Mind the (Convergence) Gap: Forward Rates Strike Back!

Andrea Berardi, Michael Markovich, Alberto Plazzi, and Andrea Tamoni*

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Abstract

Variation in expected yield changes contaminates bond risk premia information that is contained in forward rates. We show that the difference between the natural rate of interest and the current level of monetary policy stance, dubbed Convergence Gap (CG), forecasts changes in yields and helps identify whether forward rates reflect expectations of future interest rates or risk premia. Compared to a model with only forward rates, adding the CG significantly raises the R^2 in the forecasting regression of bond excess returns and delivers bond risk premia that are more countercyclical. The importance of CG remains robust out-of-sample, and in countries other than the U.S. Further, its inclusion brings significant economic gains in the context of dynamic conditional asset allocation. Overall, our results underscore the importance of revisions in monetary policy for bond predictability.

Keywords: Bond risk premia, Forward rates, Monetary policy, Natural rate of interest, Bond Predictability

JEL Classification: E0, E43, G0, G12

*Berardi is with the Department of Economics, Università Ca' Foscari Venezia, Fondamenta San Giobbe 873, 30121, Venezia, Italy, Email: andrea.berardi@unive.it. Markovich is with Credit Suisse, Kalanderplatz 1, WJSQ, 8070, Zurich, Switzerland, Email: michael.markovich@credit-suisse.com. Plazzi is with the Institute of Finance, Università della Svizzera italiana and SFI, Via Buffi 13, 6900, Lugano, Switzerland, E-mail: alberto.plazzi@usi.ch. Tamoni is with the Department of Finance, London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK, E-mail: a.g.tamoni@lse.ac.uk. We thank Riccardo Rebonato and seminar participants at Warwick Business School and Luxembourg School of Finance for comments.

1 Introduction

Starting with the seminal studies of Fama and Bliss (1987) and Campbell and Shiller (1991), the ability of interest rate variables, like the slope of the term structure, to forecast excess bond returns has received considerable attention in the fixed-income literature. Additionally, Cochrane and Piazzesi (2005) show that a combination of forward rates across different maturities explains a substantial fraction of common fluctuations in bond risk premia.

Despite the role of forward rates as natural conditioning variables, recent literature has cast doubt on the statistical and economic relevance of bond predictability. For example, Thornton and Valente (2012) use forward rates to forecast bond excess returns in the context of dynamic portfolio allocation. They conclude that deviations from the Expectations Hypothesis lead to economic gains that are small even in an in-sample setting. Analogous conclusions are drawn by Della Corte, Sarno and Thornton (2008) working at the very short end (from overnight to 3 months) of the yield curve.

In this paper, we exploit the Fama-Bliss and Campbell-Shiller identities to shed light on why yield-related forecasting variables can be successful at predicting bond returns, and why they can fail. Absent predictable patterns in yield changes, forward rates move one-to-one with bond risk premia. However, when expected yield changes are time-varying, the predictive regression suffers from an omitted variable bias. In these circumstances, a steep yield curve – forward rates higher than spot rates – may reflect either an increase in future returns, in future one-period yields, or both. Thus, the steepness in the yield curve that originates from higher expected future yields should be treated differently than steepness that is related to increasing risk-premia. Ideally, granted the goal of predicting returns, one would like to purge forward rates from any predictable movements in yield changes. This paper introduces a variable that achieves this objective.

Our proposed variable is the gap between the natural rate of interest (see Wicksell (1936), and more recently Woodford (2003, Ch. 4.1–4.2)) and the real Fed Funds rate. We dub this variable Convergence Gap (CG). The logic behind this variable is clear from a monetary policy viewpoint: if the central bank follows a targeting rule which can be traced (directly or indirectly) to the natural rate, then it may act in the Federal Funds rate market to close the gap.[1] Thus, a positive convergence gap – yields below their natural level – signals that short term yields are likely to increase in the future. This is indeed what we find in the data. Whether alone or jointly with the slope, the CG reduces by a significant 10% the root mean squared error (RMSE) when predicting future one-year yield changes compared to the benchmark random walk model. The RMSE when using the slope only is, instead, statistically indistinguishable (or even higher) than that from the random walk model at nearly every forecasting horizon.

The evidence that CG provides useful information on the path of future yields makes it a natural conditioning variable when predicting bond returns with forward rates. Following the logic above, CG would filter forward rates from predictable components in yield changes which would otherwise bias bond return forecasts. In other words, the CG allows us to take into account when a positively sloped yield curve contains expectations of higher yields, in addition to information about future risk premia. Accordingly, we follow Cochrane and Piazzesi (2005) and use CG in the construction of a bond risk factor in the predictive regression of average bond excess returns. We find that when the CG is included *together* with forward rates, the R^2 increases from 23% to 34% and the statistical significance of the forward rates is enhanced.

The convergence gap variable enters the joint regression with a negative coefficient. Thus, risk premia are lower when yields are below their natural rate, everything else equal. The intuition is that a positive CG is usually accompanied by a contemporaneous increase in forward rates that reflects expectations of higher future yields. Absent CG, this increase

[1] The turbulent Global Financial Crisis period, with the prospect of long stretches constrained by the effective lower bound, have commentators wondering whether inflation targeting regimes are the still the right approach for central banks (Williams, 2016). Accordingly, recent monetary policy discussions (see, e.g. Yellen, 2015; Kaplan, 2018) focus on the equilibrium real interest rate because it provides a gauge of a "neutral" stance of monetary policy.

would instead generate a prediction of higher risk premia and, in turn, a negative forecast error. Interestingly, the major improvements conveyed by CG occur during periods of active monetary policy intervention such as the early 1980s (Volcker experiment), the early 2000s recession, and the Great Recession.[2] Overall, these findings confirm that the success of CG comes from its ability of disentangling movements risk premia from predictable patterns in yield changes.

Another interesting aspect we document is that CG eliminates systematic patterns in forecast errors which are related to the state of the economy, and hence to the cyclical nature of risk premia. When only forward rates are included in the bond return regression, the resulting residuals tend to be countercyclical, as they are predictable by variables such as industrial production growth and the NBER dummy. When conditioning on CG, however, the residuals become nearly unpredictable. This evidence implies that controlling for CG helps restoring the countercyclicality of risk premia that is partly missed out by forward rates, as noted by Ludvigson and Ng (2009).

We next use the fitted value of the joint regression of average bond excess returns on forward rates and CG as a factor for predicting individual bond return series. We find that the statistical significance of the combined factor is preserved across the full spectrum of maturities. It is also robust to controlling for the macroeconomic factor of Ludvigson and Ng (2009). This result clarifies that CG provides complementary information about the countercyclical nature of bond risk premia with respect to other macro series or combinations thereof. In addition, other measures of gap (such as, the output gap) do not perform as well as CG in predicting bond returns, which underscores the importance of taking into account the conduct of monetary policy.

We obtain quantitatively similar results when using annual holding period returns sampled at the quarterly and annual frequency, vintage data for GDP and inflation following Ghysels et al. (2018), and monthly holding period returns. We also extend our analysis

[2]For a discussion of the Fed's policy responses in 2000-2001, see the February 2002 Monetary Policy Report available online at the Federal Reserve Board's website.

to countries other than the U.S., namely Canada, the U.K., and Germany/Eurozone. The convergence gap plays a significant role in tracking bond risk premia also in these markets, entering the regression with a negative and significant coefficient. The evidence that the role of CG remains intact in all these tests is rather reassuring of the robustness of our conclusions.

In order to measure the economic value of the documented bond return predictability, we rely on a dynamic portfolio choice problem involving a risk-free asset and a portfolio of bonds. In particular, we adopt the approach of Brandt and Santa-Clara (2006) and estimate the optimal policy for a risk-averse investor who dynamically adjusts her position based on a set of predictors. Conditioning the policy on forward rates produces a Sharpe Ratio of about 0.5 compared to a value of about 0.3 for the static (myopic) solution. The equalization fee of using forward rates equals 1.2%, and consistent with Thornton and Valente (2012) we cannot reject the hypothesis that their loadings are jointly statistically different from zero. Adding CG to the information set substantially changes the portfolio allocation and its performance. The Sharpe Ratio increases to 0.6, which makes the investor indifferent to paying an annual equalization fee of 2%. These results hold for monthly as well as annual non-overlapping bond returns, and even if we prevent the investor from taking large positions. The economic magnitude of these effects confirms that deviations from the Expectations Hypothesis are indeed quite relevant.

Finally, we investigate whether our predictability evidence holds also out-of-sample. Specifically, we look at the forecasting accuracy in predicting annual holding period returns for each month during the 1990-2017 period, when only real-time information is used. The combined factor produces a mean squared forecast error that is about one-fourth smaller than that produced by the Cochrane and Piazzesi (2005) factor alone, and further improves upon the macro factor of Ludvigson and Ng (2009).

Our findings contribute to the literature on bond returns "excess" predictability, that is, predictability achieved with variables other than current yields. Among others, Cooper and Priestley (2009) and Ludvigson and Ng (2009) propose macroeconomic factors that help tracking bond risk premia.[3] Our approach is different in scope as the importance of the convergence gap resides in its ability of purging forward rates by filtering predictable changes in future yields, rather than capturing orthogonal variation in risk premia.

The literature on bond returns "excess" predictability has recently emphasized the importance of macro trends for forecasting interest rates and bond returns (see Cieslak and Povala, 2015; Bauer and Rudebusch, 2017). Most notably, Cieslak and Povala (2015) show that augmenting yields-only predictive regressions with a trend inflation can help uncover substantial bond risk premium variation. Our findings complement this literature. Whereas macro trends remove low frequency (possibly decadal) fluctuations in yields to unmask high-frequency variation related to bond returns, our convergence gap remove (higher frequency) predictable changes in future interest rates from yields. Statistically, our proposed variable does not display trend behavior, and in general has an autocorrelation that decays faster than that of typically employed macro predictors. The differences between the convergence gap and macro trends is further confirmed by the evidence that our variable remains a significant predictor of monthly holding period bond returns.

Finally, our paper relates to the literature on hidden or unspanned factors in the term structure (see Duffee, 2013a for a review) as our proposed variable, the convergence gap, affects short-rate expectations and risk premia in an exactly offsetting way. It is, however, important to recall that our present value restrictions only require that the conditioning information set of the agent contains yields. Thus, our findings could coexist with the evidence in Cieslak (2018) that there are large and persistent errors in the way investors form expectations about the future short rate over the business cycle. In particular, it may be the case that professional forecasters underestimate or ignore the predictive power of the converge gap for future interest rates. Our approach is robust to these considerations as we we do not use agents' expectations (e.g. SPF forecasts of yields) but rather yield and macro

[3]See Table 9, in Cooper and Priestley (2009), and Table 2, in Ludvigson and Ng (2009).

data directly.

The remainder of the paper is organized as follows. Section 2 revisits the Fama-Bliss identity, and demonstrates that the importance of CG stems from being a good predictor of yields. The main empirical results concerning the predictability of U.S. bond returns and the corresponding construction of a bond risk premia factor are contained in Section 3. There, we also show that the convergence gap works for countries other than the U.S. Section 4 contains the dynamic portfolio choice exercise which exploits the extant predictability. Section 5 collects the results of the out-of-sample analysis. Finally, section 6 provides concluding remarks.

2 Present Value Restrictions and the Convergence Gap

In Section 2.1, we lay down a present value framework that is useful to study predictors of bond returns other than yields, forward rates, or combinations thereof. In Section 2.2, we define the convergence gap and detail its construction and properties. In Section 2.3, we use the present value restrictions to establish the ability of the convergence gap to predict yield changes, and in turn its capacity to help forward rates forecasting bond returns.

2.1 Fama-Bliss Identity: A Reappraisal

We start with the Fama and Bliss (1987) accounting identity. Consider a zero-coupon bond that matures at $t+n$ with a payoff of 1\$, current price $P_t^{(n)}$ and log yield $y_t^{(n)} \equiv -\frac{1}{n} p_t^{(n)}$. The superscript refers to the bond's remaining maturity. The bond's log return from t to $t+1$, when its remaining maturity is $n-1$, is $r_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)}$. Algebraically, the price $P_t^{(n)}$ of an n-year bond is the present value of the \$1 payoff discounted at the expected values of the future 1-year returns on the bond:

$$P_t^{(n)} = \exp\left(-E_t\left[r_{t+1}^{(n)}\right] - E_t\left[r_{t+2}^{(n-1)}\right] - \ldots - E_t\left[y_{t+n-1}^{(1)}\right]\right) . \tag{1}$$

Fama and Bliss (1987) show that summing the last $n-1$ expected returns in Eq. (1), and substituting the resulting expression for prices in the definition of a forward contract, gives:

$$f_t^{(n)} - y_t^{(1)} = \underbrace{\left(E_t\left[r_{t+1}^{(n)}\right] - y_t^{(1)}\right)}_{rx_{t+1}^{(n)}} + (n-1) \times \left(E_t\left[y_{t+1}^{(n-1)}\right] - y_t^{(n-1)}\right) \ . \tag{2}$$

The above Fama and Bliss (1987) identity says that the forward-spot spread contains information about either the premium for a 1-year return on an n-year bond over the 1-year spot rate, or the expected change over the next year of the yield on $n-1$ year bonds, or both. In terms of regression coefficients, Eq. (2) implies that the slope coefficients in the following system:

$$r_{t+1}^{(n)} - y_t^{(1)} = a + b_r\left(f_t^{(n)} - y_t^{(1)}\right) + e_{t+1} \tag{3}$$

$$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a + b_y\left(f_t^{(n)} - y_t^{(1)}\right) + u_{t+1} \tag{4}$$

obey the present-value restriction $b_r + b_y = 1$.[4]

Table 1 shows results from estimating regressions (3)-(4) on the Fama and Bliss tape available from CRSP over 1964-2017.[5] The table re-establishes what documented by other authors, namely that all the variation in the forward-spot spread is attributable to the 1-year

[4]This restriction is remindful of the Campbell and Shiller (1988) identity which implies that the dividend-price must forecast dividend growth and/or returns. See Cochrane (2008) for a thorough discussion of the Campbell-Shiller identity, and its implications for long-horizon regressions. Despite the similarities, two important distinctions between the Fama-Bliss and Campbell-Shiller identities are that the former: (1) is exact, whereas the latter requires a log-linear expansion, and (2) does not rely on long-horizon regressions which typically are plagued by econometric small-sample biases.

[5]Unless otherwise stated, we use overlapping observations in regressions like (3)-(4). A standard issue with these type of forecasting regressions is that the residuals are highly persistent. We follow the common practice of reporting Newey and West (1987) standard errors. However, to address the tendency for statistical tests based on Newey-West standard errors to over-reject in finite samples, we rely on recent advance discussed in Kiefer and Vogelsang (2005). In particular, Kiefer and Vogelsang (2005) develop an asymptotic theory where the bandwidth (M) of the covariance matrix estimator is modeled as a fixed proportion of the sample size (T). The authors show that their fixed-b ($b = M/T$) theory has better finite-sample properties than traditional Gaussian asymptotic theory. Unless otherwise stated, in our empirical exercise we choose a lag truncation parameter of 60 months and we compute p-values using the Kiefer and Vogelsang (2005) theory. Our choice implies a parameter $b \approx 0.1$, and 5% and 1% critical values equal to 2.24 and 2.72, respectively. Our conclusions are identical when using a lag window equal to 18 months, which corresponds to the common choice of a lag truncation parameter equal to $[1.5 \times h]$, where h is the forecasting horizon.

expected premium ($\hat{b}_r = 1$), and none to expected yield changes ($\hat{b}_y = 0$).

What is important for our purpose, and to our knowledge unexplored, is that the Fama-Bliss identity (2) also applies to other, potentially non-yield related predictors. To see this, consider a variable x_t that is orthogonal to the forward spread. Exploiting (2) we obtain:

$$\text{Cov}\left(x_t, f_t^{(n)} - y_t^{(1)}\right) = \text{Cov}\left(x_t, r_{t+1}^{(n)} - y_t^{(1)}\right) + \text{Cov}\left(x_t, (n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right)\right) ,$$

where the left-hand side is zero by construction. In words, if the variable x_t predicts interest rates changes, then it ought to predict bond returns. In terms of regression coefficients we have that:

$$r_{t+1}^{(n)} - y_t^{(1)} = a + b_{r,x} x_t + e_{t+1} \quad (5)$$

$$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a + b_{y,x} x_t + u_{t+1} \quad (6)$$

and, importantly, Eq. (2) now imposes that $b_{r,x} + b_{y,x} = 0$.[6,7]

The Fama-Bliss identity splits variation in the forward-spot spread between the 1-year expected bond premium and expected yield change. However, the forward spread combines the two sources of information and therefore is an imperfect predictor of future returns if yields are expected to change in the future. A variable x_t can help the forward spread to forecast returns if it also forecasts interest rates. We next employ this framework to study the ability of the gap between the current real monetary policy rate and its long-term pendant (referred in literature as the natural rate of interest) to forecast interest rates *and* bond returns.

[6] Analogously, the Campbell-Shiller identity also imposes restrictions on predictors other than the price-dividend ratio. These restrictions have been exploited by Cochrane (2011) to examine the predictive ability of the consumption-wealth ratio for short-run market returns, and by Bandi and Tamoni (2018) to investigate the ability of long-run uncertainty to predict long-run market returns.

[7] The restriction is also reminiscent of "hidden factor models" introduced by Duffee (2011) and Joslin, Priebsch and Singleton (2014). Our derivation based on Fama-Bliss is new, and complementary to that typically found in Gaussian no-arbitrage models of the Term Structure.

2.2 The Convergence Gap

Our candidate yield forecasting factor is the *convergence gap*, CG, defined as the difference between the natural rate of interest and the ex-ante real federal funds rate.[8] The rationale behind our choice is as follows. From a monetary policy viewpoint, the central bank may follow a targeting rule based on the natural rate and act in the Federal Funds rate market to fill the gap. This, in turn, would affect short-term rates. If periods of positive convergence gap are generally associated with an increase in future yields, as we indeed document in Section 2.3, conditioning on CG allows us to capture the expectation of higher future yield changes that is contained into a positively sloped yield curve. If instead the yield curve is steep but short maturity yields are above the natural rate (CG<0), other factors such as changes in risk premia are likely to be the underlying driving force.

A large body of research has confirmed the validity of the convergence gap as a monetary policy cycle indicator. In particular, the indicator properties of the gap for forecasting inflation and/or output have been analyzed by, e.g., Orphanides and Williams (2002) and Amato (2005) for the US, Neiss and Nelson (2003) for the UK, and Mesonnier and Renne (2007) and Garnier and Wilhelmsen (2009) for the EMU. Bomfim (1997) uses the monetary cycles identified by Romer and Romer (1989), and shows that periods of monetary tightening are consistently identified as those where policy rates are persistently above their equilibrium level. In the sample 1990–2013, Barsky et al. (2014) show that a considerable degree of wage and price inflation stabilization could have been achieved if the Federal Reserve had tracked the natural rate. Finally, Curdia et al. (2015) show that a specification of monetary policy in which the interest rate tracks the Wicksellian efficient rate as the primary indicator of real activity, fits the U.S. data better than otherwise identical Taylor rules.

More specifically, we define the natural rate of interest as the real policy interest rate consistent with a closed output gap (real GDP equal potential in the absence of transitory shocks to demand) and stationary (i.e. non-accelerating) inflation (at/around target in the

[8]See Woodford (2003) for a formalization of the convergence gap concept in the context of DSGE models.

absence of transitory shocks to supply).[9] Economic theory implies that the natural rate of interest varies over time in response to shifts to preferences and the trend (i.e. potential) growth rate of output (see, e.g. Laubach and Williams, 2003). Hence, the natural rate is related to unobservable factors, and several techniques have been adopted to estimate it, including Kalman filtering. In the main part of our work, we proxy for the natural real rate of interest with potential real GDP growth.[10] Specifically, we first extract the trend component of quarterly real GDP using a one-sided Hodrick and Prescott (1997) filter and linearly interpolate the resultant series to obtain monthly observations.[11] We then define the natural rate of interest as the year-to-year log change in monthly trend GDP. In constructing the ex-ante real rate, we proxy inflation expectations with a four quarter moving average of past inflation. Finally, the convergence gap, is obtained as the difference between the natural rate and the real funds rate.[12] Appendix A provides detailed description of the data sources for GDP, inflation, and short rate.

The solid blue line in Panel A of Figure 1 shows the CG_t series. The figure also shows the convergence gap obtained when we proxy for the natural rate with either the measure of trend growth rate of output or the Kalman filter natural-rate estimates by Laubach and Williams (2003). The graph shows that – independently of the natural rate proxy – the

[9]See Wicksell (1936), and more recently Rudebusch (2001) and Laubach and Williams (2003). For a comprehensive overview on different definitions, estimation concepts and relevant horizons associated with the natural rate of interest we refer to Giammarioli and Valla (2004).

[10]This is equivalent to assume that: (1) the natural rate of interest is primarily related to the productivity of capital, and (2) when the policy rate equals the natural rate, the output gap is zero. The interrelation between the real natural rate of interest and trend growth rate of output is also empirically confirmed by Laubach and Williams (2003).

[11]Following common practice in the economic literature, the filter is first applied to log GDP; we then take the exponential of the trend component.

[12]Our variable is defined as a gap between real (natural and short-term) rates, and it is different in nature from expected inflation which is a well-known driver of the level of nominal interest rates (see in particular Cieslak and Povala, 2015) Also, the measure of convergence gap described in this section is modestly correlated (correlation in the order of 20%) with the output gap (as measured by the deviations of the log of industrial production from a trend that incorporates both a linear and a quadratic component) proposed by Cooper and Priestley (2009). This correlation lowers further at about 10% when we replace the trend component of real GDP with other natural rate proxies like those discussed in Appendix B. In untabulated results we confirm that the *convergence* gap seems to contain fundamentally different information relative to measures of *output* gap. In particular, we find that various output gap measures have no ability to forecast bond excess returns, both when used in isolation (a result already present in Duffee, 2013b) and joint with forward rates.

convergence gap has been on average positive at about 1% throughout the sample, but with a relatively large volatility of 2.0%. The autocorrelation of about 0.96 at monthly frequency decays rapidly to 0.87 at the quarterly horizon, and is 0.57 at the annual horizon. The convergence gap was particularly negative during the high-inflationary period of the early 1980s, and positive in the early years of the 1970s, 1990s, and 2000s. Contrary to other series widely used in the literature (see, e.g. Cieslak and Povala, 2015), CG does not exhibit marked trending patterns. Thus our analysis is different from, and complementary to, the term structure literature with shifting endpoints (see Kozicki and Tinsley, 2001; Bauer and Rudebusch, 2017).

We now turn to the ability of the convergence gap to provide information about the path of future yields.

2.3 Fama–Bliss regressions and the Convergence Gap

Panel A of Table 2 reports the results from regressions (5) (leftmost part) and (6) (rightmost part) when, consistent with the framework in Section 2.1, we use as predictor x_t the component of the convergence gap that is orthogonal to the forward spread, denoted $\mathrm{CG}_t^{\perp}$. Panel B collects analogous results when the forward spread is added to the convergence gap. Panel A shows that the convergence gap has an impressive ability to forecast future changes in yields of different maturities: the R^2s are all above 10%, and the coefficients are strongly significant with t-statistic greater than three. The predictive ability of the convergence gap is decreasing in the maturity of the bond. This is intuitive since the effect of monetary policy is likely to be stronger at the short-end of the yield curve. Finally, the ability of $\mathrm{CG}_t^{\perp}$ to forecast interest rates is mirrored by its ability to forecast bond excess returns. Importantly, the sign of the coefficients is in line with the economic intuition: a positive convergence gap predicts an increase in short- and long-term rates ($b_{y,CG^{\perp}} > 0$) and, at the same time, lower prices (hence, returns) going forward ($b_{r,CG^{\perp}} < 0$).

By construction, the R^2 attained by the convergence gap in forecasting excess returns adds to the R^2 achieved by forward rates in standard Fama-Bliss regressions (c.f. Table 1). The leftmost part in Panel B shows this result: the convergence gap raises the R^2 for bonds with 2- and 3-year maturity by about 15%. Alternatively, forward rates explain about 25% of the variability in bond risk premia, once the effect of time-varying expected yield changes is properly controlled for by the convergence gap. The rightmost part of Panel B is also useful for interpreting the result: the CG_t can only raise the contribution of expected returns to forward spread variation. The convergence gap (and any other additional variables) does not shift variance attribution from returns to interest rate changes. This is why the R^2 in the rightmost part of Panel A and B are identical.[13]

2.4 Predicting yields with the Convergence Gap

As another way of tracing the forecasting ability of the convergence gap, we follow Duffee (2002a) and look at the RMSE in one-year yield predictions. Specifically, we run predictive regressions of changes in one-year yield at forecast horizons of $H = \{1, 2, 3, 4\}$ years using four different models. We collect the corresponding estimates in Table 3.

In the first model, labeled random walk ("RW"), the best forecast for the H-period ahead one-year yield is its current value. We take this model as our benchmark. The associated RMSE, reported in the second column of the table, is 1.37% for $H = 1$ and increases to 2.48% at the four-year horizon.

The second model is based on in-sample forecasts of one-year yield changes using the slope $s_t^{(5)} = y_t^{(5)} - y_t^{(1)}$. This specification generates RMSEs ranging from 1.41% for $H = 1$ to 2.21% for $H = 4$, a relatively modest improvement upon the RW model. These results are in line with those in Duffee (2002a), who shows that the random walk model is a hard-to-beat benchmark for yield predictions.

The third model, reported in the fourth column, forecasts one-year yield changes using

[13] Appendix B, with reference to Tables B.1, B.2, B.3, and B.4 show that our results are robust to alternative formulation of the convergence gap, such as changing the measure of the equilibrium rate of interest, or using alternative proxies for the real short-rate.

the convergence gap. Compared with the model based on the slope, the resultant RMSE are now smaller at 1.32% for $H = 1$ and 2.27% for $H = 4$. Thus, conditioning on CG reduces the RMSE by about 4% (5 basis points) to 10% (23 basis points) compared to the standard random walk model.

Finally, the fourth model predicts yield changes with both the slope and CG. With respect to the previous model, the RMSEs decrease further by as much as 11 basis points, the sole exception being at the one-year horizon where the performance is at par. This result further confirms the evidence that conditioning on the gap between the level of the real (short-term) interest rate and the equilibrium real rate brings additional forecasting power.

To gauge the statistical significance of these improvements in RMSE, we report in parenthesis the p-value of the Diebold and Mariano (1995) test against the RW model. At the 3-year and 4-year forecasting horizon, we can reject the null hypothesis of equality in RMSE for all models incorporating the convergence gap at the 10% significance level or better.

To summarize, we have established that CG provides information on the path of future yields, and, thus, it helps forward rates to predict bond returns. We did so by investigating maturity-by-maturity Fama-Bliss restrictions on right hand side variables. Next, we explore restrictions *across* maturities, and we investigate what are the effects of the convergence gap on the one-factor structure of expected returns first highlighted by Cochrane and Piazzesi (2005). In particular, we study ciclicality and economic significance of a factor structure in expected returns that accounts for CG_t.

3 Bond risk premia

This section collects the analysis on the effect of adding the convergence gap in the context of bond return predictability. In Section 3.1, we focus on the predictive regression of average excess returns and construct our bond risk factor. In Section 3.2, we look at the predictive regressions of individual bond excess returns and contrast our factor with competing approaches. Section 3.3 discusses several robustness checks. Finally, Section 3.4 provides evidence that the role played by the convergence gap for bond returns predictability is not limited to the U.S. but it holds also in an international setting.

3.1 Bond Risk Factors

We first analyze the role of the convergence gap in forecasting average (across maturities) one-year bond excess returns $\overline{rx}_{t+1}$. Panel A of Table 4 reports the results for various set of predictors during the 1964-2017 sample period. In specification (1), the regressors are the five forward rates as in Cochrane and Piazzesi (2005), that is we estimate regression:

$$\overline{rx}_{t+1} = \delta_0 + \delta_1' \boldsymbol{f}_t + \epsilon_{t+1} \ , \tag{7}$$

where $\boldsymbol{f}_t = [f_t^{(1)}\ f_t^{(2)}\ f_t^{(3)}\ f_t^{(4)}\ f_t^{(5)}]$. Collectively, the forward rates capture 23% of the overall variance in future excess returns over 1964-2017. We denote the fitted value from this regression as the "CP" factor, i.e. $\text{CP}_t = \widehat{\delta}_0 + \widehat{\delta}_1' \boldsymbol{f}_t$.

In specification (2), we predict $\overline{rx}_{t+1}$ with the convergence gap CG_t. The corresponding coefficient is negative at −0.40, and statistically significant with a t-statistic of −2.75 and an R^2 of about 0.06.

Consistent with the analysis in Section 2.3 and 2.4, the importance of the gap becomes more prominent when it is used as conditioning variable together with forward rates. In specification (3), we employ both forward rates and CG_t as predictors, that is we estimate regression:

$$\overline{rx}_{t+1} = \delta_0^{CG} + \delta_1^{CG\,\prime} \boldsymbol{f}_t + \delta_2^{CG} \text{CG}_t + \epsilon_{t+1}^{CG} \tag{8}$$

Several noteworthy facts emerge. First, the coefficient on CG_t now doubles at −0.79 with an associated t-statistic of −4.28. Second, its inclusion reduces the estimation error of forward rates coefficients, as documented by their increased t-statistics. Third, the associated R^2 jumps to 0.34, a nearly 50% increase with respect to the specification without the gap. The negative sign on CG_t means that a positive convergence gap reduces the expectations about future returns, everything else constant. This is due to the fact that periods when real yields are below their long-term convergence level are usually associated to expectations of increased future yields.[14] We denote the fitted value from regression (8) as the "CPG factor", i.e. $\text{CPG}_t = \widehat{\delta}_0^{CG} + \widehat{\delta}_1^{CG\,\prime} \boldsymbol{f}_t + \widehat{\delta}_2^{CG} \text{CG}_t$.

In Panel B of Figure 1, we plot the estimated loadings on forward rates from specifications (1) and (3), namely $\widehat{\delta}_1$ and $\widehat{\delta}_1^{CG}$. It is interesting to notice that, for the 1964-2017 sample considered, the coefficients from specification (1) do not have the symmetric pattern documented by Cochrane and Piazzesi (2005) over 1964-2003. Interestingly, however, conditioning forward rates on the gap makes the loadings on the former quite aligned with those from Cochrane and Piazzesi (2005), and thus more stable over time.[15]

In Panel C of Figure 1, we display the time-series of the CP and CPG factors. In the first part of the sample, which was characterized by relatively low yields, CPG tends to be lower on average than CP thus forecasting lower excess returns. The opposite is true for the late 1990s and 2000s. Some notable differences are also seen in the 2000s. The period between 2002 and 2007 is often referred to as the interest rate "conundrum" (see Greenspan, 2005), in which the increase in short-term federal fund rates did not translate into higher long-maturity yields partly because of strong demand from foreign savings. During this period, the yield curve was flat to downward sloping and the corresponding negative CP factor forecasted low or even negative bond returns.

As another way of evaluating the effect of conditioning on the gap, Panel B of Table 4 reports the results when predicting the residuals from specifications (1) and (3) of Panel A using two combinations of the following variables: inflation (CPI), Industrial Production

[14] The R-squared for the baseline regression with forward rates only in specification (1) is lower than that reported by Cochrane and Piazzesi (2005) over 1964-2003. This is due to a decline in the predictive ability of forward rates over the recent sample, which was characterized by short-term nominal interest rates close to zero and unconventional monetary policies. The importance of CG, however, is not confined to the zero lower bound (ZLB) period. Indeed, conditioning forward rates on CG over 1964/01-2008/11 brings the R-squared from 29% to 36%.

[15] We draw similar conclusions working on rolling windows, namely, the coefficients in specification (3) tend to be more stable and statistically significant than those from specification (1).

growth (IP), the Chicago Fed National Activity Index (CFNAI), and the NBER recession dummy. In the first row, we see that the CPI, IP, and NBER dummy variables capture about 11% of the variability in the residuals from specification (1), which includes forward rates only. In the second row, adding the CFNAI index in place of IP further increase the R^2 to 12%. Even more importantly, the CPI and CFNAI enter the regression with negative and significant coefficients, while the loading on NBER is positive and also significant. Hence, not only do forward rates leave some information on the table when predicting bond returns, but also such information is linked to macro conditions in a countercyclical way – i.e., bond return forecast errors are positive during recessions. This point has previously been made by Ludvigson and Ng (2009).

In the third and fourth row of the panel, we repeat these regressions on the residuals from specification (3) of Panel A, which includes forward rates and the gap. We note that the R^2 drops by about 5%. Moreover, only the NBER dummy now does enter with a statistically significant loading, but its coefficient is about half that from the first two rows. We conclude that conditioning on the gap helps capturing predictable patterns in bond risk premia that are countercyclical in nature.

In order to better trace the improvement in (in-sample) predictability stemming from CG, we look at how the difference in R-squared between specifications (1) and (3) of Panel A accrues throughout the sample. To this end, Panel A in Figure 2 displays the difference in the average squared residuals from the two specifications scaled by the variance of the dependent variable, and divided by $T-1$. That is, at each month t we display $\frac{(\widehat{\epsilon}_{t+1})^2-(\widehat{\epsilon}^{CG}_{t+1})^2}{(T-1)Var(rx_{t+1})}$. Positive values for this series indicate that the model conditioning on CG delivers lower forecast errors than those obtained using forward rates only. We observe that several positive stream occur during months that are marked as recessions by the NBER, which are presumably periods characterized by aggressive monetary policy interventions. This evidence clarifies why conditioning on CG makes bond risk premia more countercyclical.

In Panel B of Figure 2, we report the cumulative sum of the previous series. By construction, this series ends at 0.11, which is the difference between the 0.34 R^2 of specification (3) and the 0.23 R^2 of specification (1) (c.f. Table 4–Panel A). As we can see, few rather prolonged periods stand out as being characterized by an almost steady improvement in R-squared, namely 1977 to 1984 (characterized by the turmoil in the Treasury market following the Volcker's experiment), the early 2000 recession, and the financial crisis starting in mid 2007. We take this pattern as rather reassuring that the value-added of the gap is rather pervasive and not concentrated in isolated events.

3.2 Individual bond regressions

We now turn our attention to individual bonds. Panel A through D of Table 5 report the in-sample results for the predictive regression of bond excess returns with maturities ranging from two to five years, respectively, on a set of regressors X_t:

$$rx_{t+1}^{(N)} = a_N + b_N' X_t + \epsilon_{t+1} \qquad N = \{2, 3, 4, 5\} \tag{9}$$

Within each panel, we consider various combinations of X_t in order to highlight the impact of our novel bond risk factor, CPG_t.

The predictive ability of the Cochrane and Piazzesi (2005) factor is shown in specification (1). Its loading is 0.43 for two-year returns (t-statistic of 6.46) with an R^2 of about 0.19. The coefficient increases almost linearly with maturity, reaching 1.49 for five-year returns, with R^2s in the 0.22 to 0.25 range.

In specification (2), we forecast bond returns using both forward rates and the convergence gap as summarized by the CPG factor. The corresponding coefficients are comparable to those of CP but are characterized by a much stronger statistical significance and predictive power. For two-year bonds, the coefficient on CPG is 0.46 with an associated t-statistic of 9.51 and the R^2 equals 0.32 – a full 0.13 increase with respect to specification (1). Similar conclusions arise across all other maturities, with R^2s for three-, four-, and five-year bonds all above 0.30.

We next further control for Ludvigson and Ng (2009)'s $F5$ macro factor which is obtained as a linear combination (from a subset) of the first eight principal components formed from more than 130 macroeconomic and financial time series.[16] As expected, this variable enters the regression with a positive and strongly significant coefficient when combined with CP in specification (3). Interestingly, the corresponding R-squared is almost exactly at par with that from specification (2). It is thus natural to ask whether the convergence gap merely captures macroeconomic conditions that are already contained in $F5$. To this end, specification (4) includes both CPG and $F5$ as predictors. Both variables are significant predictors of bond excess returns across all maturities. More importantly, the R^2 of this model is some 5–7% higher than that of specification (3), which confirms that controlling for the convergence gap increases the forecasting ability of forward rates over and beyond macroeconomic risk.

3.3 Additional Analysis

We conduct a battery of checks to verify that our results are robust to various definitions of the convergence gap, sampling frequency, and other concerns.

Alternative Measures of Convergence Gap: In Tables B.1-B.4, we experiment several alternatives to the construction of CG. Details are provided in Appendix B.

Specifically, in Table B.1 and B.2 we use alternative measures of the natural rate of interest. In Table B.1, we replace the one-sided HP filtered trend component of real GDP with the potential GDP series obtained by Laubach and Williams (2003) using a Kalman filter. In Table B.2, we instead replace the one-sided HP filtered trend component of real GDP with the Kalman filter natural-rate estimates – denoted r_t^* – by Laubach and Williams (2003). The Laubach and Williams (2003) natural rate of interest is composed of the trend growth rate of the natural rate of output and a component that captures the households'

[16]We kindly thank Sydney Ludvigson for making the principal components available on her web site.

rate of time preference as well as other determinants of r^* unrelated to trend growth.

In Table B.3 we follow Laubach and Williams (2003) and use the forecast of the twelve-month-ahead percentage change in core PCE generated from a univariate AR(9) estimated over the prior 120 months as a proxy for inflation expectations in constructing the ex ante real interest rate.

Survey data constitute an appealing alternative to model-based expectations. They also represent a natural real-time measure of investors' expectations.[17] In Table B.4 we use CPI inflation forecasts from the Survey of Professional Forecasters (SPF) to construct the real rate from 1981 onwards, and we splice it with a four quarter moving average of past CPI inflation. We use such proxy for inflation expectations in constructing the ex ante real interest rate.

All these checks generally produce comparable, or even stronger, results than those reported in Table 2.

Alternative sampling frequencies and vintage data: In Panel A of Table 6, we report the results for average excess returns $\overline{rx}_{t+1}$ when data are sampled at the quarterly and annual frequency. The quarterly series does not require GDP data to be interpolated in the construction of CG. The yearly frequency addresses econometric concerns arising from the use of overlapping returns by relying on non-overlapping observations, at the cost of a much smaller sample size.[18] For comparison, we report in the table the R-squared (R^2_{fwd}) when using forward rates only as regressors. In both specifications, the coefficient on CG is again negative at about −1, and statistically significant with t-statistics below −4. The associated R^2s confirms that forward rates conditional on the gap capture substantial incremental time-variation in bond risk premia.

In Panel B of the table, we report analogous results when constructing the convergence

[17]In the fixed-income literature, Chun (2011) includes survey expectations in the estimation of an arbitrage-free affine term structure model, and shows that GDP growth forecasts play a crucial role in tracking bond risk premia.

[18]For the quarterly and annual regressions we set the truncation parameter in the Newey-West estimator to 18 lags. This delivers a ratio of bandwidth to sample size of about 0.09 and 0.34, respectively. Using these ratios, we recompute the p-values using the asymptotic theory of Kiefer and Vogelsang (2005).

gap using the actual vintage data for GDP and inflation. Ghysels et al. (2018) provide evidence that bond risk premia predictability using macroeconomic information is largely attenuated when taking into account the actual data release. Vintage series for inflation start being available in 1996. Starting on January 1996, we construct CG based on recursive estimations that use the vintage data available until that calendar quarter end.[19] Panel B reports the corresponding results. We note that the coefficient on CG remains negative and significant, although its magnitude is smaller than that in Panel A. The increments in R-squared compared to the forward-only specification remain large.

Forecasting Monthly Returns: Following Duffee (2011), we also predict monthly excess returns. The source is the CRSP Fama Bond Portfolio Returns tape. Panel C reports the estimates for the portfolio of bonds with two to three years to maturity over 1964-2017. In the first row, the predictors are the five forward rates, and none of them enters the regression with a significant coefficient.[20] In the second row, we add CG. The coefficient on CG is negative at -0.05 and statistically significant at the 10% level. The R^2 also increases from 0.02 to 0.03, which is a sizeable effect considering these are monthly regressions. Finally, specification (3) adds Duffee (2011)'s hidden factor, which is available until the end of 2007. This variable enters the regression with a positive and significant loading, and doubles the R^2. It does not, however, reduces the importance of the convergence gap, whose coefficient and t-statistic actually increase with respect to specification (2). We conclude that the gap also helps in predicting short-horizon returns.

[19] To fix ideas, on a given quarter, say 1998:Q3, we construct the convergence gap using the most recent GDP release as of the end of that calendar quarter (that is, September 1998), which refers to real GDP in the previous quarter (namely, 1998:Q2). For inflation, we take the one-year moving average of the vintage data, which is again lagged by one month and ignores subsequent revisions. We then move forward by one quarter, and add the last observation for CG (1998:Q4) to the series obtained using data until 1998:Q3.

[20] Also, the loadings on forward rates are not characterized by the symmetric tent shape noticed for annual returns. As documented in Duffee (2011), we verify that the CP factor is indeed no longer statistically significant for predicting monthly returns. This suggests that the tent-shaped restriction of forward rates captures mostly a low-frequency component of bond risk premia.

3.4 International evidence

So far, our analysis has relied on Fama-Bliss yield data for the U.S. Treasury market. One would expect the importance of the gap to extend also to other countries whose central banks follow explicit targeting policies.

We investigate whether our results hold internationally by estimating the regression on future average excess returns for three other countries, namely Canada, the UK, and Germany. We take the corresponding one- to five-year artificial zero coupon bond yields at the monthly frequency from the Bank of Canada, Bank of England, and Bundesbank, respectively. We construct the convergence gap as the difference between the natural rate of interest and the real interest rate. To proxy for the natural rate of interest we use the estimates from Holston et al. (2017). The short-term interest rate is: the Bank of Canada's target for the overnight rate for Canada; the Bank of England's Official Lending Rate, published by the Bank of England, for the U.K.; and the three-month rate from the Area Wide Model (Fagan et al., 2001) for Germany (Eurozone from 1999 onward). For all countries, the inflation series is constructed by splicing the core price index with an all-items price index.[21]

Table 7 collects the slope coefficients from predicting annual average bond returns for these countries using the same format as Panel A of Table 4. In specification (1), we observe that forward rates alone explain 17% of bond return variability in Canada and the UK, and as much as 28% in Germany. In specification (2), we note that the convergence gap alone enters the regression with a negative coefficient. However, it does not meet statistical significance and the associated R-squared is quite modest. As it was the case for the U.S., including both forward rates and CG leads to a sharp increase in goodness of fit and statistical significance. Indeed, the gap enters the regression with a significantly (at the 5% level or better) negative coefficient in all three countries. The in-sample R-squared increases by 30% (from 0.28 to

[21] As above, we use a four-quarter moving average of past inflation as a proxy for inflation expectations in constructing the ex ante real interest rate. Unlike the U.S., however, we note that the CG for these countries exhibits a discernible trend. We therefore use a linearly detrended convergence gap in the regressions to prevent this trend from contaminating our results.

0.36) for Germany, by 50% (from 0.17 to 0.26) for the U.K., and by 100% (from 0.17 to 0.34) for Canada.

Overall, we take the evidence that the convergence gap enhances return predictability in countries (and over different sample periods) other than the U.S. as rather reassuring of the robustness of our findings.

4 Economic Significance of Bond Predictability

Does the statistical significance of our results also translate into economic significance? We address this question by looking at the impact of our conditioning variables in the context of a dynamic portfolio strategy. Specifically, we consider the optimization problem of a quadratic utility agent who allocates her funds between a risk-free investment and a risky asset. In the fixed income literature, this approach has been previously applied by Della Corte et al. (2008) to quantify the economic significance of violations of the Expectations Hypothesis at the short-end of the maturity spectrum using daily data. Our focus, instead, is on the performance of portfolio allocation to long-term bonds at monthly and annual horizons. In our analysis, the risk-free asset is a bond with maturity equal to the investment horizon. For monthly holding-period returns, the risky asset is the monthly series of a portfolio of bonds with two to three years to maturity (the same we used in the last part of Section 3.3). For one-year holding period returns, we use the average excess return $\overline{rx}_{t+1}$, representing the return to an equally weighted portfolio of two- to five-year maturity bonds, sampled at annual frequency.

To determine the optimal conditional allocation strategy, we adopt the parametric portfolio choice approach of Brandt and Santa-Clara (2006). In their setup, the time-varying vector of relative weights w_t allocated to N risky assets is expressed as a linear function of K conditioning variables z_t, or $w_t = \theta' z_t$. In the case considered here, $N = 1$ as the only risky asset is the bond portfolio. The $K \times 1$ vector of parameters θ, to be estimated, captures the marginal impact of each variable on the portfolio weight. Brandt and Santa-Clara (2006) show that the dynamic optimization problem can be reduced to solving the static problem:

$$\max_{\theta} E_t \left[\theta' \widetilde{r}_{t+1} - \frac{\gamma}{2} \theta' \widetilde{r}_{t+1} \widetilde{r}'_{t+1} \theta \right] \tag{10}$$

where γ is a risk aversion parameter and $\widetilde{r}_{t+1} \equiv z_t \otimes r_{t+1}$ replaces the base asset with managed portfolios. Following their work, we set $\gamma = 5$. In practice, the estimates of θ are obtained by OLS in the regression of a constant term on $\widetilde{r}_{t+1}$, which allows us to use standard testing procedures for evaluating statistical significance.[22]

The smallest conditioning set is $z_t = 1$, which corresponds to the static Markowitz portfolio choice problem. The conditioning variables we include next are standardized to have mean zero and unity standard deviation, so that the coefficient on the constant can be interpreted as the time-series average allocation in the risky asset. A positive coefficient is associated to variables which either forecast higher expected returns, lower volatility, or both. The opposite is true for variables entering with a negative θ. The economic impact of the variables in z_t is then summarized by the Sharpe Ratio of the resultant optimal portfolio, and by the corresponding equalization fee, defined as the annual fee an investor is willing to pay to have access to z_t.[23]

The first specification of Table 8 reports the estimates for the two-asset unconditional portfolio allocation, or $z_t = 1$ ($K = 1$). At monthly horizon, the average return ($E(r_p)$) and standard deviation (σ_p) of the resultant optimal portfolio are about 0.07, leading to a Sharpe Ratio of 0.33. These statistics are comparable at annual frequency.

In specification (2) of the Table, the portfolio allocation is conditioned on the one- to five-year forward rates, or $z_t = [1\ \boldsymbol{f}_t]'$ ($K = 6$). The average return and standard deviation

[22]We work within a constant volatility setting since Thornton and Valente (2012) provide evidence that models with time-varying volatility do not yield significant improvement in economic value relative to the constant volatility alternative. Similarly, Duffee (2002b) and Cheridito et al. (2007) find that bond excess returns are best captured by constant volatility models, in spite of the fact that such models cannot match the time-series variation in interest rate volatility.

[23]Similar results, omitted for brevity, obtain if we use Modigliani and Modigliani (1997) performance measure.

of the managed portfolio both increase to about 0.10, with a corresponding Sharpe Ratio of about 0.50. However, none of the forward rates coefficients meets statistical significance at the monthly horizon and the p-value for the test that they all equal zero (F-test row) is 0.19 at the monthly horizon. The dynamic strategy yields an equalization fee of 1.2% at the monthly horizon, and a somewhat higher 2% fee at the annual frequency.

In specification (3), the set of conditioning variables is augmented by the convergence gap, or $z_t = [1 \, \boldsymbol{f}_t \, \mathrm{CG}_t]'$ ($K = 7$). The coefficient on CG is negative and statistically significant at both the monthly (-2.974, t-statistic of -1.9) and annual horizon (-2.015, t-statistic of -3.3). The negative sign is consistent with the evidence in the previous sections. The inclusion of CG changes substantially the loadings on forward rates and the performance of the allocation. The in-sample Sharpe Ratio raises to about 0.60 at both horizons and the equalization fee is now about 1% higher than in specification (2).

In Figure 3, we display the time-varying weight w_t implied by the monthly (top plot) and annual (bottom plot) estimates of Table 8. In the plots, the horizontal solid line represents the unconditional allocation corresponding to specification (1), the red solid line with circles corresponds to the policy that conditions on forward rates only as in specification (2), and the blue solid line tracks the portfolio weight implied by specification (3) where we condition on forward rates and the convergence gap. Several differences between the two dynamic strategies emerge. The correlation between the two weights is just 0.41 at the monthly horizon and 0.47 at the annual horizon. Both optimal policies, however, often imply taking substantial short ($w_t < 0$) or leveraged ($w_t > 1$) positions. To study their impact on performance, we follow Della Corte et al. (2008) and winsorize the weight between -1 and 2. The equalization fee from this constrained policy, reported in the last row of Table 8, is again significantly larger when CG is included, confirming that its economic relevance does not arise from taking extreme positions. As an additional check, we also found the performance measures to be quantitatively very similar or in some cases even stronger when looking at the portfolio choice of a power utility investor (not shown). In sum, after accounting for the monetary policy stance as proxied by the convergence gap, we find the time-varying component of bond risk premia to play an economically prominent role in dynamic portfolio choice. Our findings also show that departures from the Expectations Hypothesis at long maturities are indeed economically relevant.

5 Out-of-Sample Analysis

We finally conduct a recursive experiment to investigate whether our results also hold in an out-of-sample environment.[24] We consider as a burn-in sample the first 25-year period from 1964:1 to 1989:12. Using only information until the end of this period, we construct the CP and CPG factors following the methodology described in Section 3.1. Next, we regress each individual bond excess return on the lagged factors to determine their individual loadings, similarly to what reported in Table 5 for the full sample. Due to the predictive nature of the regression, the last observation in the right-hand-side variables is that of December 1988. We use the resultant coefficients and the value of the CP and CPG factors on December 1989 to produce out-of-sample forecasts of one-year excess returns for each maturity. The first forecast error obtains by comparing the excess holding period return during the January 1990 through December 1990 period and its forecast made on December 1989. We then include the January 1990 information and follow the same procedure to produce forecasts of the February 1990 through January 1991 returns, and so on until the end of 2017. Since the GDP information is available only on a quarterly basis, we keep the filtered permanent growth GDP component constant throughout the three months following a quarter's end. However, we make use of the $F5$ factor constructed on the whole sample period.[25]

In Table 9, we summarize the results of this exercise for the four individual bond returns.

[24] Out-of-sample tests are usually viewed as important tools to detect spurious, sample-specific evidence. However, as Cochrane (2008) points out poor out-of-sample predictability may arise even when the true data-generating-process is characterized by time-varying, persistent risk premia.

[25] This is likely to put our factors in an unfavorable position as the Ludvigson and Ng (2009) macro factor would make use of future information. Thus, the comparison with $F5$ should be taken as conservative.

We contrast the forecasting accuracy of Model 1, which includes the convergence gap, with that of Model 2, which excludes it. Both models incorporate a constant term. The ratio of the mean squared forecast errors $MSFE_1/MSFE_2$, reported in the third column, tells us whether the model with CG features lower (ratios less than one) or higher (ratios above one) forecast errors than the competing model. To gauge statistical significance, the column "DM test" reports the p-values for the Diebold and Mariano (1995) test on the difference in MSFEs.

In the first row of each panel, Model 2 consists of the CP factor, and, thus, it exploits information in forward rates only. Adding CG in the conditioning set leads to a surge in out-of-sample prediction accuracy across all bonds, with MSFEs declining by about 25%. This improvement is economically large, and strongly statistically significant.

In the second row, we contrast the model with CPG with a model that combines CP with the Ludvigson and Ng (2009)'s macro factor. The two models deliver comparable forecast accuracy. The ratio of MSFEs is above one for 2-year and 3-year bonds, and below one for longer maturity bonds. None of the differences in performance, however, is statistically significant.

Finally, the third row adds the $F5$ macro factor to either CPG or CP. We see that conditioning on the convergence gap improves the forecasting ability of forward rates in an out-of-sample fashion even controlling for macroeconomic conditions. Indeed, the inclusion of CG reduces the MSFE by about 10% for 2-year bond, and nearly 20% for longer maturities. This result clarifies that CG provides complementary information on bond risk premia compared to that in $F5$.

In order to further explore the provenance of these results, we investigate whether these improvements arise from a reduction in the bias or in the variance of the forecast errors through Ashley et al. (1980)'s test.[26] The p-values of these tests are reported in columns

[26]Let $\widehat{e}_{1t}$ and $\widehat{e}_{2t}$ denote the forecast errors for Model 1 and 2, respectively. Define $\Delta_t = \widehat{e}_{1,t} - \widehat{e}_{2,t}$, $\Sigma_t = \widehat{e}_{1,t} + \widehat{e}_{2,t}$ and $\overline{\Sigma}$ its time-series average. We estimate regression: $\Delta_t = \beta_0 + \beta_1(\Sigma_t - \overline{\Sigma}) + u_t$. The t-statistic for β_0 measures the bias improvement of Model 1 versus Model 2, while the t-statistic for β_1 captures reduction in the error variance. See Berardi and Torous (2005) for a paper using this test in term

“Bias” and ”Variance”, respectively. It is clear that the improvement from adding CG to the model comes from a reduction in the bias of the forecast errors. The variance reduction is, in contrast, not statistically significant at conventional levels.

6 Concluding Remarks

The Fama-Bliss identity implies that forward rates act as sufficient statistics for future returns in the absence of predictable movements in yields. In contrast, time-varying expectations of yields masquerade the predictive ability of forward rates and may bias the forward rates coefficient. Whereas most of the existing literature has been focusing on identifying factors that capture time-variation in risk premia which is missed out by forward rates, the alternative of including variables that filter forward rates from predictable components in yield changes has received comparably little attention.

In this paper, we propose one such variable: the difference between the natural rate of interest and the ex-ante real fed funds rate. We dub this variable Convergence Gap (CG). We provide evidence that periods of positive convergence gap are generally associated with an increase in future yields. As a consequence, conditioning on CG helps to identify if an upward sloping yield curve and increasing forward rates should be associated with expectations of increasing future yields or elevated risk premia.

A linear combination of forward rates and the convergence gap explains 34% of the variability in average and individual bond returns during the 1964-2017 period. This is remarkable since forward rates have been quite unstable in predicting bond returns over the past two decades. Even more important is the fact that risk premia solely based on forward rates seem to miss cyclical patterns, as also documented by Ludvigson and Ng (2009). On the contrary, after controlling for the CG factor, the resultant forecast errors are virtually uncorrelated with business cycle proxies. This implies that predictable yield changes masquerade the countercyclical behavior of bond risk premia embedded in forward

structure modeling.

rates. Our factor combining forward rates and the convergence gap remains by far the dominant forecasting element of bond returns when confronted with (other) macroeconomic and financial variables, as summarized by Ludvigson and Ng (2009) factor, and by hidden components of yields as in Duffee (2011). We also show that these results do not arise from spurious in-sample overfitting, as the combined factor brings substantial out-of-sample gains.

Overall our results emphasize the necessity to model the interaction between monetary policy and asset prices when studying countercyclical risk premia.

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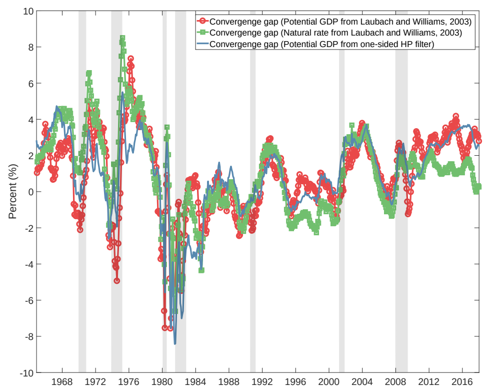
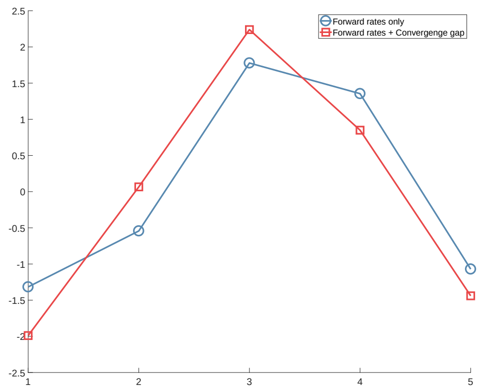
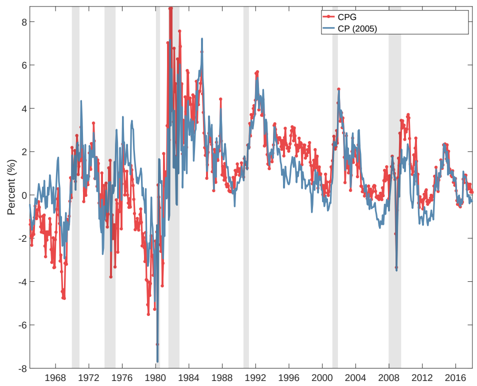


A: Convergence Gap, CG_t

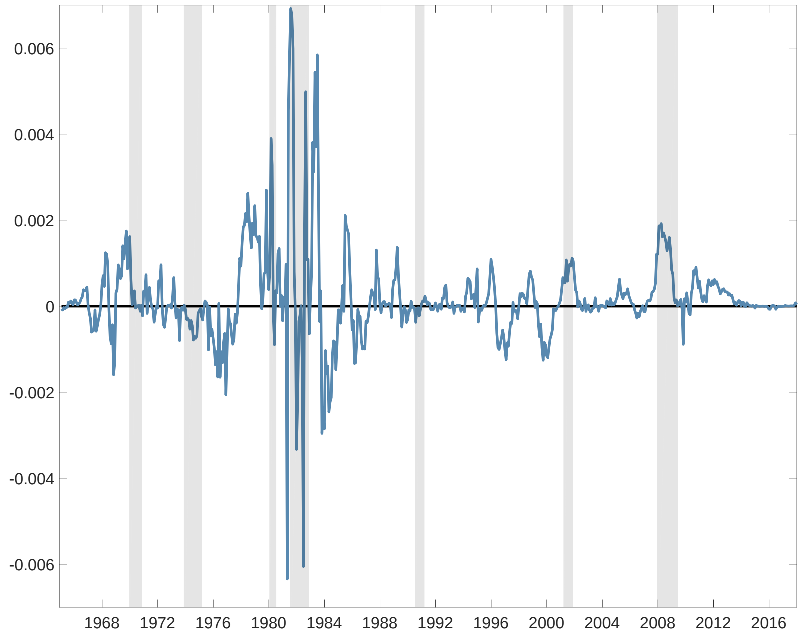


B: Regression coefficients on forward rates

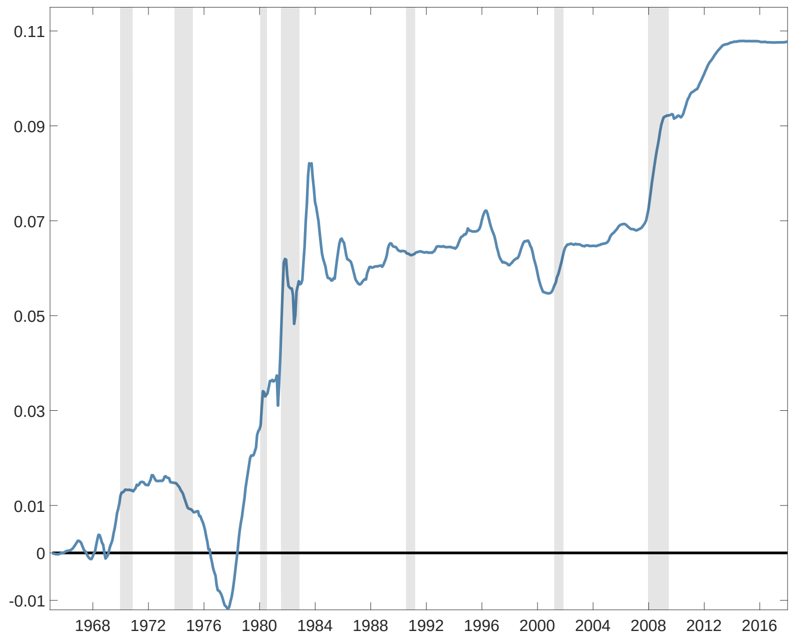


C: Bond risk factors

FIGURE 1: **Estimating the return forecasting factor**: This figure plots the time series of the convergence gap, CG (Panel A), the unrestricted coefficients from a regression of bond excess returns on all forward rates, and from a regression of bond excess returns on all forward rates and the convergence gap (Panel B), and the bond risk factors (Panel C), namely the Cochrane and Piazzesi (2005) factor CP (solid line) and the CPG factor obtained by conditioning the forward rates on the convergence gap (dotted line). The sample period is 1964/01 to 2017/12.

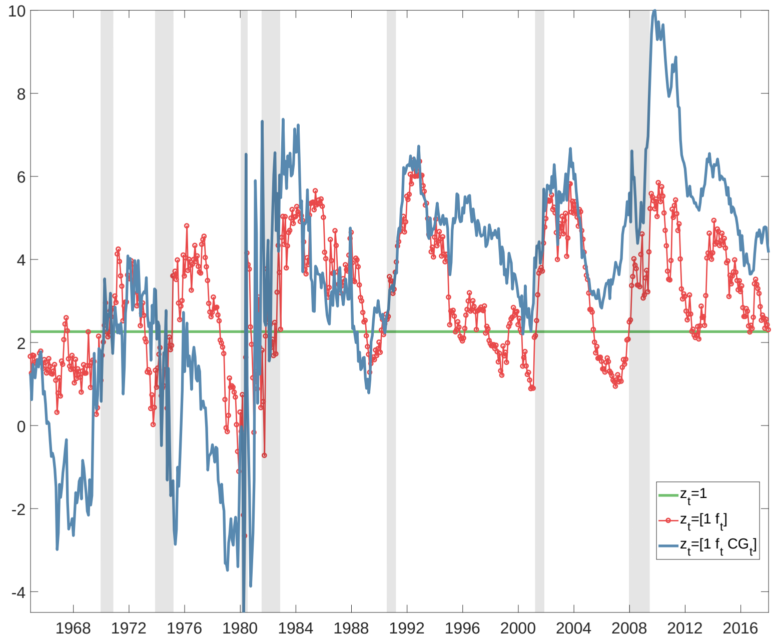


A: Scaled difference in squared residuals

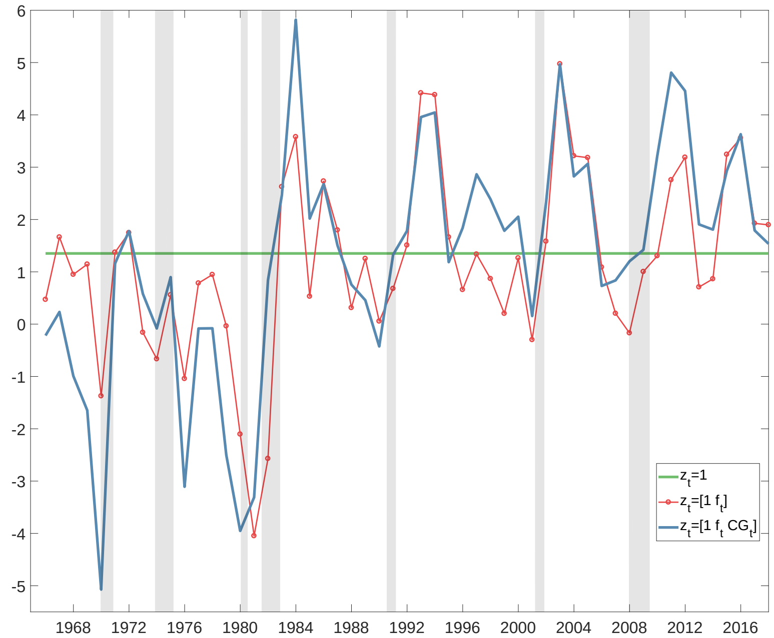


B: Cumulative scaled difference in squared residuals

FIGURE 2: **Squared forecast errors with and without CG**: Panel A of this figure plots the time series of the difference in squared residuals from Eq. (7) (whose estimates are reported in specification (1) of Table 4, Panel A) and Eq. (8) (whose estimates are reported in specification (3) of Table 4, Panel A), scaled by the variance of the dependent variable times $T-1$. That is, at each month t we display $\frac{(\widehat{\epsilon}_{t+1})^2-(\widehat{\epsilon}^{CG}_{t+1})^2}{(T-1)Var(rx_{t+1})}$. Panel B reports the cumulative sum of this series. The sample period is 1964/01 to 2017/12.



A: Monthly horizon



B: Annual horizon

FIGURE 3: **Portfolio weight on bond portfolio**: Time-series of the weight in the risky asset (bond portfolio) implied by the estimates of Table 8 at the monthly (top panel) and annual (bottom panel) horizon. The green solid line represents the unconditional allocation (specification (1) of Table 8), the solid line with circles corresponds to the policy conditional of forward rates (specification (2)), and the thick solid line tracks the portfolio weight implied by specification (3) which conditions on the forward rates and the convergence gap CG_t. The sample period is 1964/01 to 2017/12.

Table 1: **Fama-Bliss (1987) Regressions: 1964-2017**

The leftmost panel displays results for forecasting 1-year ahead excess returns on n-year bonds. The rightmost panel displays results for forecasting the 1-year ahead change in the $(n-1)$-year yield. In parentheses below the estimates we report t-statistics based on Newey and West (1987) standard errors with 60 lags. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory. The intercept estimates are omitted. The sample period is 1964/01 to 2017/12.

Maturity	$rx_{t+1}^{(n)} = a_r + b_r \left(f_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+1}$		$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_y \left(f_t^{(n)} - y_t^{(1)}\right) + u_{t+1}$	
$n =$	b_r	R^2	b_y	R^2
2	0.83***	0.12	0.17	0.01
	(4.57)		(0.92)	
3	1.13***	0.14	-0.13	0.00
	(5.21)		(-0.61)	
4	1.36***	0.16	-0.36	0.01
	(5.69)		(-1.50)	
5	1.12***	0.09	-0.12	0.00
	(4.25)		(-0.45)	

Table 2: **Forecasting regressions with Convergence Gap and Forward Rates**
This table reports regressions in the spirit of Fama and Bliss (1987). Panel A reports OLS univariate regressions using the Convergenge Gap orthogonalized with respect to the forward spread $f_t^{(n)} - y_t^{(1)}$, denoted $CG_t^{\perp}$. Panel B reports multiple regressions using the forward spread and the (orthogonalized) Convergenge Gap. The convergence gap, CG_t, is defined as the difference between the year-to-year log change in potential GDP and the real interest rate. To proxy for potential GDP we use the trend component of quarterly real GDP obtained from a one-sided Hodrick and Prescott (1997) filter. We linearly interpolate the resultant trend series to obtain monthly observations. The short-term interest rate is the annualized nominal funds rate, available from the Board of Governors. We use a four-quarter moving average of past inflation as a proxy for inflation expectations in constructing the ex ante real interest rate. In parentheses below the estimates we report t-statistics based on Newey and West (1987) standard errors with 60 lags. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory. The intercept estimates are omitted. The sample period is 1964/01 to 2017/12.

Panel A: Convergence Gap

Maturity	$rx_{t+1}^{(n)} = a_r + b_{r,CG^{\perp}} CG_t^{\perp} + \varepsilon_{t+1}$		$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_{y,CG^{\perp}} CG_t^{\perp} + u_{t+1}$	
$n =$	$b_{r,CG^{\perp}}$	R^2	$b_{y,CG^{\perp}}$	R^2
2	-0.30***	0.15	0.30***	0.17
	(-4.59)		(6.48)	
3	-0.55***	0.14	0.55***	0.16
	(-5.04)		(6.42)	
4	-0.71***	0.12	0.71***	0.14
	(-4.16)		(5.82)	
5	-0.82***	0.10	0.82***	0.11
	(-3.85)		(5.13)	

Panel B: Forward Spread and Convergence Gap

Maturity	$rx_{t+1}^{(n)} = a_r + b_r \left(f_t^{(n)} - y_t^{(1)}\right) + b_{r,CG^{\perp}} CG_t^{\perp} + \varepsilon_{t+1}$			$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_y \left(f_t^{(n)} - y_t^{(1)}\right) + b_{y,CG^{\perp}} CG_t^{\perp} + u_{t+1}$		
$n =$	b_r	$b_{r,CG^{\perp}}$	R^2	b_y	$b_{y,CG^{\perp}}$	R^2
2	0.83***	-0.30***	0.26	0.17	0.30***	0.17
	(5.42)	(-7.02)		(1.09)	(7.02)	
3	1.13***	-0.55***	0.28	-0.13	0.55***	0.16
	(4.87)	(-6.41)		(-0.57)	(6.41)	
4	1.36***	-0.71***	0.28	-0.36	0.71***	0.14
	(5.50)	(-6.12)		(-1.45)	(6.12)	
5	1.12***	-0.82***	0.19	-0.12	0.82***	0.11
	(4.18)	(-5.21)		(-0.44)	(5.21)	

Table 3: **Predicting yield changes**

This table reports the root mean squared errors (RMSE) in predicting H-year ahead one-year yields using four different models, expressed in percentage. The specification labeled "RW" (random walk) uses the current yield as best estimate of future yields. In the columns $s_t^{(5)}$, CG_t, and $[s_t^{(5)}\ \mathrm{CG}_t]$ either the slope $(y_t^{(5)} - y_t^{(1)})$, the convergence gap, or both, are used to predict future H-year changes in one-year yields. The estimates are then used to form one-year yields forecast. All predictive regressions include a constant term, whose estimate is omitted. In parentheses we report the p-value of the Diebold and Mariano (1995) test for the null hypothesis of zero difference in MSE between the random walk model and the model in the corresponding column. The sample period is 1964/01 to 2017/12.

Horizon H (years)	RW	$s_t^{(5)}$	CG_t	$[s_t^{(5)}\ \mathrm{CG}_t]$
1	1.37	1.41	1.32	1.33
		(0.06)	(0.29)	(0.40)
2	2.15	2.12	1.99	1.97
		(0.72)	(0.21)	(0.19)
3	2.47	2.25	2.24	2.13
		(0.13)	(0.08)	(0.05)
4	2.48	2.21	2.27	2.20
		(0.15)	(0.04)	(0.04)

Table 4: **Forecasting average (across maturity) bond excess returns**

Panel A of this table reports OLS slope coefficients and R^2 in the regression of average (across maturities) annual excess returns $\overline{rx}_{t+1}$ on a constant and various combinations of lagged one- to five-year forward rates and the convergence gap, CG_t. Panel B reports the OLS slope coefficients and R^2 in the regression of the residuals $\hat{\epsilon}_{t+1}$ from specification (1) and residuals $\hat{\epsilon}^{CG}_{t+1}$ from specification (3) of Panel A on a constant and two combinations of the following variables: inflation (CPI), Industrial Production growth (IP), the Chicago Fed National Activity Index (CFNAI), the NBER recession dummy, and the one-year forward rate (yield). In parentheses below the estimates we report t-statistics based on Newey and West (1987) standard errors with 60 lags. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory. The intercept estimates are omitted. The convergence gap, CG_t, is defined as in Table 2. The sample period is 1964/01 to 2017/12.

Panel A: Forecasting $\overline{rx}_{t+1}$

Spec.	$f_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$	CG_t	R^2
(1)	-1.32***	-0.54	1.78*	1.35***	-1.07*		0.23
	(-3.02)	(-0.79)	(1.98)	(3.56)	(-2.07)		
(2)						-0.40***	0.06
						(-2.75)	
(3)	-1.99***	0.07	2.24***	0.85*	-1.44***	-0.79***	0.34
	(-5.27)	(0.12)	(3.27)	(2.20)	(-3.10)	(-4.28)	

Panel B: Forecasting $\hat{\epsilon}_{t+1}$ and $\hat{\epsilon}^{CG}_{t+1}$

Panel A Spec.	CPI_t	IP_t	$CFNAI_t$	$NBER_t$	R^2
(1)	-0.31**	-0.06		0.03***	0.11
	(-2.39)	(-0.68)		(2.85)	
	-0.29*		-0.06***	0.02**	0.12
	(-2.20)		(-3.05)	(2.30)	
(3)	-0.14	-0.05		0.02**	0.06
	(-0.95)	(-0.61)		(2.46)	
	-0.13		-0.04	0.01*	0.07
	(-0.78)		(-1.72)	(2.02)	

Table 5: **Forecasting individual bond excess returns**

This table reports OLS slope coefficients and R^2 in the regressions of future annual excess returns for bonds with maturities of two years (Panel A), three years (Panel B), four years (Panel C), and five years (Panel D). The table shows (1) to (4) specifications of the regressors. CP denotes Cochrane and Piazzesi (2005) forward rates factors. CPG denotes the fitted value from specification (3) of Panel A of Table 4, where forward rates are augmented with the convergence gap. $F5$ denotes Ludvigson and Ng (2009) macro factor. In parentheses below the estimates we report t-statistics based on Newey and West (1987) standard errors with 60 lags. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory. All regressions include a constant term, whose coefficient is omitted. The sample period is 1964/01 to 2017/12.

Panel A: $rx_{t+1}^{(2)}$					Panel B: $rx_{t+1}^{(3)}$				
Spec.	CP_t	CPG_t	$F5_t$	R^2	Spec.	CP_t	CPG_t	$F5_t$	R^2
(1)	0.43***			0.19	(1)	0.83***			0.22
	(6.46)					(6.35)			
(2)		0.46***		0.32	(2)		0.86***		0.33
		(9.51)					(9.47)		
(3)	0.39***		0.43***	0.32	(3)	0.75***		0.76***	0.33
	(5.68)		(5.99)			(5.82)		(6.69)	
(4)		0.40***	0.34***	0.39	(4)		0.75***	0.59***	0.40
		(7.69)	(4.76)				(8.42)	(6.06)	
Panel C: $rx_{t+1}^{(4)}$					Panel D: $rx_{t+1}^{(5)}$				
Spec.	CP_t	CPG_t	$F5_t$	R^2	Spec.	CP_t	CPG_t	$F5_t$	R^2
(1)	1.25***			0.25	(1)	1.49***			0.23
	(6.78)					(6.71)			
(2)		1.23***		0.35	(2)		1.46***		0.32
		(10.77)					(10.58)		
(3)	1.15***		1.02***	0.36	(3)	1.36***		1.21***	0.33
	(6.20)		(7.11)			(6.12)		(6.41)	
(4)		1.10***	0.78***	0.41	(4)		1.30***	0.92***	0.38
		(9.39)	(6.53)				(9.18)	(6.11)	

Table 6: **Robustness analysis**

This table reports OLS slope estimates and associated t-statistics (in parentheses) for the regression of excess bond returns on a constant, lagged one- to five-year forward rates, and various specifications of the additional regressors and sampling frequency. In Panel A the dependent variable is the average annual excess return $\overline{rx}_{t+1}$ and the estimates are for quarterly and yearly sampled observations. The convergence gap, CG_t, is defined as in Table 2. R^2_{fwd} is the R-squared statistics for the specification with forward rates only. In Panel B, analogous results are reported when CG is constructed using real-time vintage data. In Panel C, the dependent variable is the monthly excess return on a portfolio of bonds with two to three years to maturity. RP denotes Duffee (2011)'s hidden factor. In parentheses below the estimates we report t-statistics based on Newey and West (1987) standard errors with 18 lags. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory. All regressions include a constant term, whose coefficient is omitted. The sample period is 1964/01 to 2017/12, except for the last regression for which the sample ends on 2007/12.

Panel A: Quarterly and Annual Regressions

Frequency	$f_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$	CG_t	R^2	R^2_{fwd}
Quarterly	-2.10***	-0.92	4.33***	0.36	-1.98***	-0.87***	0.41	0.29
	(-4.73)	(-1.05)	(3.59)	(0.48)	(-5.32)	(-4.28)		
Annual	-2.05**	-0.78	4.28***	0.91	-2.13***	-1.02***	0.45	0.32
	(-3.50)	(-0.75)	(4.07)	(1.55)	(-4.68)	(-5.16)		

Panel B: Quarterly and Annual Regressions, Vintage Data

Frequency	$f_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$	CG_t	R^2	R^2_{fwd}
Quarterly vintage	-1.76***	-1.08	3.98***	0.63	-1.86***	-0.54***	0.34	0.28
	(-3.33)	(-1.05)	(3.26)	(0.98)	(-4.20)	(-3.74)		
Annual vintage	-1.70***	-0.76	4.29***	1.21**	-2.02***	-0.68***	0.39	0.33
	(-4.47)	(-0.65)	(5.82)	(2.94)	(-6.34)	(-6.00)		

Panel B: Forecasting Monthly Excess Returns

Spec.	$f_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$	CG_t	RP_t	R^2
(1)	-0.06	-0.06	0.09	0.09	-0.04			0.02
	(-1.17)	(-0.53)	(0.7)	(0.7)	(-0.47)			
(2)	-0.11*	-0.02	0.12	0.06	-0.06	-0.05*		0.03
	(-1.82)	(-0.18)	(0.87)	(0.42)	(-0.85)	(-1.78)		
(3)	-0.12*	0.22	-0.24	0.00	0.10	-0.06*	2.55***	0.06
	(-1.98)	(1.57)	(-1.3)	(-0.02)	(0.88)	(-1.89)	(4.04)	

TABLE 7: **Forecasting average (across maturity) excess bond returns, international evidence**

This table reports the OLS slope coefficients and R^2 in the regression of average annual excess returns $\overline{xr}_{t+1}$ on a constant and various combinations of lagged one- to five-year forward rates and the convergence gap, CG_t. Panel A shows results for Canada, Panel B for UK, and Panel C for Germany. The convergence gap, CG, is defined as the difference between the natural rate of interest and the real interest rate. To proxy for the natural rate of interest we use the estimates from Holston et al. (2017). Finally, the convergence gap has been linearly detrended. The short-term interest rate is: the Bank of Canada's target for the overnight rate in Panel A; the Bank of England's Official Lending Rate, published by the Bank of England, in Panel B; and the three-month rate from the Area Wide Model (Fagan et al., 2001) in Panel C. For all countries, the inflation series is constructed by splicing the core price index with an all-items price index. We use a four-quarter moving average of past inflation as a proxy for inflation expectations in constructing the ex ante real interest rate. In parentheses below the estimates we report t-statistics based on Newey and West (1987) standard errors with 18 lags. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory. All regressions include a constant term, whose coefficient is omitted. The sample period is 1986/01 to 2017/12 for Canada and the UK, and 1991/01 to 2017/12 for Germany.

Panel A: Forecasting $\overline{xr}_{t+1}$, Canada: 1986/01 – 2017/12

	$f_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$	CG_t	R^2
(1)	-1.22	0.93	-3.13	9.55*	-6.04**		0.17
	(-1.71)	(0.44)	(-0.74)	(1.88)	(-2.36)		
(2)						-0.28	0.03
						(-1.10)	
(3)	-2.16***	0.15	2.60	0.59	-1.09	-0.97***	0.34
	(-2.73)	(0.07)	(0.62)	(0.12)	(-0.46)	(-5.41)	

Panel B: Forecasting $\overline{xr}_{t+1}$, United Kingdom: 1986/01 – 2017/12

	$f_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$	CG_t	R^2
(1)	-0.52	0.88	-11.70	24.29	-12.89		0.17
	(-0.49)	(0.15)	(-0.67)	(1.09)	(-1.33)		
(2)						-0.24	0.01
						(-0.69)	
(3)	-1.54	2.02	-8.10	12.26	-4.35	-1.06***	0.26
	(-1.58)	(0.38)	(-0.54)	(0.68)	(-0.58)	(-2.75)	

Panel C: Forecasting $\overline{xr}_{t+1}$, Germany and Euro Area: 1991/01–2017/12

	$f_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$	CG_t	R^2
(1)	-2.65	-0.25	18.34	-28.87	13.87		0.28
	(-2.97)	(-0.05)	(1.34)	(-1.67)	(1.72)		
(2)						-0.20	0.01
						(-0.95)	
(3)	-3.38**	0.87	15.26	-23.95	11.57	-0.66*	0.36
	(-2.81)	(0.14)	(1.07)	(-1.46)	(1.59)	(-2.17)	

Table 8: **Dynamic portfolio policies**

This table reports estimates of the portfolio policies for a quadratic utility investor with $\gamma = 5$ at the monthly and annual horizon. For each horizon, three columns are displayed corresponding to different sets of conditioning variables: (1) includes a constant term; (2) adds one- to five- year forward rates; (3) adds the convergence gap CG. All conditioning variables except the constant are standardized. The first block of the Table reports the OLS coefficient θ associated to each variable, with t-statistics in parentheses below the estimates. F-test is the p-value for the test that all slope coefficients are jointly equal to zero. The annualized mean ($E(r_p)$), annualized standard deviation (σ_p), and annual Sharpe Ratio (SR_p) of the corresponding optimal portfolio are displayed next. Equalization fee is the annual fee that the investor would pay to have access to the conditioning information. The last row reports the Equalization fee when weights are constrained between −1 and 2. The sample period is 1964/01 to 2017/12.

	Monthly returns			Annual returns		
	(1)	(2)	(3)	(1)	(2)	(3)
Const	2.261	3.000	3.377	1.352	1.165	1.256
	(2.446)	(2.405)	(2.921)	(1.910)	(1.506)	(3.061)
$f_t^{(1)}$		-3.653	-7.016		-1.061	-4.191
		(-1.238)	(-2.055)		(-0.207)	(-1.020)
$f_t^{(2)}$		-0.325	2.165		-9.391	-5.871
		(-0.050)	(0.321)		(-1.103)	(-0.806)
$f_t^{(3)}$		-0.295	0.304		8.971	9.850
		(-0.064)	(0.055)		(2.294)	(2.310)
$f_t^{(4)}$		2.725	1.078		3.756	0.612
		(0.598)	(0.211)		(0.948)	(0.166)
$f_t^{(5)}$		1.305	0.229		-2.895	-2.564
		(0.518)	(0.084)		(-1.311)	(-1.181)
CG_t			-2.974			-2.015
			(-1.902)			(-3.341)
F-test		0.190	0.013		0.000	0.000
$E(r_p)$	0.073	0.096	0.115	0.065	0.094	0.105
σ_p	0.067	0.095	0.113	0.060	0.095	0.099
SR_p	0.333	0.478	0.574	0.262	0.514	0.596
Equalization fee		0.012	0.022		0.019	0.026
Equalization fee constr.		0.002	0.010		0.012	0.017

Table 9: **Forecasting individual bond excess returns, out-of-sample analysis**

This table reports the out-of-sample accuracy in forecasting individual bond excess returns of Model 1, which includes the CPG factor, defined as in Table 5, in the first and second rows, and the CPG factor and the $F5$ factor from Ludvigson and Ng (2009) in the third row; and Model 2, which includes the CP factor in the first row, and the CP and the $F5$ factor from Ludvigson and Ng (2009) in the second and third rows. All models also include a constant term. The CP and $F5$ factors are re-estimated whenever a new observation is added to the sample. The first forecast is made in 1989/12, and the last forecast is in 2016/12, for a total of 325 (overlapping) observations. $MSFE_1/MSFE_2$ denotes the ratio between the mean squared forecast error of Model 1 to Model 2. DM reports the p-value of the Diebold and Mariano (1995) test for the null hypothesis of zero difference in MSE between the models. Bias and Variance report, respectively, the p-value of the t-statistic for the intercept and slope in the regression of the difference in forecast errors on the demeaned sum of forecast errors of the two models. The full sample period is 1964/01 to 2017/12.

Model 1	Model 2	$MSFE_1/MSFE_2$	DM test	Bias	Variance
		Panel A: $xr_{t+1}^{(2)}$			
CPG	CP	0.779	0.005	1.41e-11	0.409
CPG	[CP $F5$]	1.047	0.531	0.865	0.526
[CPG $F5$]	[CP $F5$]	0.891	0.060	8.50e-12	0.569
		Panel B: $xr_{t+1}^{(3)}$			
CPG	CP	0.760	0.004	8.65e-12	0.305
CPG	[CP $F5$]	1.006	0.927	0.728	0.703
[CPG $F5$]	[CP $F5$]	0.861	0.033	6.49e-12	0.587
		Panel C: $xr_{t+1}^{(4)}$			
CPG	CP	0.733	0.003	5.64e-12	0.235
CPG	[CP $F5$]	0.966	0.550	0.258	0.743
[CPG $F5$]	[CP $F5$]	0.816	0.019	4.83e-12	0.593
		Panel D: $xr_{t+1}^{(5)}$			
CPG	CP	0.744	0.004	2.88e-12	0.213
CPG	[CP $F5$]	0.952	0.346	0.239	0.993
[CPG $F5$]	[CP $F5$]	0.825	0.017	3.48e-12	0.622

A Data

We require data for real GDP, inflation, and the short-term nominal interest rate, as well as a procedure to compute inflation expectations to calculate the ex ante real short term interest rate. The GDP data is obtained from the Federal Reserve Bank of St.Louis dataset and starts in 1957, which allows a burn-in seven-year period to estimate the 1964 trend component. The inflation measure is the growth rate of the price index for personal consumption expenditure (PCE) excluding food and energy, referred to as core PCE inflation. The short-term interest rate is the annualized nominal funds rate, available from the Board of Governors. Because the federal funds rate frequently fell below the discount rate prior to 1965, we use the Federal Reserve Bank of New York's discount rate prior to 1965, reported by the IMF. For our benchmark measure we use a four-quarter moving average of past inflation as a proxy for inflation expectations in constructing the ex ante real interest rate. This is the same approach used in Holston et al. (2017).

Canadian, UK, and Euro Area data is from Holston et al. (2017). We refer the reader to their detailed data appendix.

B Robustness

Table B.1 and B.2 carry out the same analysis of Table 2 using alternative measures of the natural rate of interest. In particular, in Table B.1, we replace the one-sided HP filtered trend component of real GDP with the potential GDP series obtained by Laubach and Williams (2003) using a Kalman filter. In Table B.2, we instead replace the one-sided HP filtered trend component of real GDP with the Kalman filter natural-rate estimates - denoted r_t^* - by Laubach and Williams (2003). The Laubach and Williams (2003) natural rate of interest is composed of the trend growth rate of the natural rate of output and a component that captures the households' rate of time preference as well as other determinants of r^* unrelated to trend growth.

In general Panel A in both tables show that the relative difference between the current level of monetary stance yield and its long-term convergence level provide information on the path of future yields. The R^2 are very close to those reported in Table B.2. Hence our results are robust to alternative choices of the equilibrium real interest rate.

Tables B.3 and B.4 carries out the same analysis of Table 2 using alternative measure of the ex ante real interest rate. In particular, in Table B.3 we proxy inflation expectations with the forecast of the twelve-month-ahead percentage change in the price index for personal consumption expenditures excluding food and energy ("core PCE prices") generated from a univariate AR(9) of inflation estimated over the prior 120 months. This measure is similar to the measure of inflation expectations used by Laubach and Williams (2003). In Table B.4 we instead use CPI inflation forecasts from the Survey of Professional Forecasters (SPF) to construct the real rate. The SPF data are quarterly beginning in 1981Q2.[27] We use the median across the respondents, but results are identical when we use the mean. A survey at quarter t reports k quarter ahead consensus predictions of CPI inflation for $k = 1, \dots, 4$. We use these forecasts to calculate predictions of inflation over the next year. To go further back in time, we splice this series with the four-quarter moving average of past CPI inflation, in order to obtain a final series spanning the period 1964-2017.

Panel A show that the relative difference between the current level of monetary stance yield and its long-term convergence level provide information on the path of future yields. The R^2 are very close to those reported in Table B.2. We conclude that using Survey consensus forecasts of future inflation or a yearly moving average of past inflation does not alter our conclusions.

[27]PCE forecasts are available only starting from 2007:Q1, hence we switch to CPI as our inflation proxy despite the fact that the Fed pays more attention to the PCE for policy purposes.

TABLE B.1: **Forecasting regressions with Convergence Gap and Forward Rates**

This table reports regressions in the spirit of Fama and Bliss (1987). **Panel A:** reports simple regressions using the Convergenge Gap orthogonalized with respect to the forward spread $f_t^{(n)} - y_t^{(1)}$. **Panel B:** reports multiple regressions using the forward spread and the (orthogonalized) Convergenge Gap. The convergence gap, CG_t, is defined as the difference between the year-to-year log change in potential GDP and the real interest rate. To proxy for potential GDP we use the Laubach and Williams (2003) estimates obtained by Kalman filter. We linearly interpolate the Laubach and Williams (2003) series to obtain monthly observations. We linearly interpolate the resultant trend series to obtain monthly observations. The short-term interest rate is the annualized nominal funds rate, available from the Board of Governors. We use a four quarter moving average of past inflation as a proxy for inflation expectations in constructing the ex ante real interest rate. In parentheses below the estimates we report t-statistics based on Newey and West (1987) standard errors with 60 lags. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory. The intercept estimates are omitted. The sample period is 1964/01 to 2017/12.

Panel A: Convergence Gap

Maturity	$rx_{t+1}^{(n)} = a_r + b_{r,CG} CG_t + \varepsilon_{t+1}$		$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_{y,CG} CG_t + u_{t+1}$	
$n =$	$b_{r,CG}$	R^2	$b_{y,CG}$	R^2
2	-0.33***	0.16	0.33***	0.18
	(-3.28)		(5.29)	
3	-0.61***	0.15	0.61***	0.17
	(-3.33)		(4.46)	
4	-0.76***	0.12	0.76***	0.14
	(-2.73)		(3.53)	
5	-0.82**	0.09	0.82***	0.09
	(-2.42)		(2.96)	

Panel B: Forward Spread and Convergence Gap

Maturity	$rx_{t+1}^{(n)} = a_r + b_r \left(f_t^{(n)} - y_t^{(1)}\right) + b_{r,CG} CG_t + \varepsilon_{t+1}$			$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_y \left(f_t^{(n)} - y_t^{(1)}\right) + b_{y,CG} CG_t + u_{t+1}$		
$n =$	b_r	$b_{r,CG}$	R^2	b_y	$b_{y,CG}$	R^2
2	0.83***	-0.33***	0.28	0.17	0.33***	0.18
	(4.60)	(-4.94)		(0.92)	(4.94)	
3	1.13***	-0.61***	0.28	-0.13	0.61***	0.17
	(4.65)	(-4.62)		(-0.54)	(4.62)	
4	1.36***	-0.76***	0.28	-0.36	0.76***	0.15
	(5.29)	(-3.86)		(-1.40)	(3.86)	
5	1.12***	-0.82***	0.17	-0.12	0.82***	0.10
	(4.07)	(-3.02)		(-0.43)	(3.02)	

Table B.2: **Forecasting regressions with Convergence Gap and Forward Rates**

This table reports regressions in the spirit of Fama and Bliss (1987). **Panel A:** reports simple regressions using the Convergenge Gap orthogonalized with respect to the forward spread $f_t^{(n)} - y_t^{(1)}$. **Panel B:** reports multiple regressions using the forward spread and the (orthogonalized) Convergenge Gap. The convergence gap, CG_t, is defined as the difference between the natural rate of interest and the real interest rate. To proxy for the natural rate of interest we use the Laubach and Williams (2003) estimates obtained by Kalman filter. We linearly interpolate the Laubach and Williams (2003) series to obtain monthly observations. The short-term interest rate is the annualized nominal funds rate, available from the Board of Governors. We use a four quarter moving average of past inflation as a proxy for inflation expectations in constructing the ex ante real interest rate. In parentheses below the estimates we report t-statistics based on Newey and West (1987) standard errors with 60 lags. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory. The intercept estimates are omitted. The sample period is 1964/01 to 2017/12.

Panel A: Convergence Gap

Maturity	$rx_{t+1}^{(n)} = a_r + b_{r,CG}CG_t + \varepsilon_{t+1}$		$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_{y,CG}CG_t + u_{t+1}$	
$n =$	$b_{r,CG}$	R^2	$b_{y,CG}$	R^2
2	-0.27***	0.13	0.27***	0.15
	(-2.85)		(3.81)	
3	-0.52***	0.13	0.52***	0.15
	(-3.31)		(3.66)	
4	-0.65***	0.11	0.65***	0.13
	(-3.16)		(3.47)	
5	-0.76***	0.10	0.76***	0.10
	(-3.22)		(3.49)	

Panel B: Forward Spread and Convergence Gap

Maturity	$rx_{t+1}^{(n)} = a_r + b_r\left(f_t^{(n)} - y_t^{(1)}\right) + b_{r,CG}CG_t + \varepsilon_{t+1}$			$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_y\left(f_t^{(n)} - y_t^{(1)}\right) + b_{y,CG}CG_t + u_{t+1}$		
$n =$	b_r	$b_{r,CG}$	R^2	b_y	$b_{y,CG}$	R^2
2	0.83***	-0.27***	0.25	0.17	0.27***	0.15
	(5.69)	(-3.71)		(0.83)	(3.71)	
3	1.13***	-0.52***	0.27	-0.13	0.52***	0.15
	(5.34)	(-3.67)		(-0.50)	(3.67)	
4	1.36***	-0.65***	0.27	-0.36	0.65***	0.14
	(6.06)	(-3.45)		(-1.17)	(3.45)	
5	1.12***	-0.76***	0.18	-0.12	0.76***	0.11
	(4.26)	(-3.49)		(-0.33)	(3.49)	

Table B.3: **Forecasting regressions with Convergence Gap and Forward Rates**

This table reports regressions in the spirit of Fama and Bliss (1987). **Panel A:** reports simple regressions using the Convergenge Gap orthogonalized with respect to the forward spread $f_t^{(n)} - y_t^{(1)}$. **Panel B:** reports multiple regressions using the forward spread and the (orthogonalized) Convergenge Gap. The convergence gap, CG_t, is defined as the difference between the year-to-year log change in potential GDP and the real interest rate. To proxy for potential GDP we use the trend component of quarterly real GDP obtained from a one-sided Hodrick and Prescott (1997) filter. We linearly interpolate the resultant trend series to obtain monthly observations. The short-term interest rate is the annualized nominal funds rate, available from the Board of Governors. We use the forecast of the twelve-month-ahead percentage change in core PCE generated from a univariate AR(9) estimated over the prior 120 months as a proxy for inflation expectations in constructing the ex ante real interest rate. In parentheses below the estimates we report t-statistics based on Newey and West (1987) standard errors with 60 lags. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory. The intercept estimates are omitted. The sample period is 1964/01 to 2017/12.

Panel A: Convergence Gap

Maturity	$rx_{t+1}^{(n)} = a_r + b_{r,CG} CG_t + \varepsilon_{t+1}$		$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_{y,CG} CG_t + u_{t+1}$	
$n =$	$b_{r,CG}$	R^2	$b_{y,CG}$	R^2
2	-0.30***	0.15	0.30***	0.17
	(-4.53)		(7.14)	
3	-0.56***	0.15	0.56***	0.17
	(-5.02)		(6.51)	
4	-0.74***	0.13	0.74***	0.16
	(-4.13)		(5.87)	
5	-0.84***	0.11	0.84***	0.12
	(-3.81)		(5.11)	

Panel B: Forward Spread and Convergence Gap

Maturity	$rx_{t+1}^{(n)} = a_r + b_r \left(f_t^{(n)} - y_t^{(1)}\right) + b_{r,CG} CG_t + \varepsilon_{t+1}$			$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_y \left(f_t^{(n)} - y_t^{(1)}\right) + b_{y,CG} CG_t + u_{t+1}$		
$n =$	b_r	$b_{r,CG}$	R^2	b_y	$b_{y,CG}$	R^2
2	0.83***	-0.30***	0.27	0.17	0.30***	0.17
	(5.36)	(-7.09)		(1.08)	(4.56)	
3	1.13***	-0.56***	0.28	-0.13	0.56***	0.17
	(4.83)	(-6.53)		(-0.57)	(6.53)	
4	1.36***	-0.74***	0.28	-0.36	0.71***	0.16
	(5.42)	(-6.24)		(-1.43)	(6.24)	
5	1.12***	-0.84***	0.19	-0.12	0.82***	0.12
	(4.14)	(-5.21)		(-0.44)	(5.21)	

TABLE B.4: **Forecasting regressions with Convergence Gap and Forward Rates**

This table reports regressions in the spirit of Fama and Bliss (1987). **Panel A:** reports simple regressions using the Convergenge Gap orthogonalized with respect to the forward spread $f_t^{(n)} - y_t^{(1)}$. **Panel B:** reports multiple regressions using the forward spread and the (orthogonalized) Convergenge Gap. The convergence gap, CG_t, is defined as the difference between the year-to-year log change in potential GDP and the real interest rate. To proxy for potential GDP we use the trend component of quarterly real GDP obtained from a one-sided Hodrick and Prescott (1997) filter. We linearly interpolate the resultant trend series to obtain monthly observations. The short-term interest rate is the annualized nominal funds rate, available from the Board of Governors. We use CPI inflation forecasts from the Survey of Professional Forecasters (SPF) to construct the real rate from 1981 onwards, and we splice it with a four quarter moving average of past CPI inflation. We use such proxy for inflation expectations in constructing the ex ante real interest rate. In parentheses below the estimates we report t-statistics based on Newey and West (1987) standard errors with 60 lags. Significance: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory. The intercept estimates are omitted. The sample period is 1964/01 to 2017/12.

Panel A: Convergence Gap

Maturity	$rx_{t+1}^{(n)} = a_r + b_{r,CG} CG_t + \varepsilon_{t+1}$		$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_{y,CG} CG_t + u_{t+1}$	
$n =$	$b_{r,CG}$	R^2	$b_{y,CG}$	R^2
2	-0.30***	0.15	0.30***	0.17
	(-3.76)		(5.52)	
3	-0.55***	0.14	0.55***	0.16
	(-3.78)		(4.69)	
4	-0.71***	0.12	0.71***	0.14
	(-3.48)		(4.36)	
5	-0.82***	0.10	0.82***	0.11
	(-3.33)		(4.01)	

Panel B: Forward Spread and Convergence Gap

Maturity	$rx_{t+1}^{(n)} = a_r + b_r \left(f_t^{(n)} - y_t^{(1)}\right) + b_{r,CG} CG_t + \varepsilon_{t+1}$			$(n-1) \times \left(y_{t+1}^{(n-1)} - y_t^{(n-1)}\right) = a_y + b_y \left(f_t^{(n)} - y_t^{(1)}\right) + b_{y,CG} CG_t + u_{t+1}$		
$n =$	b_r	$b_{r,CG}$	R^2	b_y	$b_{y,CG}$	R^2
2	0.83***	-0.30***	0.26	0.17	0.30***	0.17
	(5.19)	(-5.46)		(1.04)	(5.46)	
3	1.13***	-0.55***	0.27	-0.13	0.55***	0.16
	(5.23)	(-4.74)		(-0.61)	(4.74)	
4	1.36***	-0.71***	0.28	-0.36	0.71***	0.15
	(5.94)	(-4.55)		(-1.57)	(4.55)	
5	1.12***	-0.82***	0.19	-0.12	0.82***	0.11
	(4.08)	(-4.05)		(-0.43)	(4.05)	