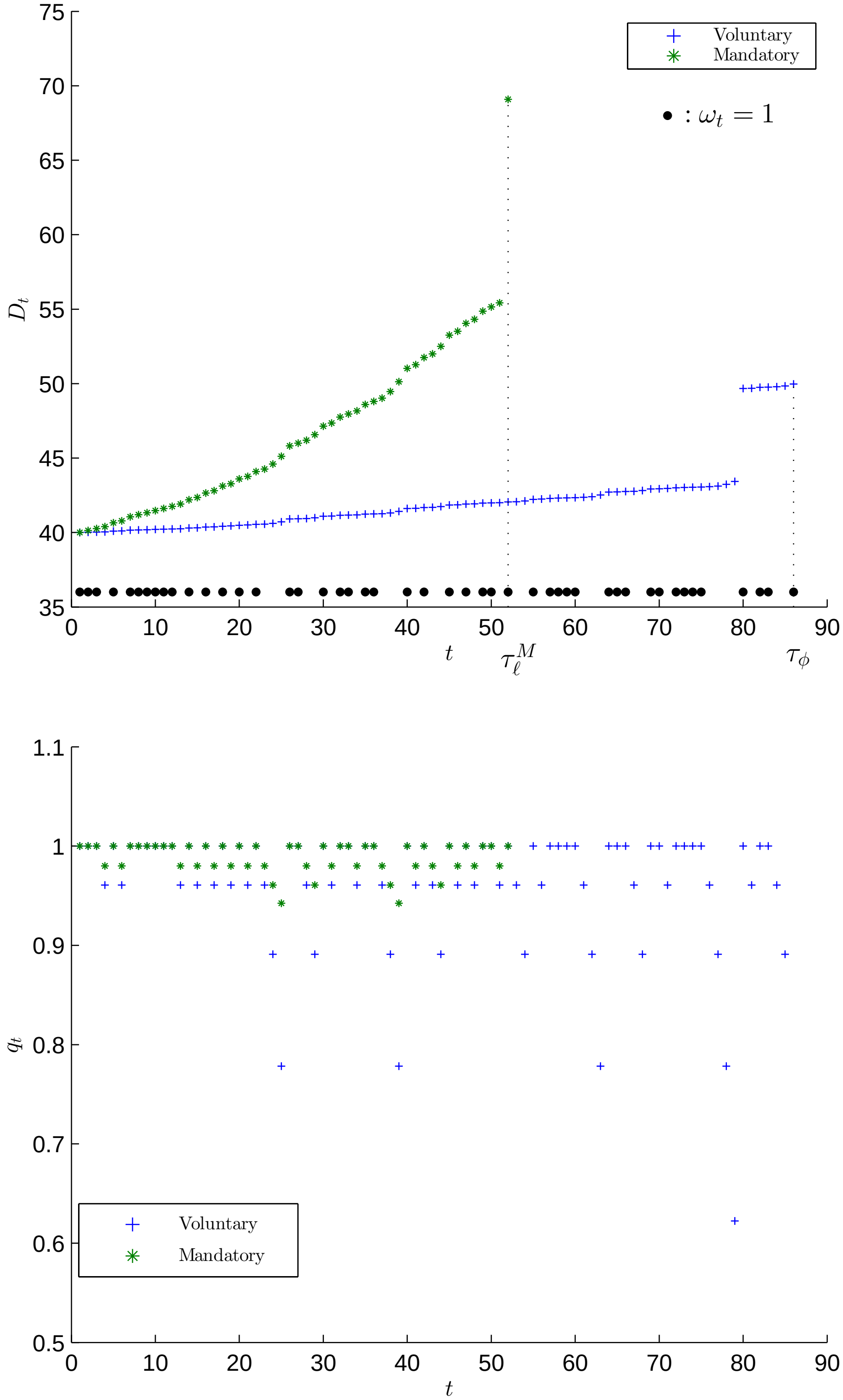


Figure 5: Debt and beliefs dynamics in a voluntary disclosure-induced run.



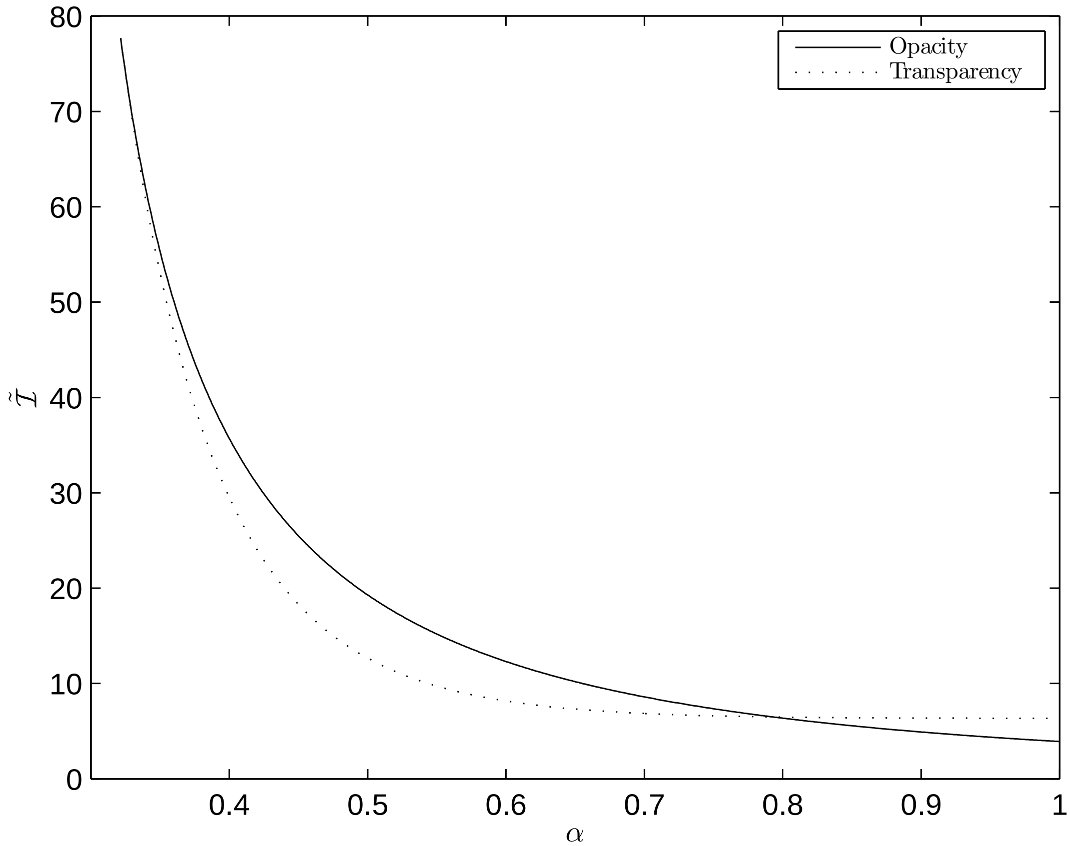
$\phi = \lambda^{BG} = 1 - \lambda^{GG} = 2\%$, other parameters are at their baseline value.

Figure 6: Debt and beliefs dynamics in a mandatory disclosure-induced run.



$\phi = \lambda^{BG} = 1 - \lambda^{GG} = 2\%$, other parameters are at their baseline value.

Figure 7: Inefficiency as a function of the liquidity parameter α.



Continuous-time version with $\phi = 0.1$, $p_c = p_r = 0.04$ and $D_0 = 25$.

$\tilde{\mathcal{I}} = \frac{1}{1-\alpha}\mathcal{I}$ designates the expected value of the asset conditional on premature liquidation and preserves the ordering between opacity and transparency given by $\mathcal{I}$. Note that under $\mathcal{I}$ both curves join at 0 when $\alpha = 1$, in which case there is no loss of value upon liquidation.

For low α, the short-term protection of opacity lasts less because debt capacities are low. Moreover runs, when they occur on good banks, are particularly harmful in terms of efficiency. The reverse holds for α close to 1.