When Do Currency Unions Benefit From Default?*
(Job Market Paper)

Xuan Wang †

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Abstract

Since the Eurozone Crisis of 2010-12, a key debate on the viability of a currency union has focused on the role of a fiscal union in adjusting for country heterogeneity. However, a fully-fledged fiscal union may not be politically feasible. This paper develops a two-country international finance model to examine the benefits of the bankruptcy code of a capital markets union - in the absence of a fiscal union - as an alternative mechanism to improve the financial stability and welfare of a currency union. When domestic credit risks are present, I show that a lenient union-wide bankruptcy code that allows for default in the cross-border capital markets union removes the pecuniary externality of banking insolvency, so it leads to a Pareto improvement within the currency union. Moreover, the absence of floating nominal exchange rates removes a mechanism to neutralise domestic credit risks; I show that softening the union-wide bankruptcy code can recoup the lost benefits of floating nominal exchange rates. The model provides the financial stability and welfare implications of bankruptcy within a capital markets union in the Eurozone.

Keywords: Default, bankruptcy code, fiscal union, capital markets union, financial stability, bank credit, inside money, price-level and exchange rate determinacy, liquidity-intermediary asset pricing

JEL Codes: E42, F33, G15, G21

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†PhD (DPhil) candidate in Financial Economics, Saïd Business School and Exeter College, University of Oxford.

1 Introduction

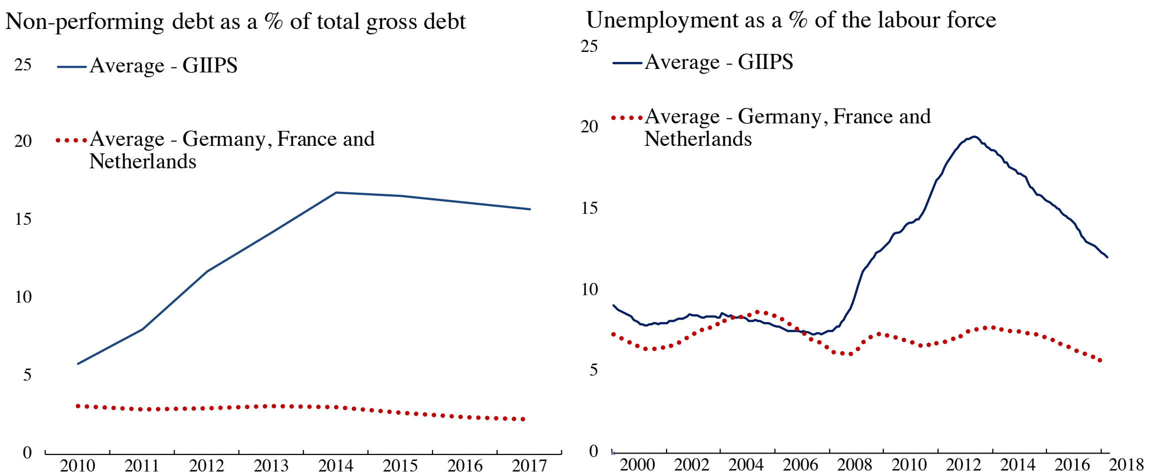
The establishment of a currency union in Europe has long begged the question of what constitutes a fiscal union that is capable of making cross-country transfers within the currency union (see Friedman 1997; Goodhart 1997, 1998). One argument is that a single union-wide monetary authority may prove inadequate because countries in the Eurozone exhibit economic and financial heterogeneity; thus, a fiscal union is needed to adjust for this heterogeneity and improve financial stability. Without such fiscal integration, Friedman (1997) raised concerns that the adoption of the euro could create divergence among member countries and, in turn, lead to political disunity.

The Eurozone Crisis may appear to validate these concerns, as following the crisis the core and the peripheral Eurozone have exhibited diverging financial stability and economic fundamental profiles, as illustrated by the non-performing loan rates and unemployment rates in Figure 1. Thus, 20 years since the creation of the euro, a key debate following the Eurozone Crisis has thus centred on the ability of a fiscal union to improve the viability of sharing a single currency. Meaningful work on fiscal unions has been timely produced (see Farhi and Werning 2017; Kehoe and Pastorino 2017). However, pragmatically, a fully-fledged fiscal union may not be politically feasible (see a detailed discussion in the Nobel Lecture by Sargent 2012). The question then arises: when such a fiscal union is absent, what else can be done to improve the welfare and financial stability of currency unions?

Figure 1: Country heterogeneity in the Eurozone

(a) Non-performing loans

(b) Unemployment rate



Source: Statistical Data Warehouse, ECB and OECD Economic Surveys: Euro Area 2018. Average of first three quarters. GIIPS: Greece, Ireland, Italy, Portugal and Spain.

Source: Eurostat database and OECD Economic Surveys: Euro Area 2018, calculated as unweighted average. GIIPS: Greece, Ireland, Italy, Portugal and Spain.

The goal of this paper therefore is to design the cross-border bankruptcy code of a capital markets union as an alternative mechanism to improve the financial stability and social welfare of a currency union, in the absence of a fiscal union.[1] This question is relevant as it sheds light on an ongoing debate on the capital markets union as

[1]In this paper, a banking union is one manifestation of a fiscal union. The model takes the view that a banking union needs fiscal coordination, and that it is essentially a common fiscal entity making transfers across banks in different countries within the currency union.

a close substitute for a fiscal union to improve the viability of the Eurozone (see Martinez et al. 2019). Furthermore, the perspective on cross-border bankruptcy is also timely. In 2016, the European Commission proposed a legal directive of a lenient cross-border insolvency law or bankruptcy code in Europe, as a key foundation of the capital markets union. However, there has been little economic study that analyses the welfare implication of such bankruptcy code adjustment and its relevance to the financial stability of the Eurozone.[2]

For this purpose, I develop a three-period nominal international finance model of a two-country and two-good endowment economy with uncertainty. My innovation is to relate the cross-border insolvency reform or the bankruptcy code adjustment to the functioning of the capital markets union in improving financial stability of a currency union. This model has the unique features of a single currency, banking, and bankruptcy codes. I show that in the presence of moral hazard, when the union-wide bankruptcy code[3] is sufficiently lenient to allow for some degree of state-dependent default in the cross-border capital markets, the currency union-specific pecuniary externality of banking insolvency is removed from the system. Therefore, a Pareto improvement is obtained despite the social cost of default. As such, bankruptcy code leniency of a capital markets union can be a close substitute for a fiscal union. This is because the endogenous default on cross-border financial securities that ensues from softening the union-wide bankruptcy code generates a liquidity transfer from the country in the good state to the country in the bad state. This liquidity transfer via default adjusts for country heterogeneity and in turn shields the domestic banking sector from insolvency. However, if the union-wide bankruptcy code is punitive, I show that member countries will suffer from internal devaluation due to the currency union-specific pecuniary externality of banking insolvency.

Moreover, to respond to the loss of floating nominal exchange rates in a currency union, I also consider the case of *credible*[4] national currencies and the role of nominal exchange rates. I show that under relatively general conditions, competitive floating exchange rates can indeed neutralise credit risks across states and alleviate domestic banking stress. In a currency union, however, softening the union-wide bankruptcy code may function to recoup the lost benefits of flexible nominal exchange rates. The key point is, when countries join a currency union without a fiscal union, and the nominal exchange rate mechanism is removed to neutralise credit risks, then the bankruptcy code needs to adjust.

To formalise this theory, I model the issuance of the single currency via banks in a two-country and two-good endowment economy. As the focus is on monetary unions, the model is nominal. Within the currency union, in each country there are a continuum of households and a domestic commercial banking sector. Each country's households are risk averse and are endowed with one type of consumption good in the first two periods. The endowment at the second period is state contingent. Households borrow

[2]Section 6.3.2 provides a brief history and the institutional details of recent cross-border insolvency reforms in Europe.

[3]I use the terms "union-wide bankruptcy code" and "cross-border bankruptcy code" interchangeably in this paper. This bankruptcy code proxies for the cross-border insolvency law.

[4]"Credible" in this context means the sovereign does not intervene in the foreign exchange markets via quantitative measures and that exchange rate targeting is not in the national central bank's mandate.

commercial loans from their domestic commercial banking sector to get money for all transactions. Households trade goods as they consume both home goods and foreign goods. They also trade *nominal* financial securities via the capital markets union for risk sharing. These modelling components are akin to those featured in Geanakoplos and Tsomocos (2002) and Peiris and Tsomocos (2015). More importantly, in my model there exists a union-wide central bank that issues the single currency as the only stipulated means of exchange via lending interbank loans to the commercial bank sectors in the two countries.[5]

This model features a particular type of nominal rigidity. The nominal rigidity here stems from the transaction means of money and the single currency denomination of nominal prices, asset payoffs and the face value of loans. This modelling choice allows me to exclusively focus on the unique feature of sharing a single currency and to isolate its impact on welfare. The two key frictions in the model are the endogenous domestic credit risks of commercial bank loans and their interaction with sharing a single common currency.

The backbone of the model is the role of bank's balance sheets in creating and circulating the single currency against credit. This feature builds on the theory of inside money and outside money à la Shapley and Shubik (1977) and Dubey and Geanakoplos (1992, 2003b, 2006). *Inside money* is defined as money endogenously issued against an offsetting bank credit, and *outside money* refers to the initial monetary endowment that is free and clear of any debt obligation.[6] In my model, inside money is issued in the common currency the moment households apply for loans from their respective domestic commercial banking sectors. The domestic commercial banking sectors ultimately obtain the common currency from the union-wide central bank via the interbank loan contract.

For simplicity, the only role of the banking sector that I consider here is liquidity creation against credit (see Diamond and Rajan 2001 and Hart and Zingales 2014). In my model, bank liquidity creation helps to establish the price-level determinacy and inflation determinacy, and therefore, the nominal exchange rate determination in the subsequent model extension in which I consider national currencies. Consequently, this model is able to generate real effects from nominal and financial forces. In equilibrium, domestic commercial bank sectors end up splitting the seigniorage with the union-wide central bank. As argued by Reis (2013), such a seigniorage split is a distinct feature of the central bank balance sheet of a currency union. Additionally, the domestic commercial banks can obtain central bank reserves in the common currency via interbank borrowing. Therefore, these commercial banks can meet the liquidity demands of domestic households at any point during the timeline subject to interest rates, even though the assets of these banks have a long-term and inter-period maturity.

As credit risks and banking fragility were at the forefront of the Eurozone Crisis

[5]As in Farhi and Werning (2017); Kehoe and Pastorino (2017), as well as in the optimal currency area literature, I take the existence of a currency union as given and do not endogenise the formation of the currency union in the first place. Papers by Perotti and Soons (2019); Fuchs and Lippi (2006) explicitly model the endogenous adoption of a currency union. As suggested in Farhi and Werning (2017), consensus seems to be lacking on the benefits of currency unions and the economic reasons for joining a currency union. Thus, taking the existence of a currency union as an exogenous constraint serves a useful starting point here.

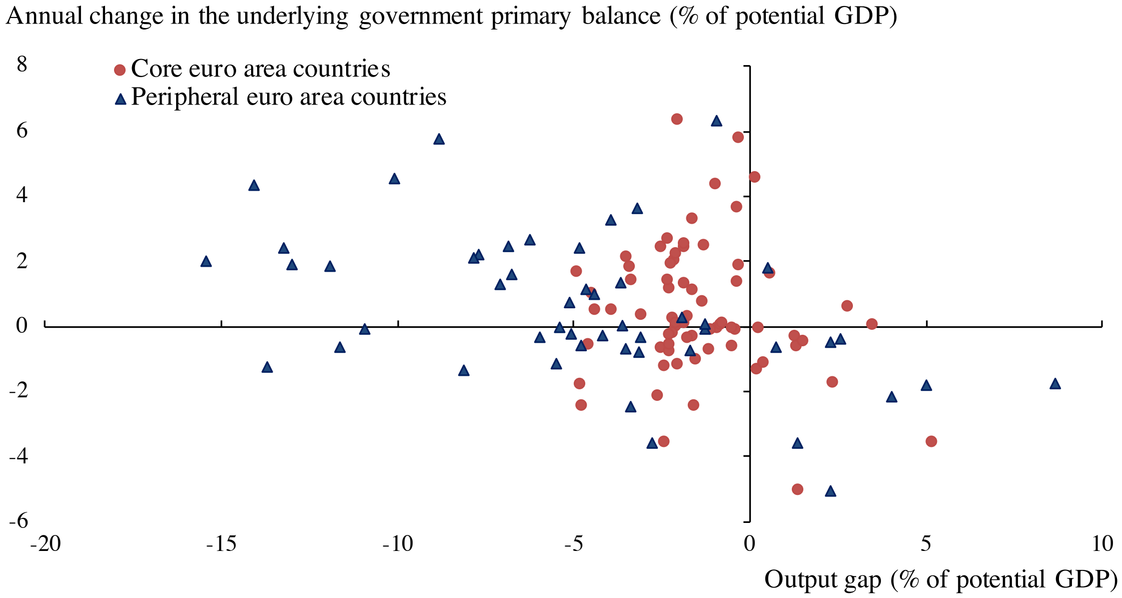
[6]These distinctions can be traced at least as far as back to Gurley and Shaw (1960).

(Figure 1a), the second key ingredient of the model is non-bank borrowers' moral hazard and endogenous credit risks that give rise to non-performing loans. Upon loan repayment, households may default but will suffer a default cost. Subject to the default cost, households may choose to default fully, partially, or repay fully, depending on the states of nature. As in Shubik and Wilson (1977), Zame (1993), and Dubey et al. (2005), I model the default cost as a non-pecuniary penalty cost. The stance of bankruptcy code is modelled as the harshness of the penalty per unit of default. With moral hazard present, I show that the currency union removes foreign exchange markets, which leads to an increase of credit risk volatilities across states, a key friction that leads to the pecuniary externality of banking insolvency causing welfare loss. The essence of my model therefore is to explore the efficacy of softening the union-wide bankruptcy code of the capital markets union to "undo" this friction.

With these two key features, I design three regimes in a currency union which I call hereafter Regimes A (internal devaluation), B (fiscal union), and C (bankruptcy leniency), as well as a national currency regime which I call Regime D. The design of these regimes is mainly achieved by varying the relative stance between the domestic bankruptcy code for bank lending and the union-wide bankruptcy code of the cross-border capital markets union. For each regime, I show analytically and numerically the implication of regime characteristics for allocation efficiency within state, risk sharing, inflation, and asset prices.

Let us start by considering **Regime A** as the baseline. It is a currency union that rules out a fiscal union and sets a punitive union-wide bankruptcy code of the capital markets union. It resembles the status quo of the Eurozone that it lacks a fully-fledged fiscal or banking union and meanwhile the stance for cross-border default is tough. I show that non-performing loans arise endogenously in the bad state, and that the domestic commercial bank sector fails and has to be costly bailed out using the national bailout tax. Regime A proves to be among the least desirable in all regimes, because the bailout cost causes a pecuniary externality and both the bailout cost and the domestic credit risk premium distort risk sharing and asset allocations. Since the bailout cost is due to national taxation that would be levied in the bad state when the non-performing loan rate is high, such national fiscal action resembles the actual fiscal austerity measures adopted in the Eurozone after crises. Indeed, as Figure 2 shows, during bad times in the peripheral countries, government primary balance actually rose, suggesting fiscal austerity after crises. Prices in this regime would turn out suppressed due to higher transaction costs. Therefore, Regime A is also referred to as the *internal devaluation* regime throughout the paper.

Figure 2: Fiscal austerity



Source: OECD (2018), OECD Economic Outlook: Statistics and Projections (database), annual data 2008-2017.

Although the main research question is about the benefits of softening the union-wide bankruptcy code as an alternative to a fiscal union for the viability of a currency union, it is still of interest to study the equilibrium with a fiscal union for welfare comparisons. **Regime B** therefore is modelled as a currency union featuring cross-country transfers via a fiscal union. It would resemble a hypothetical Eurozone with a fully-fledged fiscal union. Regime B's closest existing real-world equivalences are the US and China.[7] When the fiscal union makes cross-country transfers, I show that these transfers can ensure that the domestic banking sectors survive. Consequently, the pecuniary externality of banking bailout is removed and welfare improves.

When a fiscal union is absent however, **Regime C** is considered. In this regime, the union-wide bankruptcy code of the capital markets union is set more leniently than the domestic bankruptcy code.[8] As a result, domestic households may default on the cross-border financial securities in the bad state. The option to default in the cross-border capital market provides extra liquidity, acting like "cross-country transfers" from the rich to the poor to alleviate the stress of the otherwise failing banks. Consequently, there is no bailout cost or pecuniary externality of banking insolvency, and via the associated price effects, welfare improves in a Pareto sense. Thus, the internal devaluation effect via the bailout cost dissipates entirely, and the transaction costs in both countries decrease.

[7] In the US, different states use the same US dollar as the only stipulated means of exchange, and in China, different provinces share the same Chinese RMB. Both countries, if seen as currency union blocks in their own right, have their federal government or central government as the "fiscal union" to make cross-state or cross-province fiscal transfers (see the discussion about the US case in Sargent 2012).

[8] In practice, the default cost for unsecured lending takes a myriad of forms such as market/credit exclusion, sanctions, immediate liquidation, or harshness of the terms of debt restructuring. Take market/credit exclusion as an example, an ultra-tough bankruptcy code could mean when borrowers default, they are excluded from credit markets forever. A somewhat lenient bankruptcy code could mean upon default, per unit of default, defaulters are excluded from credit markets only for a certain period of time.

Nevertheless, a caveat of bankruptcy leniency remains: there exists a lower bound for the union-wide bankruptcy code. If it is set too leniently, then for all states of nature, no households in the currency union would ever repay cross-border borrowing, and the capital markets union would collapse. This scenario is inferior even to the internal devaluation regime because such an ultra-lenient union-wide bankruptcy code impedes cross-border risk sharing.

To corroborate the role of Regime C (bankruptcy leniency) in improving the viability of currency unions, the theory needs to explain why cross-border default via bankruptcy code adjustment is *particularly* vital for currency unions. To understand this question, it is important to know what benefits a currency union has given up and whether cross-border default can serve to recoup these benefits. Therefore, I extend the model to consider currency union dissolution and national currencies, a question largely unaddressed in the existing literature.

Regime D is such an extension. It considers national currencies and competitive floating exchange rates. I prove that under very general conditions, competitive floating exchanges indeed adjust for and neutralise domestic credit risks. Accordingly, banks survive and welfare improves. However, if a currency union is the *a priori* arrangement of member countries, in a parameterisation of the model I show that Regime C (bankruptcy leniency) obviates such needs for floating exchange rates to neutralise domestic credit risks. Essentially, removing nominal exchange rates implies rigidities in the country-level inflation. Since countries cannot rely on inflation as a form of "soft" default, the capital markets union should allow for actual default and acknowledge the underlying credit risks. Encouraging some degree of cross-border default by softening the union-wide bankruptcy code therefore provides a compensation for the lost benefits of nominal exchange rates.

The rest of the paper is structured as follows: Section 2 reviews the related literature. Section 3 presents the currency union model, various regime equilibrium analyses and the analytical results. Section 4 conducts a welfare and numerical analysis and provides the impetus for policy considerations. Section 5 investigates the national currency case. Section 6 discusses the results, policy implications, and two layers of institutional details (one on the Eurozone TARGET2 system and the other on the cross-border insolvency reforms in practice). Section 7 is a conclusion.

2 Related Literature

A burgeoning emergence of academic endeavour has started tackling this issue of currency union viability.[9] Broadly, there are three types of proposals, albeit not orthogonal to one another and non-exhaustive: 1) fiscal unions (e.g., Farhi and Werning 2017; Kehoe and Pastorino 2017), 2) banking unions (e.g.,Martinez et al. 2019), and 3) union-wide safe assets (e.g., Brunnermeier et al. 2016). The welfare improvement hinges on whether these proposals can reduce transaction costs and improve risk sharing; however, pragmatically, the aforementioned proposals may encounter political resistance from nation states within the currency union. Therefore, the political economic consideration of these three proposals is likely to spark off further exciting research (see Foarta 2018). My work contributes to existing work by considering an

[9] See Brunnermeier and Reis (2015) for an excellent summary of recent theories.

alternative financial regime, i.e. capital markets union (see Martinez et al. 2019). The key step forward of my work is that I consider the economics of bankruptcy within the capital markets union in the presence of credit risks, which are omitted in Martinez et al. (2019). This financial regime is plausible because the forces work through the invisible hand of the markets, which might encounter less political resistance in terms of implementation.

My work is also related to the rich body of literature on optimal currency areas that starts with Mundell (1961); McKinnon (1963); Kenen (1969). Subsequently the new open economy macro literature (salient examples include Obstfeld and Rogoff 2000; Gali and Monacelli 2005) builds micro-foundations and is applied to provide concrete welfare analysis on the monetary and fiscal issues in currency unions (see Gali and Monacelli 2008; Ferrero 2009; Aguiar et al. 2015; Farhi and Werning 2017; Kehoe and Pastorino 2017; Adam and Grill 2017). My work complements this body of literature by introducing the finance elements and explicitly modelling the endogenous determination of the value of currencies. Moreover, I incorporate credit risks and moral hazard, which were at the forefront of the eurozone debt crisis. Rather than assuming a continuum of atomistic countries, I take the two-country international finance approach, which is a suitable laboratory to analyse the north-and-south dynamics in the Eurozone.

In terms of the broader message, my paper shares a similar kindred spirit to Adam and Grill (2017) and Goodhart et al. (2018) that cross-border default can conditionally benefit currency unions. However, these two papers do not explain why cross-border default is particularly vital to sustain a currency union. For example, the friction in Adam and Grill (2017) is the non-state contingent bond, which is not necessarily a friction that is specific to currency unions. It is known in the general equilibrium theory with incomplete markets that strategic default can improve risk sharing by increasing the asset span (see Zame 1993; Dubey et al. 2005). Thus, the welfare improvement result found in Adam and Grill (2017) is expected and should hold qualitatively, even assuming away currency unions. Different from Goodhart et al. (2018) and Adam and Grill (2017), my work explicitly models the arrangement of sharing the common currency. Meanwhile, my model makes available state-contingent nominal financial securities in order to show that cross-border default is needed for risk sharing due to being bound by the common currency in the presence of domestic credit risks. Therefore, my model is able to conduct the counterfactual experiment on currency union dissolution and compare and contrast a currency union with the case of national currencies and competitive flexible exchange rates.

To model a currency union with banking fragility (e.g. the Eurozone Crisis), the model should explicitly include the currency, banks, liquidity, and credit. Therefore, I choose an international finance modelling framework based on the seminal papers by Geanakoplos and Tsomocos (2002); Tsomocos (2008); Peiris and Tsomocos (2015). Geanakoplos and Tsomocos (2002) model a general equilibrium to unify international trade and finance. Their model is rich enough to include multiple goods, multiple countries, multiple consumers in each country, multiple time periods, multiple credit markets, and multiple currencies. The authors prove the existence of the equilibrium. Because of the role of money and the heterogeneity of markets and agents, the authors prove that fiscal and monetary policy both have real effects even under flexible prices. Parallel to Geanakoplos and Tsomocos (2002), Tsomocos (2008) proves generic determinacy and money non-neutrality of international monetary equilibria. The author obtains price-level determinacy and the endogenous determination of exchange rates in a rich general equilibrium. Further enriching Geanakoplos and Tsomocos (2002), Peiris and Tsomocos (2015) develop an international finance model with incomplete markets and relax the assumption of fully committed debt repayment. The authors prove the equilibrium existence and obtain a non-trivial role for monetary policy with incomplete markets and credit risks. These frameworks incorporate money and financial frictions into international trade, sharing a similar spirit to Manova (2012).

In this paper, I simplify and modify Peiris and Tsomocos (2015) to consider the special case of a currency union. Rather than assuming each country has one independent central bank as in Peiris and Tsomocos (2015), I assume that countries share the same central bank and I also consider the risk-shifting of domestic commercial banks, which are not present in Peiris and Tsomocos (2015). These modifications allow me to isolate the impact of sharing a common currency on the seigniorage split between domestic commercial banks and the union-wide central bank (see Reis 2013).

Money and liquidity creation via bank credit are key features of this paper. This mechanism was much emphasised by early economists when the banking sector was just booming. Classic works by Macleod (1866), Wicksell (1906), Hahn (1920), Hawtrey (1923), Schumpeter (1954), Keynes (1931), Tobin (1963) and Minsky (1977) have all provided insight into this monetary operation and its macro-financial implications. The early formalisation of this mechanism can be found in the general equilibrium theory of money. In this literature, there is an assumed requirement that money must be used to carry out transactions formalised through cash-in-advance constraints similar to Grandmont and Younes (1972, 1973); Lucas Jr and Stokey (1987). Inside money enters the economy against an offsetting obligation that guarantees its departure, and it is issued when borrowing agents apply for loans from the banks. As in Tsomocos (2003), commercial banks can be viewed as creators of “money” à la Tobin (1963). Some quantity of money, called outside money, is present as agents' initial monetary endowment that is used to pay for loan interest. The banking sector therefore can be either an intermediary of existing money or a creator of new inside money, as in Dubey and Geanakoplos (1992, 2003b, 2006), Bloise et al. (2005), Bloise and Polemarchakis (2006), Tsomocos (2003), and Goodhart et al. (2006, 2013).

Following the 2007-2009 Global Financial Crisis, there has been a revival of inside money modelling due to the renewed interest in banks' balance sheet transformation for credit extension and liquidity creation and the associated macro-financial outcomes. Recent advances include and are not limited to Bigio and Weill (2016), Brunnermeier and Sannikov (2016) , Faure and Gersbach (2017), Donaldson et al. (2018), Kumhof and Wang (2018), Bianchi and Bigio (2018), Piazzesi and Schneider (2018), McMahon et al. (2018), Kiyotaki and Moore (2018a), Kiyotaki and Moore (2018b), and Tsomocos and Wang (2019). Sharing a similar spirit to inside money provision against bank credit, liquidity creation is also much emphasised in the literature on banking (see Gorton and Pennacchi 1990; Diamond and Rajan 2001; Stein 2012; Hart and Zingales 2014; DeAngelo and Stulz 2015) and safe assets (see J Caballero and Farhi 2017).

In addition to banks and liquidity, the second key ingredient of my work is endogenous default, and it connects with a large body of literature on strategic sovereign default. Although I do not explicitly model the default decision by a separate government, in my model the default decision of the atomistic households in a given country is interpreted as the aggregate default at the country level. Typically there are two ways of thinking about default at the country level: 1) strategic default via explicit default costs (e.g. Eaton and Gersovitz 1981; Aguiar and Gopinath 2006; Arellano 2008; Arellano and Ramanarayanan 2012; Na et al. 2018) and 2) default without explicit costs but driven by political considerations (e.g.Guembel and Sussman 2009; D'Erasmo and Mendoza 2016). My paper belongs to the first group. As argued in Eaton and Gersovitz (1981), strategic default is suitable to analyse the trade-off of country-level default, because any negative net worth criterion for a country-level default is essentially irrelevant.

At the country level, default punishment can take a myriad of forms that range from credit or market exclusion (e.g. Eaton and Gersovitz 1981; Aguiar and Gopinath 2006; Arellano and Ramanarayanan 2012; Na et al. 2018) to sanctions (see the discussion in Bulow and Rogoff 1989), and from the loss of insurance opportunities (e.g. Bloise et al. 2017) to internal devaluation (e.g. Regime A of this paper). In light of this consideration, in this paper I do not model the various specific forms of punishment but assume a non-pecuniary default penalty à la Shubik and Wilson (1977) and Dubey et al. (2005). The intensity parameter λ of the default penalty is interpreted as the bankruptcy code in my model. Unlike Eaton and Gersovitz (1981); Aguiar and Gopinath (2006); Arellano and Ramanarayanan (2012); Na et al. (2018) that model default as a binary decision, my paper emphasises that the social cost of default depends on the severity of default; hence, partial default is also considered. Modelling partial default is also found in Calvo (1988); Bolton and Jeanne (2007); Corsetti and Dedola (2013); Adam and Grill (2017) and is in line with empirical evidence (see Trebesch and Zabel 2017) and quantitative findings (see Gordon and Guerron-Quintana 2018). I acknowledge that an alternative way of modelling default punishment would be to collateralise lending. However, because I model aggregate debt positions at a country level, seizing "collaterals" at a country level would imply further political frictions outside the scope of this paper. In light of this issue, I have only considered uncollateralised lending.

Moreover, the model extension of this paper connects with the body of literature on the cost and benefit of flexible exchange rates and the nexus between nominal exchange rates and default. For example, Neumeyer (1998) acknowledges that the general belief that "excessive" exchange rate variability harms the economy is difficult to prove in a formal setting. However, the author shows when the excess exchange rate risk is driven by political factors that influence monetary affairs, flexible exchange rate causes inefficiency. Guembel and Sussman (2004) use a market microstructure approach to obtain optimal exchange rates, and the authors assume markets are incomplete so that the cost of flexible exchange rates stems from its volatility that impedes risk sharing. In the model extension of my paper in which national currencies are considered, I deliberately choose not to model any cost of flexible exchange rates but only consider the potential benefits. This is because I want to pin down the upper bound of the lost benefits by removing nominal exchange rates and see how much cross-border default in currency unions can recoup the lost benefits of flexible exchange rates. Indeed, a key benefit of flexible exchange rate in my model extension is to neutralise domestic credit risks such that banks remain solvent. The role of nominal exchange rate therefore is to provide a buffer for country-level default, an insight reminiscent of a key point from Uribe (2006).

Finally, as my modelling environment features agent heterogeneity, banking and liquidity, and financial assets, it follows that default risk premium, rents extracted by the banking sector, bailout costs, and the value of the currency all affect the stochastic discount factor (SDF) for asset pricing. The exact specification for SDF depends on each regime considered. Such asset pricing formulae complement the growing body of theoretical and empirical literature on intermediary asset pricing (see He and Krishnamurthy 2013; Adrian et al. 2014; He et al. 2017; Bongaerts et al. 2017; Kondor and Vayanos 2019).

3 The Model - Currency Union

The model is a simple two-country endowment economy with uncertainty, and both aggregate endowment risks and idiosyncratic income risks are present. There are two types of consumption goods available for international trade, and each country has only one type of consumption good. In each country reside a domestic commercial banking sector and a continuum of households. A union-wide central bank acts as lender of last resort of issuing the common currency to the two national commercial banking sectors. The common currency is fiat because it does not enter utility functions. Households borrow from commercial banking sectors to obtain the common currency for transactions.

3.1 Model Description

The economy has three periods, $t \in T = \{0, 1, 2\}$, with date $t = 1$ having S states of nature which I index with $\boldsymbol{s} \in S = \{1, .., S\}$. Including date $t = 0$, there are $S + 1$ date-events in the set $S^* = \{0, 1, .., S\}$. Consumption happens at $t = 0, 1$, and date $t = 2$ is for any outstanding loan settlement. For simplicity there is no discounting. The two countries are indexed by $H \in \{I, J\}$ where trade occurs at prices denominated in a common currency. Country I has a measure 1 of households i and the commercial banking sector i, and country J has a measure 1 of households j and the commercial banking sector j.

Households in both countries are risk-averse and consumption goods are all perishable. In country I, households i are endowed with outside money m^i in the common currency and domestic consumption good e^i_{I0} at $t = 0$. At $t = 1$, households i are endowed with state contingent domestic consumption goods $e^i_I = (e^i_{I1}, .., e^i_{I\boldsymbol{s}}, .., e^i_{IS}) \in \mathbb{R}^S_+$. Similarly, in country J, households are endowed with outside money m^j in the common currency and domestic consumption good c^j_{J0} at $t = 0$. At $t = 1$, households j are endowed with state contingent domestic consumption goods $e^j_J = (e^j_{J1}, .., e^j_{J\boldsymbol{s}}, .., e^j_{JS}) \in \mathbb{R}^S_+$. In every state of nature, the two types of goods are traded at nominal spot prices $p_I = (p_{I0}, p_{I1}, ..., p_{I\boldsymbol{s}}, ...p_{IS}) \in \mathbb{R}^{S^*}_+$ and $p_J = (p_{J0}, p_{J1}, ..., p_{J\boldsymbol{s}}, ..., p_{JS}) \in \mathbb{R}^{S^*}_+$ in the common currency. Given two types of contingent endowments and households' preferences, both aggregate endowment risks and idiosyncratic income risks can be captured.

To link cross-country trade and capital flows, I make available state-contingent *nominal* financial securities. These financial securities are akin to Arrow securities, but the payoff of the financial security for state $\boldsymbol{s}$ is 1 unit of the common currency, rather than 1 unit of good I or J. I assume the number of these financial securities is the same as the number of states, and I call these securities as the nominal Arrow securities. The set of state prices is denoted as $\pi = (\pi_1, ..\pi_l, .., \pi_S) \in \mathbb{R}^S_+$. These financial securities are traded on exchanges, so I have in mind this huge anonymous international capital market in a currency union. Therefore, cross-country lenders and borrowers do not have one-on-one interactions.

In addition to the financial contracts, there are inter-period domestic loan contracts and interbank loan contracts that provide liquidity in the common currency as the only stipulated means of exchange. The union-wide central bank lends interbank loans (μ^i_{CB}, μ^j_{CB}) to provide the common currency to the commercial banking sectors in the two countries. In each country, there is a domestic commercial bank sector that extends loans (μ^i_I or μ^j_J) to provide liquidity in the common currency to its respective domestic households, as in Assumption 1.

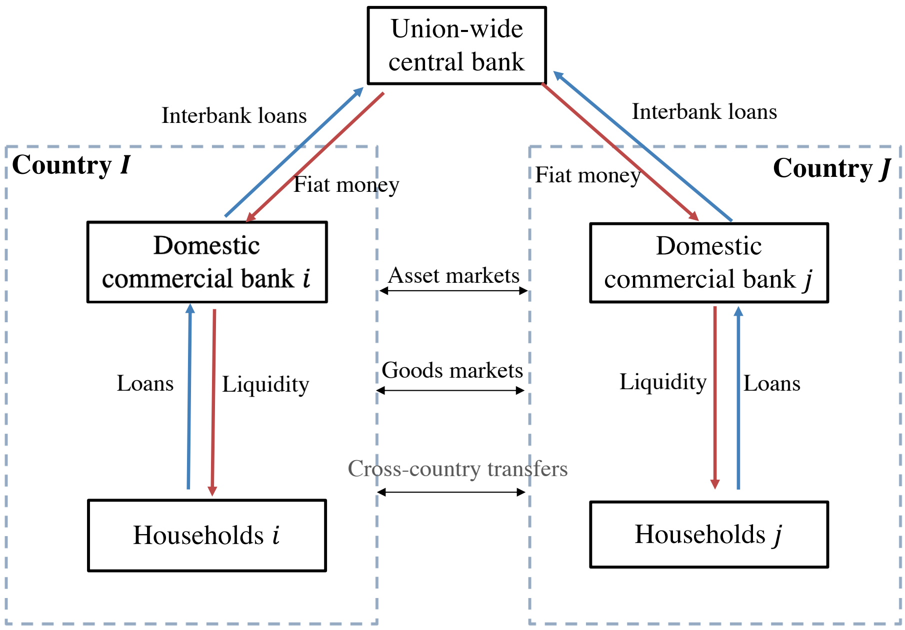
Assumption 1 (bank lending). *In terms of loans to non-bank sectors, commercial banks only grant loans to domestic non-bank sectors, but not foreign non-bank sectors.*

This assumption is based on the strong "home bias" of bank lending in the Eurozone widely documented in empirical literature (see Acharya and Steffen 2015; Becker and Ivashina 2017; Gabrieli and Labonne 2018; Ongena et al. 2018). It also reflects the doom-loop in the Eurozone à la Brunnermeier et al. (2016) and Farhi and Tirole (2017) that banks in the eurozone hold disproportionately large amount of national debt or bonds issued by their own sovereigns.[10]

The capital markets union takes the form of financial asset markets that facilitate cross-country capital flow. Following Shubik and Wilson (1977) and Dubey et al. (2005), market participants choose how much to deliver for asset payoffs, and the asset market is assumed an anonymous market with promises between different sellers not allowed to be distinguished even though they may deliver differently. This assumption implies that the expected delivery rates of the financial securities denoted as K are macro variables taken as given by the households, in the same tradition as the competitive market environment. All deliveries are pooled and buyers of the pool for each financial security receive a pro rata share of the net deliveries. Each ownership share of the pool of the financial security $\boldsymbol{s}$ receives a fraction $K_{\boldsymbol{s}} \in [0, 1]$ of the promised delivery in state $\boldsymbol{s}$.

[10] As many existing works have endogenised this home bias or relationship lending either in the eurozone context or in a broader context (see Acharya and Rajan 2013; Gennaioli et al. 2014; Uhlig 2014; Acharya et al. 2014; Farhi and Tirole 2017); therefore, I do not seek to provide further microfoundations for Assumption 1 in this paper.

Figure 3: Nominal flows of the economy



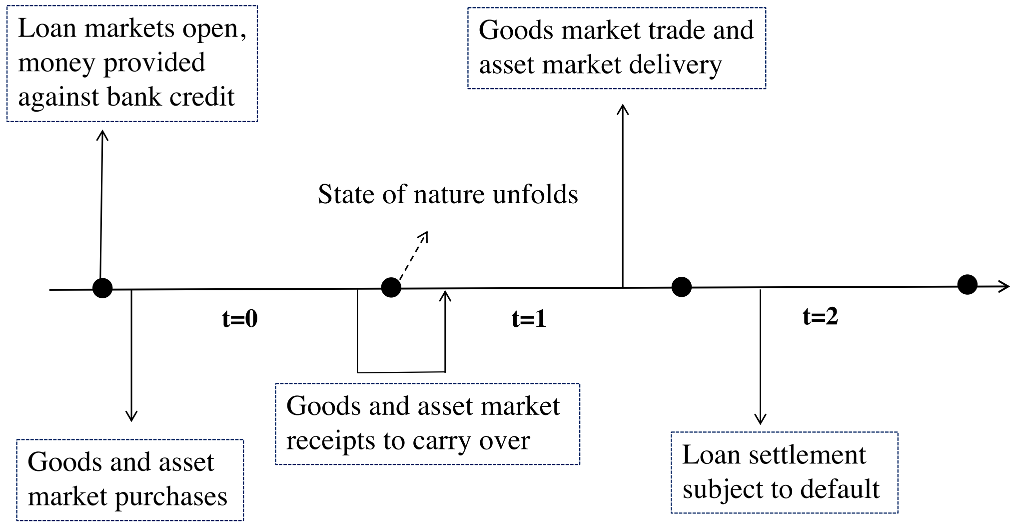
The model structure and agents' interactions are depicted in Fig (3). Linking the two member countries are the union-wide central bank and interbank markets, capital asset markets, goods markets, and possibly cross-country transfers via a fiscal/banking union.

Assumption 2 (means of transaction). *Money is used to facilitate transactions due to a high searching cost and lack of double coincidence of wants.*

Assumption 2 together with the banking structure illustrated in Fig (3) implies that all transactions are carried out in the common currency, and that households face cash-in-advance constraints akin to Lucas Jr and Stokey (1987). This assumption helps to make explicit the issuance of the single currency against credit.[11] Figure (4) shows the timeline. At $t = 0$, loan markets open so that fiat money in the common currency is issued against bank loans. Households use money to buy assets and imports, and they carry the monetary proceeds of selling assets and exports to $t = 1$. Uncertainty unfolds at $t = 1$, assets deliver nominal payoffs and goods are traded. At $t = 2$, households use money at hand to settle outstanding loans. I make the sequence precise when I formally describe the budget constraints and the flow of funds.

[11]For a detailed characterisation of money and credit in a more general setting, please see Gu et al. (2016) who incorporate frictions such as spatial or temporal separation, imperfect information and limited commitment.

Figure 4: Timeline



3.2 Country I

Country I's modelling is described in detail. The modelling of country J is exactly symmetric to that of country I and the endowment shocks are asymmetric (see country J in Appendix A).

Households i

Households i consume at $t = 1, 2$ and derive utility from the two tradable goods, i.e. the domestic consumption good of $c^i_{Is^*}$ and the foreign consumption good of $c^i_{Js^*}$. Additionally, households i will suffer a non-pecuniary penalty if they default.

Let $(\forall \boldsymbol{s} \in S)$,

$v^i_{\boldsymbol{s}} \equiv$ the households' choice of repayment rate on domestic loans (i.e., the NPL rate is $1 - v^i_{\boldsymbol{s}}$),

$\theta^i_{\boldsymbol{s}} \equiv$ the long position in the asset markets,

$\phi^i_{\boldsymbol{s}} \equiv$ the short position in the asset markets,

$D^i_{\boldsymbol{s}} \equiv$ the choice of asset delivery,

$q^i_I \equiv$ the quantity of exports,

$b^i_J \equiv$ the amount of money spent on imports,

$p_{\boldsymbol{s}} \equiv$ the union-wide price level index at $t = 1$,

$[d^i_{\boldsymbol{s}}]^+ \equiv$ the size of default on domestic loans in real terms, where

$$[d^i_{\boldsymbol{s}}]^+ = \frac{max[(1 - v^i_{\boldsymbol{s}})\mu^i_I, 0]}{p_{\boldsymbol{s}}},$$

$[f^i_{\boldsymbol{s}}]^+ \equiv$ the size of default in asset markets in real terms, where

$$[f^i_{\boldsymbol{s}}]^+ = \frac{max[\phi^i_{\boldsymbol{s}} - D^i_{\boldsymbol{s}}, 0]}{p_{\boldsymbol{s}}}.$$

Formally households' preference is given as follows:

$$\underbrace{Max}_{\mu^i_I,\theta^i,\phi^i,c^i_I,c^i_J,q^i_I,b^i_J,v^i,D^i} \quad E_0\Big\{U^i\Big(c^i_{I0}, c^i_{J0}, c^i_{I\boldsymbol{s}}, c^i_{J\boldsymbol{s}}\Big) - \lambda^i[d^i_{\boldsymbol{s}}]^+ - \lambda[f^i_{\boldsymbol{s}}]^+\Big\},$$

where the preference over consumption goods $U(\cdot)$ is assumed to be homothetic, strictly increasing, concave, and differentiable. The disutility from default is separable from consumption utility and is linear in the amount of default. The λ^i denotes the domestic default penalty harshness, which is interpreted as the domestic bankruptcy code throughout the paper. I denote the union-wide bankruptcy code as λ. Default can be either strategic or due to ill fortune, but creditors cannot observe why borrowers default. The households evaluate their own marginal benefit from default and marginal cost of default. If the former is larger than the latter, households default strategically even if there are resources at hand.

Households i choose the amount of domestic loans of μ^i_I to borrow, the quantity of nominal Arrow securities of θ^i to buy, the quantity of nominal Arrow securities of ϕ^i to sell, the quantity of domestic goods of c^i_I to consume, the amount of importing goods of c^i_J to consume, the amount of exporting goods of q^i_I, the amount of money of b^i_J to spend on imports, the loan repayment rate of v^i, and total asset delivery of D^i.

Let Δ denote any unused money from the corresponding flow of funds constraint, let $\eta^i_0, \eta^i_{1\boldsymbol{s}}, \eta^i_{2\boldsymbol{s}}$ be the shadow price of the corresponding constraint, let r_I be the domestic loan rate and $\tau_{I\boldsymbol{s}}$ be the domestic tax rate, let $K_{\boldsymbol{s}}$ be the aggregate delivery rates of the nominal Arrow security $l = \boldsymbol{s}$, and let $\delta^i_{\boldsymbol{s}}$ be any *potential* cross-country transfer. When a fiscal union is absent, then $\delta^i_{\boldsymbol{s}}$ is simply set to 0 for all states. Households i are subject to the following budget sets and flow of funds constraints.

At $t = 0$:

$$b^i_{J0} + \sum_{l=1}^{S} \pi_l \theta^i_l \leq \frac{\mu^i_I}{1 + r_I} + m^i. \qquad \eta^i_0 \quad (1)$$

At $t = 1$ $\forall \boldsymbol{s} \in S$:

$$b^i_{J\boldsymbol{s}}(1 + \tau_{I\boldsymbol{s}}) + (D^i_{\boldsymbol{s}} - K_{\boldsymbol{s}}\theta^i_{\boldsymbol{s}}) + \phi^i_{\boldsymbol{s}}\tau_{I\boldsymbol{s}} \leq \Delta(1) + \sum_{l=1}^{S} \pi_l \phi^i_l + p_{I0}q^i_{I0} + p_{I\boldsymbol{s}}q^i_{I\boldsymbol{s}} + \delta^i_{\boldsymbol{s}}. \qquad \eta^i_{1\boldsymbol{s}} \quad (2)$$

At $t = 2$:

$$v_s^i \mu_I^i \leq \Delta(2)_{\boldsymbol{s}}. \qquad \eta_{2\boldsymbol{s}}^i \quad (3)$$

And for $\boldsymbol{s}^* \in S^*$, the feasibility constraints are satisfied, i.e., $c_{I\boldsymbol{s}^*}^i \leq e_{I\boldsymbol{s}^*}^i - q_{I\boldsymbol{s}^*}^i$ and $c_{J\boldsymbol{s}^*}^i \leq \frac{b_{J\boldsymbol{s}^*}^i}{p_{J\boldsymbol{s}^*}}$.

Condition (1) states that at $t = 0$, households apply for an inter-period loan[12] of μ_I^i from the domestic commercial banking sector at the loan rate r_I to obtain inside money. Households i use the money inflow from the domestic commercial banking sector, plus any outside money of m^i to buy nominal Arrow securities and imports (*money as a means of transaction*). At the same time, households i receive monetary income from selling securities and exports for a total of $\sum_{l=1}^{S} \pi_l \phi_l^i + p_{I0} q_{I0}^i$ and carry it over into $t = 1$ (*money as a store of value*).

Condition (2) states that at $t = 1$, households i use the monetary income from $t = 0$ and export income of $t = 1$ plus any unused money $\Delta(1)$ and cross-country fiscal transfer of $\delta_{\boldsymbol{s}}^i$ (if any) to spend on the imports of $b_{J\boldsymbol{s}}^i$ and to deliver the net monetary payoff of $D_{\boldsymbol{s}}^i - K_{\boldsymbol{s}} \theta_{\boldsymbol{s}}^i$ for the security $l = \boldsymbol{s}$. Moreover, import expenditures and cross-country borrowing are subject to a state-contingent tax levied by the national government for a possible bailout fund.

At $t = 2$, households use the residual money from $t = 1$ to settle the domestic loan and choose how much to repay or default (see Condition (3)). This loan settlement constraint is equivalent to the transversality condition in infinite horizon models.

Domestic Commercial Banking Sector i

Bank i is the domestic commercial banking sector in country I. Bank i extends loans to domestic households and provides liquidity for the households to make purchases. To ensure the liquidity bank i provides would have a one-to-one convertibility to the common currency the union-wide central bank issues, bank i needs to borrow interbank loans from the union-wide central bank to meet the liquidity demands from domestic households. In this sense, commercial banks act as the "creators of money" à la Tobin (1963), with the central bank being the ultimate fiat money issuer.[13]

Bank i needs to make the following choices. It needs to choose how much domestic liquidity of $\mu_I^i/(1 + r_I)$ to supply to the household, how much interbank loans μ_{CB}^i to borrow from the union-wide central bank to obtain the fiat money denominated in the common currency, how much interbank liquidity L^i to make available to ensure that the liquidity supplied to domestic households has a one-to-one convertibility to the common currency obtained from the central bank. Bank i maximises its franchise value, defined as the average payoff across states weighted by the risk-neutral probabilities. Formally,

[12] The modelling of the inter-period loan reflects the reality that the bank's asset is typically less liquid than its liability, i.e. money in this case.

[13] In practice, when individual commercial banks supply loans they immediately write deposits as IOU notes for the borrowers, but the deposits are convertible to central bank reserves with the central bank being the lender of the last resort. Therefore, ultimately fiat money is issued by the central bank, and the commercial banks are a risk-shifting "pass-through" of central bank fiat money.

$$\underbrace{Max}_{\mu^i_{CB},\mu^i_I,L^i,\omega^i} \sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}}\omega^i_s,$$

where ω^i_s is bank i's nominal profits for state $\boldsymbol{s}$ and $z_{\boldsymbol{s}}$ is the risk-neutral probability for state $\boldsymbol{s}$. Let ρ be the interbank rate, and let $R^i_{\boldsymbol{s}}$ be the bank's expected repayment rate of the households. Bank i is subject to the following flow of funds constraints:

$$L^i \leq \frac{\mu^i_{CB}}{1+\rho}, \tag{4}$$

$$\frac{\mu^i_I}{1+r_I} \leq L^i, \tag{5}$$

$$\omega^i_{\boldsymbol{s}} = \Delta(4) + \Delta(5) + R^i_{\boldsymbol{s}}\mu^i_I - \mu^i_{CB}. \tag{6}$$

At $t = 0$, bank i borrows interbank loans from the union-wide central bank and obtains fiat money in the common currency, ready to be extended as interbank liquidity of L^i. This is shown in Condition (4).

Meanwhile when bank i extends commercial loans of μ^i_I to the households, it must ensure the liquidity bank i provides against the bank loans has a one-to-one convertibility to the fiat money issued by the central bank. This is shown in Condition (5) and Lemma 2, which shall prove that Condition (5) is binding whenever $\rho > 0$. Eq (6) states at $t = 2$, $\forall \boldsymbol{s} \in S$, bank i uses the households' loan repayment to pay back the interbank loans[14] to the union-wide central bank, and the difference between these two repayments adds to bank i's net cash flow, i.e. profits.

Depending on the NPL rate of $1 - v^i_{\boldsymbol{s}}$ for state $\boldsymbol{s} \in S$, bank i's nominal profits $\omega^i_{\boldsymbol{s}}$ could be negative and that bank i becomes insolvent. Given the characteristics of different regimes to be specified in Proposition 2, there may be cross-country transfers of $\delta^{bI}_{\boldsymbol{s}}$ or domestic government bailout funds of $T_{I\boldsymbol{s}}$ injected to bank i. I define $\omega^{i'}_{\boldsymbol{s}}$ as the after-bailout net cash inflow to bank i, $\forall \boldsymbol{s} \in S$, i.e., $\omega^{i'}_{\boldsymbol{s}} = \omega^i_{\boldsymbol{s}} + \delta^{bI}_{\boldsymbol{s}} + T_{I\boldsymbol{s}}$.

National Government i

National government i collects taxes from domestic households to build a state-contingent bailout fund of $T_{I\boldsymbol{s}}$. Assumption 3 implies that national government will levy tax to bail out the domestic banking sector, should the government foresee domestic banking insolvency in a particular state. The households and the domestic commercial banking sector are assumed uninformed at $t = 0$ of the national government's contingent action at $t = 1$. At $t = 1$, they take the government's action as given.

Assumption 3 (bailout). *The insolvency of the domestic banking sector incurs a high social cost.*

[14] Interbank loans are modelled as non-defautable. When the Euro Crisis emerged, the authorities arranged the form of the rescue to make sure that there was no default on the interbank loans that French and German banks had provided to the Greeks.

The social cost in Assumption 3 can be interpreted in two dimensions. When the domestic commercial banking sector incurs negative profits and becomes insolvent, it is unable to pay back the interbank loans to the union-wide central bank. One dimension of the social cost is that domestic banking insolvency means defaulting on the union-wide central bank. The implication is that the country as a whole may lose the membership of being in the currency union. The other dimension is that domestic banking insolvency would require huge resources for the national government to restore its domestic banking system. Given such considerations, the national government will bail out the domestic banking system should it foresee domestic banking insolvency in a certain state.

To collect the bailout fund, the national government levies taxes based on import expenditures and cross-country borrowing as in Eq (7),[15] reflecting the point that in a bad state, the government resorts to fiscal austerity to bailout the domestic banking system.

$$T_{I\boldsymbol{s}} = p_{J\boldsymbol{s}} c^{i}_{J\boldsymbol{s}} \tau_{I\boldsymbol{s}} + \phi^{i}_{\boldsymbol{s}} \tau_{I\boldsymbol{s}}. \tag{7}$$

National government i uses the bailout funds to rescue the domestic commercial banking sector whenever the banking sector's nominal profits (adjusting for possible cross-country transfers) would drop to negative, i.e. the bank fails. In short, in the bad state the national government makes a state-contingent transfer to ensure $\omega^{i'}_{\boldsymbol{s}} = \omega^{i}_{\boldsymbol{s}} + \delta^{bI}_{\boldsymbol{s}} + T_{I\boldsymbol{s}} = 0$.

3.3 Union-wide Central Bank

The union-wide central bank lends interbank loans of μ^{h}_{CB}, $\forall h \in \{i, j\}$, and provides fiat money in the common currency to the two national commercial banking sectors. The union-wide central bank sets the interbank target rate of ρ as the policy rate.

To guarantee the determinacy of price level, the union-wide central bank, through the flow of funds of the banking system, collects households' outside money as the seigniorage, but it does not redistribute the seigniorage within the same period. In this sense, the treatment of seigniorage in this model is non-Ricardian (Sims 1994; Buiter 1999). This approach follows Dubey and Geanakoplos (1992, 2006) and Tsomocos (2003). It resonates the institutional separation between a central bank and a government, and takes the view that price-level determinacy in equilibrium reflects the central bank mandate on price stability.

3.4 Equilibrium

The currency union equilibrium is defined as an allocation $(c^{i}_{Is^*}, c^{i}_{Js^*}, c^{j}_{Is^*}, c^{j}_{Js^*}, b^{j}_{Is^*}, b^{i}_{Js^*}, q^{i}_{Is^*}, q^{j}_{Js^*}, \theta^{h}_{l}, \phi^{h}_{l}, D^{h}_{\boldsymbol{s}}, \mu_I, \mu_J, \mu_{CB})$ with prices $(p_{I\boldsymbol{s}^*}, p_{J\boldsymbol{s}^*}, \pi_l, r_I, r_J, v^{h}_{\boldsymbol{s}}, R^{h}_{\boldsymbol{s}})$, given bankru--ptcy codes (λ^h, λ) and policy rate and fiscal rules $(\rho, \tau_H, \delta^h, \delta^{bH})$, $\forall \boldsymbol{s}^* \in S^*$, $\forall \boldsymbol{s} \in S$, $\forall l \in S$, $h \in \{i, j\}$, $H \in \{I, J\}$ such that agents maximise subject to liquidity-in-advance constraints and budget constraints, markets clear, and expectations are rational.

[15] Using tax rates rather than a lump-sum leads to clearer analytical expressions for propositions. The results are also robust to a lump-sum tax levy instead.

- Goods markets:

$$p_{I\boldsymbol{s}^*} q^i_{I\boldsymbol{s}^*} = b^j_{I\boldsymbol{s}^*},$$

$$p_{J\boldsymbol{s}^*} q^j_{J\boldsymbol{s}^*} = b^i_{J\boldsymbol{s}^*},$$

- Asset markets for the nominal Arrow security $\boldsymbol{s} = l$:

$$\sum_{h\in\{i,j\}} \phi^h_l = \sum_{h\in\{i,j\}} \theta^h_l,$$

- Domestic loan markets:

$$\frac{\mu^h_H}{1+r_H} = L^h,$$

- Interbank loan and money market:

$$1+\rho = \frac{\mu^i_{CB} + \mu^j_{CB}}{M},$$

- Rational expectation:

$$K_{\boldsymbol{s}} = \left\{ \begin{array}{cc} \frac{\sum_{h\in\{i,j\}} D^h_{\boldsymbol{s}}}{\sum_{h\in\{i,j\}} \phi^h_{\boldsymbol{s}}} & \text{if } \sum_{h\in\{i,j\}} \phi^h_{\boldsymbol{s}} > 0 \\ \text{arbitrary} & \text{if } \sum_{h\in\{i,j\}} \phi^h_{\boldsymbol{s}} = 0 \end{array} \right\},$$

$$R^h_{\boldsymbol{s}} = \left\{ \begin{array}{cc} v^h_{\boldsymbol{s}} & \text{if } 1 - v^h_{\boldsymbol{s}} > 0 \\ \text{arbitrary} & \text{if } 1 - v^h_{\boldsymbol{s}} = 0 \end{array} \right\}.$$

3.5 Equilibrium and Regime Characterisation

This subsection characterises the equilibrium and regimes. Suppose state s is a good state for country I and a bad state for country J, and state s' is a bad state for country I and a good state for country J, i.e. $e^i_{Is} > e^i_{Is'}$, $e^j_{Js} < e^j_{Js'}$. The subsequent analysis focuses on such asymmetric endowment shocks. Let $\gamma_{\boldsymbol{s}}$ the probability state $\boldsymbol{s}$ occurs, $\forall \boldsymbol{s} \in S$.

To ensure both nominal and real determinacy, Lemmas 1-3 prove the binding conditions of flow of funds constraints. Lemma 4 states the shadow price of the flow of funds constraint at $t = 1$. Lemma 5 and Proposition 1 characterise the equilibrium and Proposition 2 designs regimes of a currency union.

Lemma 1. Binding conditions of the Liquidity-in-advance constraints.

If $r_I > 0$, then $\Delta(1) = 0$.

If $\rho > 0$, then $\Delta(4) = 0$.

Proof. See Appendix C.1.

Lemma 2. Interbank liquidity and the single currency convertibility.

If $\rho > 0$, then $\Delta(5) = 0$.

Proof. See Appendix C.2.

Remark: That (5) binds means that the interbank liquidity the domestic banking sector i extends to the households is pegged one-to-one to the common currency issued by the union-wide central bank. This is not imposed *a priori* but rather a result of the non-arbitrage conditions from the interbank market.

Lemma 3. No worthless money at end.

If $r_I > 0$, then $\Delta_{\boldsymbol{s}}(3) = 0$.

If $\rho > 0$, then $\Delta_{\boldsymbol{s}}(6) = 0$.

Proof. See Appendix C.3.

Lemma 4. Heterogeneous tightness of nominal constraints.

In a currency union with trades in goods market and asset market, if $r_I, r_J > 0$ and no full default on loans, then $\eta^i_{1s} \neq \eta^i_{1s'}$ and/or $\eta^j_{1s} \neq \eta^j_{1s'}$.

Proof. See Appendix C.4.

Lemma 5. (zero credit risks and the loss of exchange rate): If in the currency union $\forall \boldsymbol{s} \in S$, $h \in \{i, j\}$, $v^h_{\boldsymbol{s}} = 1$, given markets are complete, domestic banking sectors break even for all states, i.e. $\omega^h_{\boldsymbol{s}} = 0$.

Proof. See Appendix C.5.

Claim. *With domestic credit risks, the loss of floating nominal exchange rates (i.e., currency unions) translates into a currency crisis, disguised as a banking debt crisis, i.e.,* $\omega^h_{\boldsymbol{s}} < 0, \exists \boldsymbol{s} \in S, h \in \{i, j\}$.

The above claim implies that the banking sectors in a currency union become more vulnerable due to losing the flexibility of exchange rates. Having a floating exchange rate might neutralise domestic credit risks and prevent such crises. Not to jump ahead of myself, I shall revisit this claim with a formal argument and proof in Proposition 4 of the equilibrium analysis of the currency union and Proposition 6 in Section 5 in which I consider national currencies and the role of nominal exchange rates.

Given that in a currency union, zero domestic credit risks in all states of nature as in Lemma 5 is unlikely to hold in reality, in the subsequent analysis, I only focus on the cases when domestic credit risks are present in a currency union. I also do not consider the case of 100 % non-performing loans where there exists a state in which the household defaults on domestic loans completely. Formally, let,

$$\overline{\Lambda}^h = \{\lambda^h : v^h_{\boldsymbol{s}} = 1, \forall \boldsymbol{s} \in S, h \in \{i, j\}\},$$
$$\underline{\Lambda}^h = \{\lambda^h : v^h_{\boldsymbol{s}} = 0, \exists \boldsymbol{s} \in S, h \in \{i, j\}\}.$$

Thus, $\overline{\Lambda}^h$ covers the cases of full delivery of domestic loans in all states, and $\underline{\Lambda}^h$ covers the cases in which there exists a state of full default on domestic loans. In *all* the subsequent analysis I restrict λ^h to be an intermediate default penalty for domestic loans, i.e. for $h \in \{i, j\}$, $\lambda^h \notin \overline{\Lambda}^h$ and $\lambda^h \notin \underline{\Lambda}^h$.

Proposition 1. (the Fisher effect, Quantity Theory of Money, and money non-neutrality):

- **The Fisher effect**: Suppose for households i, $b^i_{J\boldsymbol{s}^*} > 0$, $\forall \boldsymbol{s}^* \in S^*$. Suppose further that households i have some money left over the moment the domestic loan comes due at $\boldsymbol{s}$, then in equilibrium,

$$1 + r_I = \Big(E_0(\frac{U^i_{c^i_{J\boldsymbol{s}}}}{U^i_{c^i_{J0}}})(\frac{p_{J0}}{p_{J\boldsymbol{s}}})\frac{1}{(1+\tau_{I\boldsymbol{s}})}\Big)^{-1},$$

where $U^i_{c^i_{J0}}$ and $U^i_{c^i_{J\boldsymbol{s}}}$ are household i's marginal utilities of consuming imports at $t = 0$ and in state $\boldsymbol{s}$. A similar expression of the Fisher effect obtains for country J as well.

Taking the logarithm of the above Fisher equation and interpreting it loosely, the nominal interest rate equals the real interest rate plus the expected inflation adjusted by any bailout tax. Any tax needed for bank bailout in a currency union also distorts real allocation and inflation. As the bailout tax puts downward pressure on inflation and the real interest rate, and it resembles fiscal austerity, I call this distortionary effect the *internal devaluation* effect.

- **Quantity Theory of Money**: If $\rho > 0$, the aggregate income of the currency union, namely the nominal value of consumption goods sales is equal to the total stock of bank money and outside money, adjusted by asset trades and the bailout tax levy. Let $\Delta^i_{\boldsymbol{s}} = \Delta(2) - p_{I\boldsymbol{s}}q^i_{I\boldsymbol{s}}$, and likewise for household j,

$$p_{I0}q^i_{I0} + p_{J0}q^j_{J0} = M - \sum_{h\in\{i,j\}}\sum_{l=1}^{S}\pi_l\theta^h_l + \sum_{h\in\{i,j\}} m^h,$$

$$p_{I\boldsymbol{s}}q^i_{I\boldsymbol{s}} + p_{J\boldsymbol{s}}q^j_{J\boldsymbol{s}} = M + \sum_{h\in\{i,j\}} m^h - \sum_{H\in\{I,J\}} T_{H\boldsymbol{s}} - \sum_{h\in\{i,j]} \Delta^h_{\boldsymbol{s}}.$$

- **Money non-neutrality**: Suppose $\rho > 0$, any change in ρ results in a different equilibrium in which some households' consumption is different.

  Remark: Even with flexible prices, money and default render monetary policy non-neutral.

  Proof. See Appendix C.6.

Corollary 1.1. (credit risks and the term-structure of interest rates): Suppose $\rho > 0$, in a currency union with idiosyncratic credit risks, suppose $v^i_s > \sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}}v^i_{\boldsymbol{s}} > v^i_{s'}$ and $v^j_s < \sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}}v^j_{\boldsymbol{s}} < v^j_{s'}$, $\forall \boldsymbol{s} \in S$, the term structure of interest rates incorporates credit risks, and $\omega^i_s, \omega^j_{s'} > 0$.

In state s:

$$\rho M + (\frac{v_s^j}{\sum_{s=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^j} - 1)\mu_{CB}^j + T_s^j = \sum_{h\in\{i,j\}} m^h - \omega_s^i. \tag{8}$$

In state s':

$$\rho M + (\frac{v_{s'}^i}{\sum_{s=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^i} - 1)\mu_{CB}^i + T_{s'}^i = \sum_{h\in\{i,j\}} m^h - \omega_{s'}^j. \tag{9}$$

Proof. See Appendix C.7.

Corollary 1.2. (Monetary policy rate pass-through): For $H \in \{I, J\}, h \in \{i, j\}, \forall \boldsymbol{s} \in S$,

$$1 + r_H = \frac{1+\rho}{\sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h}. \tag{10}$$

Corollary 1.1 states that both the liquidity creation by banks and the credit risks of households affect the term structure of the interest rates. The left-hand side of the Eqs (8) (9) is the union-wide central bank's interest rate revenue for issuing the common currency. It equates the total outside money minus the rents extracted by commercial banks, to be collected by the union-wide central bank at $t = 2$. Note that the union-wide central bank does not collect all the outside money as profits. This amount of profits collected by the central bank is called seigniorage. No matter how small it is, it serves to obtain price-level determinacy.[16] Isomorphically, the term structure of interest rate in relation to the seigniorage can be interpreted as the nexus between fiat money and the fiscal sovereign. Indeed, Goodhart (1998) argues that seigniorage is part of the government's taxation plan, and as Tsomocos (2003) puts it, by collecting the seigniorage, "the government compels the acceptance of fiat money as a final discharge of debt".

Corollary 1.2 or Eq (10) shows the imperfect pass-through of the union-wide monetary policy hampered by credit risks. It states the borrowing cost at the national level equates the union-wide monetary policy rate adjusted for the expected domestic NPL rates. The implication is that a fall in the union-wide monetary policy rate does not necessarily translate to a loosened monetary condition at the national level, because the monetary policy pass-through is augmented with terms of financial contracts at the national level.

Apart from the classic results above, the key step forward of this international finance model is that it includes regime designs of a currency union by varying the domestic and cross-country bankruptcy codes ($\lambda^h, \lambda, h \in \{i, j\}$) and then their respective welfare properties are ranked. Which regime the currency union falls under is endogenous to the relative harshness of domestic and union-wide bankruptcy code. In different regimes, the terms in the term structure equations shall take on different values, and the term structure equations summarise the driving force of the specific structural

[16] For a general proof of determinacy, please see Dubey and Geanakoplos (2006) and Tsomocos (2008).

assumptions in the various regimes considered. Proposition 2 formalises the regime design.

Proposition 2. (domestic and union-wide bankruptcy codes):

- If the union-wide bankruptcy code is harsher than the domestic bankruptcy code, households fully deliver on financial assets. i.e., for $h \in \{i, j\}$,
  - if $\lambda > \lambda^h$, $D^h_{\boldsymbol{s}} = \phi^h_{\boldsymbol{s}}$ at state s.
- If the union-wide bankruptcy code is more lenient, households may default on financial assets. i.e., for $h \in \{i, j\}$,
  - if $\lambda < \lambda^h$, $0 \leq D^h_{\boldsymbol{s}} \leq \phi^h_{\boldsymbol{s}}$ at state s.

Proof. See Appendix C.8.

Proposition 2 states when the union-wide bankruptcy code is harsher than domestic bankruptcy codes, default in the cross-border capital markets does not occur; when the domestic bankruptcy code is harsher than the union-wide bankruptcy code, default in the cross-border capital markets may occur in equilibrium. Proposition 2 establishes the foundation for the design of the following three regimes. Formally, define $CA^H_{\boldsymbol{s}}$ as country H's current account net flow and $FA^H_{\boldsymbol{s}}$ as country H's capital account net flow at $t = 1$, i.e.,

$$\begin{aligned}
CA^I_{\boldsymbol{s}} &= p_{I\boldsymbol{s}} q^i_{I\boldsymbol{s}} - p_{J\boldsymbol{s}} c^i_{J\boldsymbol{s}}, \\
FA^I_{\boldsymbol{s}} &= K_{\boldsymbol{s}} \theta^i_{\boldsymbol{s}} - D^i_{\boldsymbol{s}}, \\
CA^J_{\boldsymbol{s}} &= p_{J\boldsymbol{s}} q^j_{J\boldsymbol{s}} - p_{I\boldsymbol{s}} c^j_{I\boldsymbol{s}}, \\
FA^J_{\boldsymbol{s}} &= K_{\boldsymbol{s}} \theta^j_{\boldsymbol{s}} - D^j_{\boldsymbol{s}}.
\end{aligned}$$

A positive CA means current account is running surplus and a positive FA means international capital inflow, and vice versa. With these definitions, I state the following regime designs.

- **Regime A (baseline)**: $\lambda > \lambda^h$, $\delta^h_{\boldsymbol{s}} = \delta^{bH}_{\boldsymbol{s}} = 0$, $T_{H\boldsymbol{s}} = -\omega^h_{\boldsymbol{s}}$ whenever $\omega^h_{\boldsymbol{s}} < 0$, and $T_{H\boldsymbol{s}} = 0$ whenever $\omega^h_{\boldsymbol{s}} \geq 0$, where $h \in \{i, j\}$, $H \in \{I, J\}$, $\forall \boldsymbol{s} \in S$.

  Regime A is the baseline currency union in which a punitive union-wide bankruptcy code prevents default in the cross-border capital markets, and a fiscal union is also ruled out. The domestic bailout tax is levied in the respective bad state to bailout the domestic banking system.

- **Regime B (fiscal union)**: A currency union supported by a fiscal union, and a punitive union-wide bankruptcy code prevents default in the cross-border capital markets, i.e., $\lambda > \lambda^h, T_{H\boldsymbol{s}} = 0$, for $h \in \{i, j\}$, $H \in \{I, J\}$, $\forall \boldsymbol{s} \in S$. I consider two cases for Regime B as follows.
  - **Regime B.a**: A fiscal union that makes cross-country fiscal transfers of $\delta^h_{\boldsymbol{s}}$ directly between households, and $\delta^h_{\boldsymbol{s}} = -FA^H_{\boldsymbol{s}} - CA^H_{\boldsymbol{s}}$ and $\delta^{bH}_{\boldsymbol{s}} = 0$. It

follows that $\sum_{h\in\{i,j\}} \delta^h_{\boldsymbol{s}} = 0$.[17]

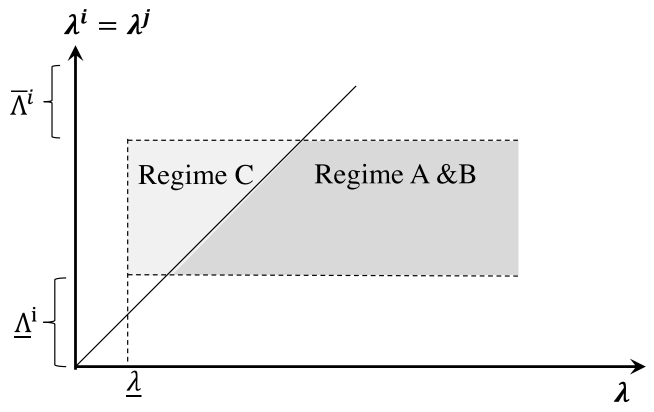
- **Regime B.b**: A fiscal union that makes cross-country fiscal transfer of $\delta^{bH}_{\boldsymbol{s}}$ directly between domestic commercial banks, such that $\omega^h_{\boldsymbol{s}} + \delta^{bH}_{\boldsymbol{s}} \geq 0$ and $\sum_{H\in\{I,J\}} \delta^{bH}_{\boldsymbol{s}} = 0$, $\delta^h_{\boldsymbol{s}} = 0$. Regime B.b can be interpreted as a banking union supported by a common fiscal entity.

- **Regime C (bankruptcy leniency)**: $\underline{\lambda} < \lambda < \lambda^h$, $\delta^h_{\boldsymbol{s}} = \delta^{bH}_{\boldsymbol{s}} = 0$, and $T_{H\boldsymbol{s}} = 0$.

  Regime C is a currency union with a more lenient union-wide bankruptcy code, but a fiscal union is ruled out and no domestic bailout tax is levied. In this regime, the bankruptcy code can induce endogenous default in the cross-border capital markets to emerge in equilibrium.

Note that the lower bound $\underline{\lambda}$ of the union-wide bankruptcy code in Regime C ensures the financial markets do not collapse. This is because if the union-wide bankruptcy code is too lenient, households in both countries would fully default on the financial assets; hence, assets would not be traded at $t = 0$. To sum up, Fig (5) illustrates the regions of default penalty harshness and the corresponding regimes of the currency union. The horizontal axis denotes the union-wide default penalty harshness λ, the north-pointing vertical axis denotes domestic default penalty harshness λ^i and λ^j. For the ease of illustration, $\lambda^i = \lambda^j$, but the equality does not need to hold in general. Focusing on the intermediate domestic default penalty harshness, Regime C belongs to the region where domestic bankruptcy codes are harsher than the union-wide bankruptcy code, and Regimes A and B belong to the region where the union-wide bankruptcy code is tougher than domestic bankruptcy codes.

Figure 5: Regimes and bankruptcy codes



3.6 Equilibrium Analysis

In this subsection, I show the welfare properties of each regime for allocations, risk sharing, and asset prices. In particular, propositions are given demonstrating the mechanism in which a lenient bankruptcy code for the capital markets union could improve welfare. A caveat is also given that if certain conditions are not met, the possibility of cross-border default could impede international risk sharing.

[17]Note that country H's Balance of Payment ($BoP^H_{\boldsymbol{s}}$) in state $\boldsymbol{s}$ is $BoP^H_{\boldsymbol{s}} = CA^H_{\boldsymbol{s}} + FA^H_{\boldsymbol{s}} + \delta^h_{\boldsymbol{s}}$.

The intuition of the potential benefit of cross-border default in the capital markets is that it provides extra liquidity for the borrower in the bad state such that risk sharing improves. Before formalising welfare improvement, we need to understand the mechanism how a lenient union-wide bankruptcy code can incentivise the borrower to grab the option to strategically default in the bad state. Suppose we are in Regime C with a relatively lenient cross-country bankruptcy code. The households in the bad state may fully default on its cross-country borrowing whereas the other households in the other country may fully repay, if they both have short positions on the Arrow security of this state. This is because the poor households have a high marginal utility of consumption, which would outweigh the marginal cost of default on Arrow securities, given a lenient cross-country bankruptcy code. However, the other households are rich in this state, and the marginal utility of consumption is low, which would push down the marginal benefit of default. When the marginal benefit of default is less than the marginal cost of default, this rich households would fully deliver despite the poor households' full default. Therefore, although the poor households would fully default, the aggregate default rate on the nominal Arrow security of that state actually would fall between 0 and 1.

Moreover, the poor households may enter both the short and long positions of the nominal Arrow security of the bad state. The poor households would buy this Arrow security to insure against the bad shock, but they may also sell this Arrow security at the same time because selling gives the option to default fully. The option to default on Arrow securities provides extra liquidity leading to a possible increase in consumption or a higher domestic loan repayment rate. This implies an increase in the households' utility. An interior solution can be obtained because although selling more of the Arrow security leads to extra liquidity due to default on the one hand, it implies this poor households would also need to buy more of this Arrow security such that market clears, and buying on the other hand incurs more cost of liquidity. Proposition 3 formalises the mechanism of cross-border default. Later on, Proposition 5 builds on Proposition 3 and proves Pareto improvement as a result of endogenous default in the cross-border capital markets.

Proposition 3. (strategic default on financial securities):

- When the union-wide bankruptcy code is lenient enough, households in the bad state may long and short the Arrow security of that state at the same time, and fully default on this Arrow security.
- Consider the case where $S = \{1, 2\}$, let $\gamma_1 = \gamma_2$, $e_1^i > e_2^i$, and $e_1^j < e_2^j$. Suppose that in equilibrium $\lambda < p_2\eta_{12}^i < \lambda^i$, $\eta_{12}^j < \frac{\lambda}{p_2}$ and $\eta_{11}^i < \eta_{12}^i$ holds. Then, $\phi_2^i, \phi_2^j, \theta_2^i > 0$, $\theta_2^j = 0$, $D_2^i = 0, D_2^j = \phi_2^j$, and $0 < K_2 < 1$ whenever $\Big(K_2 - \pi_2\Big(v_2^i(1 + r_I) - 1\Big)\Big)/p_{I2} > \pi_2 r_I/p_{I1}$. Similar logic follows for the other state.[18]

Proof. See Appendix D.1.

Corollary 3.1. When the union-wide bankruptcy code is too lenient, it impedes international risk sharing in the currency union.

Corollary 3.2. As domestic bankruptcy codes become more lenient, the room to

[18]In Section 4, an equilibrium with these characteristics is obtained.

adjust the union-wide bankruptcy code in the capital markets union decreases.

The insight of Proposition 3 is reminiscent of Example 2 in Dubey et al. (2005). A lenient cross-country bankruptcy code encourages the households in the bad state to default fully. Even though the poor households have nominal inflows on hand for delivery, they do not deliver anything while the rich households deliver fully! Default in this case is strategic and makes asset payoffs endogenous. Assets are still traded despite strategic default: the households in the poor country enter both long and short positions of Arrow securities and the households in the rich country only shorts Arrow securities of that state.

Note that financial securities are voluntarily traded despite the possibility of default, and no market participants are forced to buy or sell the financial securities. In this sense, the invisible hand of the markets provides "voluntary liquidity transfers" via endogenous default. This mechanism is in principle different from Regime B where a fiscal union employs a visible hand to move nominal resources directly. However, a caveat remains for Regime C (Corollary 3.1). Suppose now the union-wide bankruptcy code λ is set ultra-low, i.e. $\lambda < p_{s'}\eta^j_{1s'}$ or $\lambda < p_s\eta^i_{1s}$, then Arrow securities are not traded. The currency union loses risk sharing altogether. Therefore, there exist a lower bound and an upper bound for the union-wide bankruptcy code of the capital markets union.

Moreover, for the currency union to retain risk sharing and for the aforementioned default to occur in the cross-border capital markets in equilibrium, the union-wide bankruptcy code λ must fall into the interval $(p_{s'}\eta^j_{1s'}, \lambda^i) \cap (p_s\eta^i_{1s}, \frac{\lambda^j}{a_s})$ in equilibrium. As the domestic bankruptcy code λ^i or λ^j decrease, $||(p_{s'}\eta^j_{1s'}, \lambda^i) \cap (p_s\eta^i_{1s}, \lambda^j)||$ decreases. Thus, as domestic bankruptcy codes become more lenient, the range to set cross-country bankruptcy code shrinks (Corollary 3.2).

A key condition for Proposition 3 to go through is $\eta^j_{12} < \frac{\lambda}{p_2}$ in equilibrium (and its equivalent for state 1), which says the union-wide bankruptcy code is strict enough to prevent default of the rich households. This condition ensures that defaultable Arrow securities are still traded in equilibrium even when the households of the poor country in the bad state fully defaults. I call this condition *within-union standard*. When the two countries' fundamentals differ exceptionally or when domestic bankruptcy code(s) are too lenient or discretionary, the "within-union standard" may fail to satisfy. In this case Regime C causes asset trades to collapse.

In contrast to Regime C, Regime B.a and Regime B.b advocate using the visible hand of a common fiscal entity to make cross-country transfers. Whereas a fully-fledged fiscal union in a currency union may be highly controversial and politically infeasible, in practice, there have been small steps towards building union-wide transfer funds, for example, the concept of a banking union in the Eurozone. Therefore, it is of interest to investigate the properties of Regimes B.a and B.b.

Let $var(1 - v^{h,B.a}_{\boldsymbol{s}})$ be the variance of non-performing loan rate of households h in Regime B.a, let $var(1-v^{h,B.b}_{\boldsymbol{s}})$ be that of households h in Regime B.b, and $var(1-v^{h,A}_{\boldsymbol{s}})$ that of Regime A, where $h \in \{i, j\}$. Lemma 6 says the domestic credit risk volatility across states is smaller in Regime B.a than in Regime B.b and Regime A.

Lemma 6 (credit risk volatility):

- A fiscal union that mediates transfers between households can reduce domestic credit risk volatility across states.
- In Regime B.a, for $h \in \{i, j\}, H \in \{I, J\}$, suppose $\delta_{\boldsymbol{s}}^h = -FA_{\boldsymbol{s}}^H - CA_{\boldsymbol{s}}^H$, it follows that $\delta_{\boldsymbol{s}}^i + \delta_{\boldsymbol{s}}^j = 0$, and moreover,

$$var(1 - v_{\boldsymbol{s}}^{h,B.a}) \leq var(1 - v_{\boldsymbol{s}}^{h,B.b}),$$
$$var(1 - v_{\boldsymbol{s}}^{h,B.a}) \leq var(1 - v_{\boldsymbol{s}}^{h,A}).$$

Proof. See Appendix D.3.

Proposition 3 and Lemma 6 equip the currency union with distinct institutional features for the common objective to reduce domestic banking stress. Proposition 4 formalises the mechanisms whereby this objective is achieved.

Proposition 4. (capital flow and banking crisis): Suppose $\lambda^h \notin \overline{\Lambda}^h$ and $\lambda^h \notin \underline{\Lambda}^h$, for $h \in \{i, j\}, H \in \{I, J\}, \forall \boldsymbol{s} \in S$.

- In Regime A, the volatility of domestic credit risks and international capital flow can lead to domestic banking insolvency.
  - If $\lambda > \lambda^h$ and $\delta_{\boldsymbol{s}}^h = \delta_{\boldsymbol{s}}^{bH} = 0$, whenever $v_{\boldsymbol{s}}^h < \sum_{\boldsymbol{s}=1}^S z_{\boldsymbol{s}} v_{\boldsymbol{s}}^h$, then $\omega_{\boldsymbol{s}}^h < 0$ and $T_{H\boldsymbol{s}} = -\omega_{\boldsymbol{s}}^h$.
- In Regime B.a, international capital flow does not drive domestic banking insolvency.
  - If $\lambda > \lambda^h$, $\delta_{\boldsymbol{s}}^{bH} = 0$, and $T_{H\boldsymbol{s}} = 0$, setting $\delta_{\boldsymbol{s}}^h = -FA_{\boldsymbol{s}}^H - CA_{\boldsymbol{s}}^H$, then $\omega_{\boldsymbol{s}}^h = 0$.
- In Regime B.b, the banking union funds alleviate domestic banking stress.
  - If $\lambda > \lambda^h$, $\delta_{\boldsymbol{s}}^h = 0$, and $T_{H\boldsymbol{s}} = 0$, as long as $\omega_{\boldsymbol{s}}^j + \omega_{\boldsymbol{s}}^i \geq 0$, a banking union fund of $\delta_{\boldsymbol{s}}^{bH}$ can be set to transfer between bank i and bank j such that $\omega_{\boldsymbol{s}}^h + \delta_{\boldsymbol{s}}^{bH} \geq 0$ and $\sum_{H \in \{I,J\}} \delta_{\boldsymbol{s}}^{bH} = 0$.
- In Regime C, default in the cross-border capital markets may prevent domestic banking insolvency.
  - If $\underline{\lambda} < \lambda < \lambda^h$, $\delta_{\boldsymbol{s}}^h = \delta_{\boldsymbol{s}}^{bH} = 0$, and $T_{H\boldsymbol{s}} = 0$, under the conditions in Proposition 3 on strategic default, $\omega_{\boldsymbol{s}}^h = 0$.

Proof. See Appendix D.4.

Corollary 4.1. Under Proposition 4, Regimes B and C obviate the need for national bailout taxes whereas Regime A needs it.

Proposition 4 shows whether international capital flow may lead to domestic banking stress under various regimes in a currency union. In Regime A, if domestic bailout funds are unavailable, the domestic commercial banking sector fails in the bad state.

In Regime B.a, the visible hand of a fiscal union sets the amount of cross-country fiscal transfers mediated directly between households in different countries. Such transfers remove the stress from capital flow on member countries' domestic banking system. Domestic commercial banks survive even in the bad state. In Regime B.b, the fiscal union makes transfers directly between commercial banks of different countries, subject to the total banking union fund constraint $\omega_{\boldsymbol{s}}^{i} + \omega_{\boldsymbol{s}}^{j} \geq 0$. However, Regime B.a faces no such constraint. This constraint is in line with Bolton and Oehmke (2018) that study bank resolution of global banks and show that the loss-absorbing capital is shared across jurisdictions but faces implementation constraints.

In Regime C, despite cross-country fiscal transfers being unavailable, a softened union-wide bankruptcy code can help domestic banks bypass the pressure from international capital flow such that banks survive. The intuition is that a softened union-wide bankruptcy code gives the domestic households the choice to default on their cross-country borrowing in the bad state. Because the domestic bankruptcy code is tougher than the union-wide bankruptcy code in this regime, the marginal cost of default on domestic bank loans is higher than the household's marginal benefit of default. Rationally, the households choose not to default on domestic loans even in the bad state. The takeaway is the relative stance of domestic and cross-country bankruptcy codes can change incentives on the margin and shift domestic credit risks to international capital markets, relieving domestic banks from distress. Adjusting the union-wide bankruptcy code, cross-border default provides a "voluntary" liquidity transfer via the capital markets, in the absence of a fiscal union.

With different extent of domestic banking stress and accordingly distinct needs for national level bailout tax, the regimes designed above bear different implications for allocation efficiency, risk sharing and asset prices. Under the conditions of Propositions 3 and 4, the corollaries below formalise these implications for $h \in \{i, j\}$.

Corollary 4.2 (allocation efficiency within state):

- In Regime A, optimal allocation does not obtain in $t = 0$ due to domestic credit risks and the cost of liquidity; optimal allocation within state does not obtain in $t = 1$ due to the national bailout tax causing the internal devaluation effect.

  At $t = 0$,

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}} \frac{1}{(1 + r_I)(1 + r_J)},$$

  and at $t = 1$, $\boldsymbol{s} \in S$

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{U^i_{c^i_{J\boldsymbol{s}}}} = \frac{U^j_{c^j_{I\boldsymbol{s}}}}{U^j_{c^j_{J\boldsymbol{s}}}} \frac{1}{(1 + \tau_{I\boldsymbol{s}})(1 + \tau_{J\boldsymbol{s}})}.$$

- In Regime B, optimal allocation efficiency does not obtain at $t = 0$ due to domestic credit risks and the cost of liquidity; optimal allocation within state obtains at $t = 1$.

At $t = 0$,

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}} \frac{1}{(1 + r_I)(1 + r_J)},$$

and at $t = 1$, $\boldsymbol{s} \in S$

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{U^i_{c^i_{J\boldsymbol{s}}}} = \frac{U^j_{c^j_{I\boldsymbol{s}}}}{U^j_{c^j_{J\boldsymbol{s}}}}.$$

- In Regime C, default in the cross-border capital markets obviates the need for national bailout tax, optimal allocation does not obtain at $t = 0$ due to the cost of interbank liquidity; optimal allocation within state obtains at $t = 1$.

  At $t = 0$,

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}} \frac{1}{(1 + \rho)^2},$$

  and at $t = 1$, $\boldsymbol{s} \in S$

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{U^i_{c^i_{J\boldsymbol{s}}}} = \frac{U^j_{c^j_{I\boldsymbol{s}}}}{U^j_{c^j_{J\boldsymbol{s}}}}.$$

Proof. See Appendix D.5.

We can observe that the wedge between the households' marginal rate of substitution across goods distorts allocation efficiency within state. Regime A has the highest wedge due to domestic borrowing costs and bailout tax rates, and the domestic borrowing cost incorporates the cost of liquidity and commands domestic credit risk premia. Regime C has the lowest wedge only resulting from the interbank transaction cost. Note that the transaction cost of money is Regime C is just the interbank transaction cost, and it is lower than the borrowing cost in Regime B. This is because the borrowing cost in Regime B commands the credit risk premium of domestic loans, but that of Regime C precludes it owing to the shield of default in the cross-border capital markets.

Corollary 4.3. (risk sharing):

- In Regime A, optimal risk sharing does not obtain due to domestic credit risks, the cost of liquidity, and the bailout tax.

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{1}{(1 + \tau_{Js})(1 + \tau_{Is})(1 + r_I)(1 + r_J)},$$

$$\frac{U^i_{c^i_{Js}}}{U^i_{c^i_{Js'}}} = \frac{U^j_{c^j_{Js}}}{U^j_{c^j_{Js'}}} \frac{1}{(1+\tau_{Is'})(1+\tau_{Js'})(1+r_I)(1+r_J)}.$$

- In Regime B, optimal risk sharing does not obtain due to borrowing costs which commands the domestic credit risk premium and incorporates the cost of liquidity.

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{1}{(1+r_I)(1+r_J)},$$

$$\frac{U^i_{c^i_{Js}}}{U^i_{c^i_{Js'}}} = \frac{U^j_{c^j_{Js}}}{U^j_{c^j_{Js'}}} \frac{1}{(1+r_I)(1+r_J)}.$$

- In Regime C when default in the cross-border capital markets obviates the need for national bailout tax, optimal risk sharing does not obtain due to two sources of inefficiency: the cost of interbank liquidity and the default premium in the capital markets union.

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{K_s K_{s'}}{(1+\rho)^2},$$

$$\frac{U^i_{c^i_{Js}}}{U^i_{c^i_{Js'}}} = \frac{U^j_{c^j_{Js}}}{U^j_{c^j_{Js'}}} \frac{K_s K_{s'}}{(1+\rho)^2}.$$

Proof. See Appendix D.6.

The above expressions tell us that the wedge between the households' marginal rate of substitution across states distorts risk sharing, and that the wedge in Regime A is the highest because of the extra distortion stemming from the bailout tax. Between Regime B and Regime C, however, it is not obvious whose wedge is higher. Both wedges in these two regimes incorporate the interbank transaction cost and credit/default risk premia.

Corollary 4.4. (asset prices): Financial, monetary, and fiscal factors all affect the stochastic discount factor.

- In Regime A, state prices are affected by the domestic credit risks, the cost of liquidity, and the bailout tax directly.

$$\pi_s = \gamma_s \frac{U^i_{c^i_{Js}}/p_{Js}}{U^i_{c^i_{I0}}/p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}/p_{Is}}{U^j_{c^j_{J0}}/p_{J0}(1+\tau_{Js})(1+r_J)},$$

$$\pi_{s'} = \gamma_{s'} \frac{U^i_{c^i_{Js'}}/p_{Js'}}{U^i_{c^i_{I0}}/p_{I0}(1+\tau_{Is'})(1+r_I)} = \gamma_{s'} \frac{U^j_{c^j_{Is'}}/p_{Is'}}{U^j_{c^j_{J0}}/p_{J0}}.$$

- In Regime B state prices are only directly affected by the domestic credit risks and the cost of liquidity.

$$\pi_s = \gamma_s \frac{U^i_{c^i_{Js}}/p_{Js}}{U^i_{c^i_{I0}}/p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}/p_{Is}}{U^j_{c^j_{J0}}/p_{J0}(1+r_J)},$$

$$\pi_{s'} = \gamma_{s'} \frac{U^i_{c^i_{Js'}}/p_{Js'}}{U^i_{c^i_{I0}}/p_{I0}(1+r_I)} = \gamma_{s'} \frac{U^j_{c^j_{Is'}}/p_{Is'}}{U^j_{c^j_{J0}}/p_{J0}}.$$

- In Regime C state prices are affected by the interbank transaction cost and the cross-border default premium directly.

$$\pi_s = \gamma_s \frac{U^i_{c^i_{Js}}/p_{Js}}{U^i_{c^i_{I0}}/p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}/p_{Is}K_s}{U^j_{c^j_{J0}}/p_{J0}(1+\rho)},$$

$$\pi_{s'} = \gamma_{s'} \frac{U^i_{c^i_{Js'}}/p_{Js'}K_{s'}}{U^i_{c^i_{I0}}/p_{I0}(1+\rho)} = \gamma_{s'} \frac{U^j_{c^j_{Is'}}/p_{Is'}}{U^j_{c^j_{J0}}/p_{J0}}.$$

Proof. See Appendix D.7.

Asset prices are typically suppressed by the transaction wedge. The interbank transaction cost, domestic loan credit risk premium, cross-border default premium, and the bailout tax all constitute the transaction wedge. We can observe from Corollary 4.4 that the state prices in Regime A are typically lower than those in Regime B and Regime C, because Regime A is distorted by the bailout tax as the extra transaction wedge. Since the total transaction cost in Regime A is the highest, in this regime one unit of currency tomorrow is worth the lowest level of consumption goods today. Thus, using internal devaluation to sustain a currency union would also put downward pressure on asset prices. This is confirmed by the numerical analysis in Section 4.2.

4 Welfare and Numerical Analysis

4.1 Default and Welfare

The analysis so far suggests that both cross-country transfers and default in the capital markets union can reduce the transaction wedge distorting allocations and prices. Table 1 provides a summary of the transaction wedge in Corollaries 4.1-4.4. The dash "-" indicates the relative degree of inefficiency. The more dashes assigned, there are more sources of distortions to allocations, risk sharing, and asset prices.

Table 1: Regime comparison

	Regime A benchmark	**Regime B** fiscal union	**Regime C** default
Allocation within state	- - -	- -	-
Risk sharing	- - -	-	-
Asset prices	- - -	-	-
Austerity tax	yes	no	no
Banking crises	yes	no	no

[a] "-" indicates the relative degree of inefficiency.

Let us observe that compared with Regime A, Regime C's allocations, risk sharing, and asset prices are all much less distorted. This is because bankruptcy leniency in the capital markets union can reduce the transaction cost for both countries in the currency union; thus, naturally one expects Pareto improvement. Proposition 5 formalises this claim.

Proposition 5. (Endogenous default and Pareto improvement):

- In the absence of a fiscal union, endogenous cross-border default can Pareto improve a currency union.
- Consider the case where $S = \{1, 2\}$, let $\gamma_1 = \gamma_2$, $e_1^i > e_2^i$, and $e_1^j < e_2^j$. Consider Regime A equilibrium, i.e. $\lambda^i, \lambda^j < \lambda$ and $v_2^i, v_1^j < 1$. Now reduce λ to λ' such that $\lambda^i, \lambda^j > \lambda'$, whenever $\Big(1 - \pi_2\big(v_2^i(1 + r_I) - 1\big)\Big)/p_{I2} > \pi_2 r_I/p_{I1}$ and $\Big(1 - \pi_1\big(v_1^j(1 + r_J) - 1\big)\Big)/p_{J1} > \pi_1 r_J/p_{J2}$, it leads to a utility increase for both households i and j on the margin, while the expected utilities of domestic banking sectors remain unchanged.

Proof. See Appendix D.8.

Interestingly, the philosophy underpinning the Pareto improvement here is to introduce an additional friction. Typically when a friction is added, there is one more imperfection, and one might expect a worse outcome. Here, it is exactly the opposite. A currency union typically bear two imperfections: the interbank transaction cost of money and the possibility to default on domestic loans, i.e. domestic credit risks. Suppose removing these two imperfections using brutal force is not possible, Regime C simply introduces the possibility to default on financial assets as a third imperfection. The third imperfection is introduced in such a way that the most "harmful" imperfection, i.e. state-varying domestic credit risks, is simply not manifested in equilibrium, leading to a superior outcome. Indeed, the possibility to default on financial assets can exactly shift borrowers' incentive on the margin to bypass the state-varying domestic credit risks, such that banks survive and no bailout tax is levied, as in Regime C.

I hasten to add that the conditions identified in Proposition 5 are only sufficient but not necessary. The proof of Proposition 5 does not require setting the bailout tax rate to zero, which would provide an additional liquidity boost at $t = 1$ and increase consumption. However, as the softened bankruptcy code shifts the default incentive on the margin, eventually the economy shifts to Regime C equilibrium where the bailout tax is not even needed. Therefore, one can expect to further relax the conditions in Proposition 5 for Pareto improvement to occur. As analytic solutions are not available, the exact sufficient and necessary conditions are difficult to pin down. Therefore, I provide numerical analysis in the next subsection.

4.2 Numerical Analysis

This subsection assumes two states (state s and state s') in $t = 1$ and solves the model numerically. Table 2 exhibits the exogenous parameters and functional forms.

Table 2: Exogenous variables

Preference	$U^i = \alpha_0^i ln(c_{I0}^i) + (1 - a_0^i) ln(c_{J0}^i) + \beta^i E_0[\alpha_{\boldsymbol{s}}^i ln(c_{I\boldsymbol{s}}^i) + (1 - a_{\boldsymbol{s}}^i) ln(c_{J\boldsymbol{s}}^i)]$
	$U^j = \alpha_0^j ln(c_{J0}^j) + (1 - a_0^j) ln(c_{I0}^j) + \beta^j E_0[\alpha_{\boldsymbol{s}}^j ln(c_{J\boldsymbol{s}}^j) + (1 - a_{\boldsymbol{s}}^j) ln(c_{I\boldsymbol{s}}^j)]$
	$\alpha_0^i = \alpha_0^j = 0.5 \quad \alpha_s^i = \alpha_{s'}^j = 0.7 \quad \alpha_{s'}^i = \alpha_s^j = 0.3$
Prob. of states	$\gamma_s = \gamma_{s'} = 0.5$
Discount factor	$\beta^i = \beta^j = 1$
Endowment	$e_{I0}^i = e_{J0}^j = 10 \quad e_{Is}^i = e_{Js'}^j = 15 \quad e_{Is'}^i = e_{Js}^j = 5$
Bankruptcy code	$\lambda^i = \lambda^j = 0.1036 \quad \lambda^C = 0.098 \quad \lambda^A = \lambda^B = 0.12$
Outside money	$m^i = m^j = 0.1$
Policy rate	$\rho = 0.01$

4.2.1 Cross-border Default and Pareto Improvement

Table 3 shows the numerical solutions of the endogenous variables of Regime A, Regimes B.a and B.b, and Regime C.[19] Let W^I denote country I's utility of consuming goods in both periods, SU^I denote country I's social utility, i.e. the consumption utility minus the social cost of default, and df denote the default rate. Regime A leads to the least desirable equilibrium among the four regimes considered: the default rate on domestic loans in the bad state is high, asset prices are suppressed, domestic commercial bank loan rates are much higher than the policy rate due to a high credit risk premium, and bailout tax turns out positive as the domestic bank would become insolvent in the bad state. Hence, both the allocation welfare and social utility of Regime A are quite low.

[19] As country J is symmetric to country I, here I only display the prices and allocations of country I for conciseness.

Table 3: Endogenous variables

Regime A Equilibrium (Baseline)					
$W^I = 3.2921$	$SU^I = 3.2496$	$df^i_{s'} = 13.02\%$	$\pi_s = 0.4807$	$r^I = 8.03\%$	$\tau_{Is'} = 6.45\%$
Regime B.a Equilibrium					
$W^I = 3.2943 \uparrow$	$SU^I = 3.2879 \uparrow$	$df^i_s = 3.8\% \downarrow$	$\pi_s = 0.4875 \uparrow$	$r^I = 4.49\% \downarrow$	$\tau_{Is'} = 0\% \downarrow$
Regime B.b Equilibrium					
$W^I = 3.2943 \uparrow$	$SU^I = 3.2711 \uparrow$	$df^i_{s'} = 6.75\% \downarrow$	$\pi_s = 0.4889 \uparrow$	$r^I = 4.53\% \downarrow$	$\tau_{Is'} = 0\% \downarrow$
Regime C Equilibrium					
$W^I = 3.2947 \uparrow$	$SU^I = 3.2916 \uparrow$	$df_{s'} = 1.63\% \downarrow$	$\pi_s = 0.4934 \uparrow$	$r^I = 1\% \downarrow$	$\tau_{Is'} = 0\% \downarrow$

[a] Arrows indicate the direction of travel compared with Regime A.

Regime B.a is a fiscal union with no banking union. Default rates on domestic loans are the same across states, and are lower than Regime A, the internal devaluation regime; hence, the domestic commercial bank loan rates are lower. Moreover, asset prices, social utility and allocation welfare all improve upon Regime A, and the bailout tax is not needed. Regime B.b, a fiscal union with a banking union, exhibits a similar improvement as Regime B.a.

Regime C assumes away cross-country fiscal transfers but a lenient union-wide bankruptcy code allows for cross-border default in the capital markets. In Regime C, the default rate is of asset payoffs and it is significantly lower than the default rate on domestic loans in Regime A. Since the cross-country bankruptcy code induces the default risks to move away from domestic loans for the purpose of alleviating domestic banking stress, the domestic loan rate is equal to the monetary policy rate. Similar to Regime B.a and Regime B.b, allocation welfare, social utility, and asset prices are all higher than in Regime A, and the bailout tax is also not needed in Regime C.

Figure 6: Allocation welfare (shock variance)

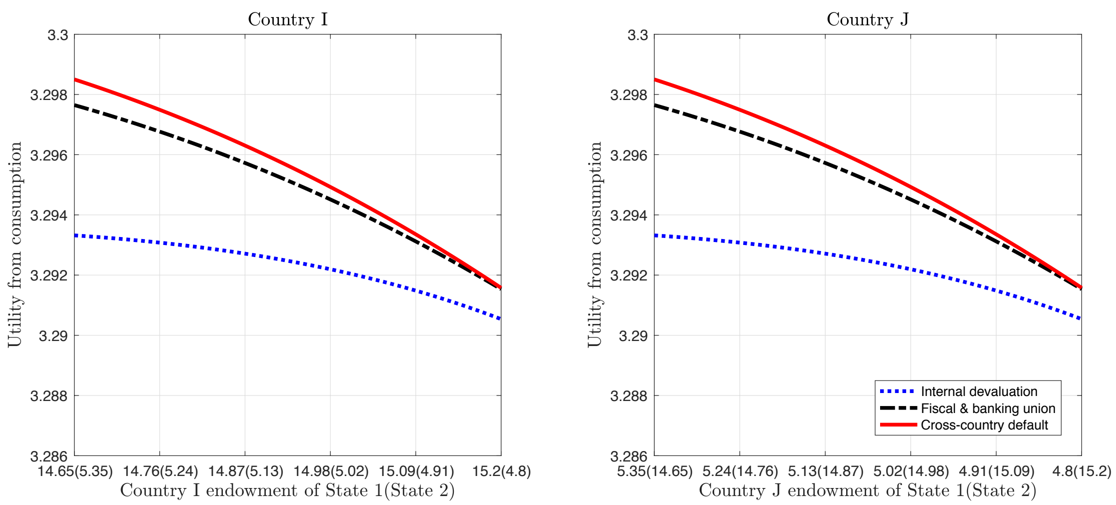


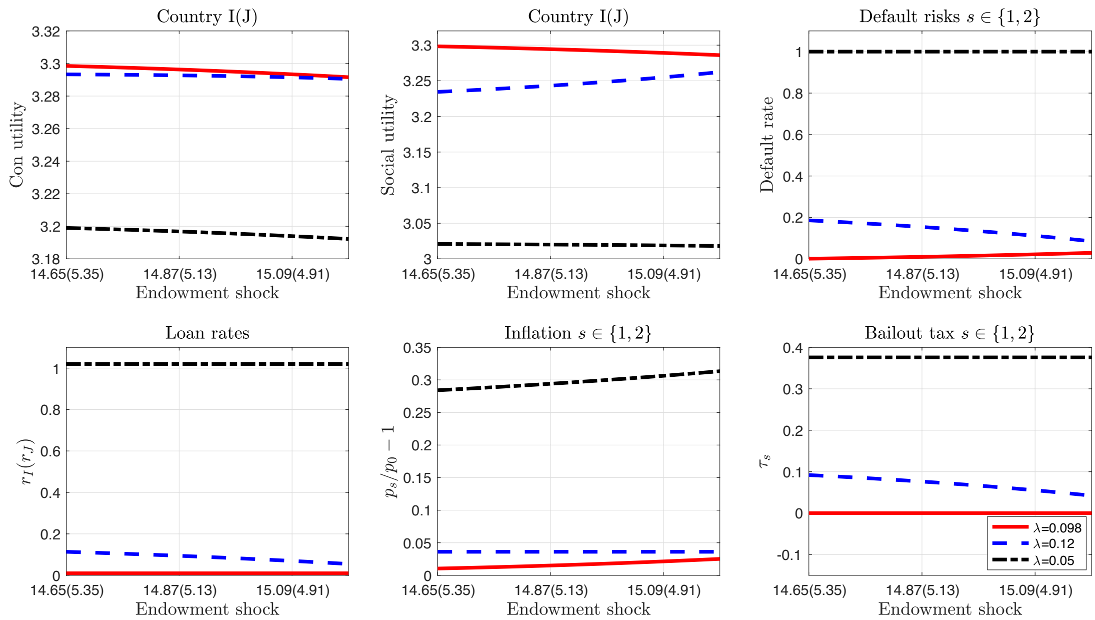
Figure 6 conducts the robustness check of the numerical solution in Table 3 by varying income risks around the initial parameterisation of $e^h_{Hs}(h \in \{i, j\}, H \in \{I, J\})$ set out in Table 2. The horizontal axis represents the state-contingent endowments of the two states in each country. The further to the right, the greater the variance of the state-contingent endowments while the mean remains unchanged at 10. The vertical axis represents the households' utility from consuming home and foreign goods, which I call allocation welfare. As might be expected, Regime C with cross-border default in the capital markets union Pareto dominates the Regime A which suffers from internal devaluation. Regime B.b, the fiscal-banking union, is superior to Regime A as well but is slightly worse off than Regime C. This is because the transaction cost due to default on financial securities is lower than the transaction cost due to default on commercial bank loans.

Interestingly, as the variance of state-contingent endowments widens, the welfare improvement of Regime C upon Regime A decreases. This is due to the substitution effect between default and consumption goods being dominated by the income effect, which reduces default risks in Regime A equilibrium. The widening variance implies a lower endowment in the bad state, a higher marginal benefit of defaulting on domestic loans. The substitution effect thus would increase default rate. Meanwhile, a lower endowment in the bad state increases the value of this good and its relative price increases, which leads to an increase in liquidity inflow from goods sale to repay domestic bank loans. Default rate hence decreases. The overall decrease in default rates push down the borrowing costs and the bailout tax rates in Regime A. Thus, Regime A's allocation welfare deteriorates less as the variance widens, and the gap between Regime A and Regime C narrows. Adjusting the allocation welfare with the social cost of default, Figure 13 in Appendix E.1 replaces the vertical axis with social utility (i.e. consumption utility minus the social cost of default) and shows a similar picture. For more robustness check, I also replace the horizontal axis with country I's state 2 endowment e^i_{I2} and shock the economy asymmetrically. Figure 14 in Appendix E.2 displays the social utility of both countries while changing e^i_{I2}. Again, cross-border default Pareto improves upon the internal devaluation regime.

4.2.2 When Does Cross-border Default Not Work?

Nevertheless, a caveat remains on the welfare improving role of default in the cross-border capital markets. As Corollary 3.1 states, when the union-wide bankruptcy code λ is set too low, no households in either countries would ever repay cross-country borrowing, leading to a collapse of international financial markets. Figure 7 provides numerical support to this claim. As in Figure 6, the horizontal axis represents the state-contingent endowments of the two states in each country. The solid line represents the economy when the union-wide bankruptcy code is set to 0.098, only slightly more lenient than the domestic bankruptcy codes ($\lambda^h = 0.1036, h \in \{i, j\}$). The dashed line shows the economic allocation and prices when the union-wide bankruptcy code is set to 0.12, much tougher than the domestic bankruptcy codes. The dashed line essentially represents Regime A. The dash-dotted line represents financial autarky, where an ultra-low union-wide bankruptcy code ($\lambda = 0.05$) causes the collapse of financial markets and impedes risk sharing.

Figure 7: Union-wide VS domestic bankruptcy codes



As illustrated in Figure 7, the case with a slightly softened union-wide bankruptcy code obtains the best allocation and social utility. In this case, assets are defaultable but are still traded, domestic borrowing costs in both countries are low, and neither country needs to levy bailout taxes since banks do not experience insolvency even in the bad state. A tougher union-wide bankruptcy code, represented by the dashed line, shifts the equilibrium to Regime A. Domestic borrowing costs are higher and bailout taxes are levied. The worst case among those considered here is financial autarky. It is when cross-border default backfires due to an ultra-lenient union-wide bankruptcy code. In this case, no financial assets are traded for risk sharing, and the limited risk sharing is provided by money and state-dependent inflation, which acts as a shock "absorber". Because of the impediment to risk sharing, countries in the bad state are poorer than the case with risk sharing, hence, their marginal utility of consumption increases, implying a high marginal benefit of defaulting on domestic loans. It turns out quite dramatically that, given the domestic bankruptcy codes as $\lambda^h = 0.1036, h \in \{i, j\}$, both countries default 100 % on their domestic loans in their respective bad states. Banks would become insolvent if not for the national governments' bailout. As a consequence, borrowing costs and default risks are exorbitantly high, bailout tax rates increase to around 37.6 %. Because limited risk sharing implies a collapse of goods trade at $t = 1$, and the Quantity Theory of Money holds, inflation in both states soars up.

4.2.3 Union-wide Bankruptcy Stance

It is fair to say the case of financial autarky is engineered by an unrealistically ultra-lenient union-wide bankruptcy code, because it would imply that a currency union has an extremely weak institution to enforce repayment and implement default punishment. But what if the union-wide bankruptcy code is just slightly softened ($\lambda = 0.098$), as the solid line represents in Figure 7, and then we additionally soften the union-wide bankruptcy by small increment, how would the economy behave? Figure 8 conducts such an experiment.

Figure 8: Cross-country bankruptcy code and default

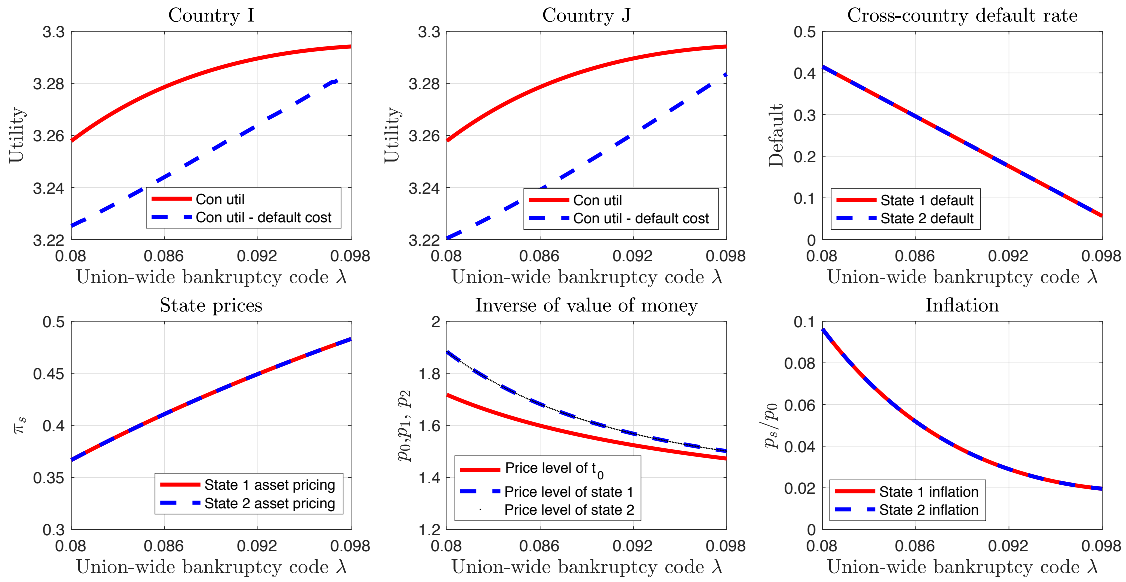
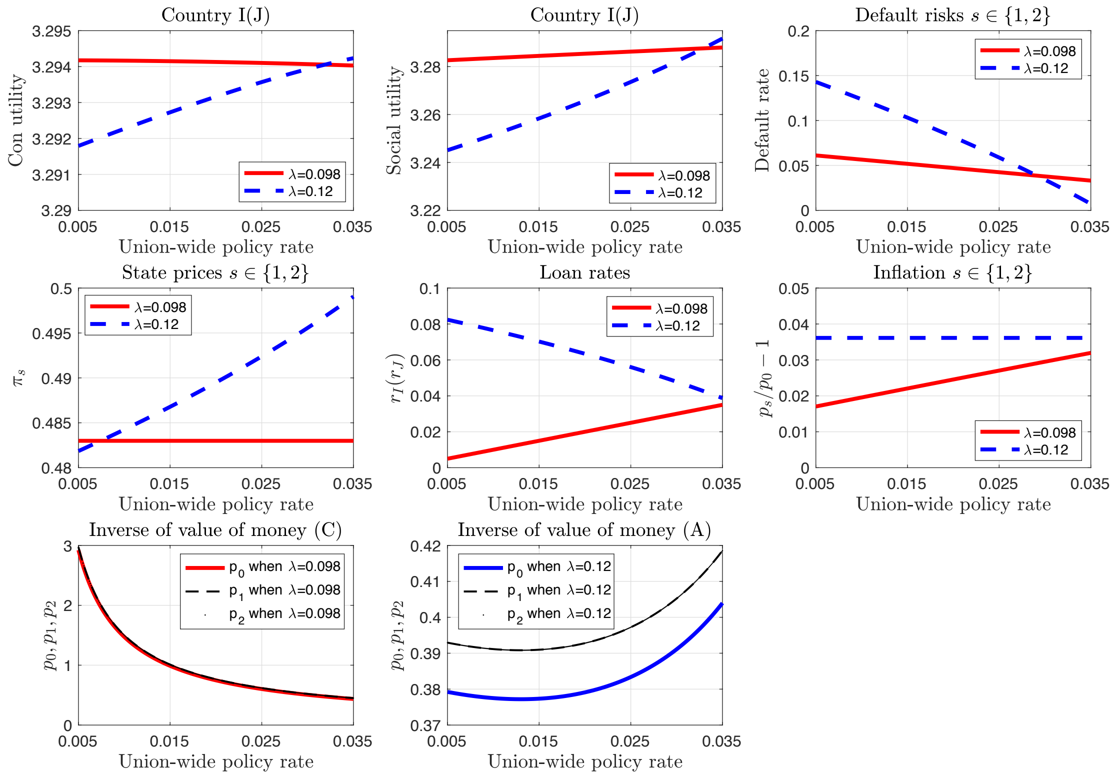


Figure 8 varies the union-wide bankruptcy code (i.e. the horizontal axis λ) by small amounts but ensures it is slightly more lenient than domestic bankruptcy codes. Therefore, what we observe is the comparative statics within Regime C with default in the cross-border capital markets. As λ decreases, the further left horizontally, the marginal cost of default decreases and the substitution effect dominates in equilibrium. Further to the left, therefore, default risks in the capital markets union increase, pushing down asset prices and generating inflationary pressure. As default risks in the cross-border capital markets increase, the value of the common currency decreases. The modelling of inside money and strategic default naturally enables the model to produce the endogenous relationship between default and the value of money.

4.2.4 Union-wide Monetary Policy

Albeit a currency union has a single monetary authority, and the "one size" of its policy tool does not fit all, it remains to be of interest to study the properties of such monetary regime, particularly the interplay of credit risks and value of the currency. This is a relevant exercise because after the eurozone crisis, the ECB has kept rates low based on the classic argument about stimulating aggregate demand; however, the union-wide output recovery has turned out sluggish and goods prices remain subdued despite large quantities of reserve injection via asset purchase programmes. This phenomenon has sparked off a heated debate on "monetary hysteresis". Figure 9 conducts such policy experiment. It suggests that when the eurozone has limited cross-country transfers and a tough stance on cross-border default, the classic demand effect of low policy rates may not prevail, because we also need to consider the related impact on the demand and supply of credit and liquidity.

Figure 9: Union-wide monetary policy



The first two rows in Figure 9 display the equilibrium of Regime C (the solid line) and that of Regime A (the dashed line), while varying the union-wide monetary policy rate (the horizontal axis). Counterintuitively, when the currency union is in the internal devaluation regime (Regime A), a lower policy rate actually worsens allocation and tends to push down price level of goods (see the last subplot of Figure 9), which might help explain the slow growth and subdued prices in the eurozone after the crisis. In Regime A, a lower policy rate, i.e. further to the left on the horizontal axis, means a lower repayment pressure on domestic banks to the union-wide central bank. On the margin, this lower repayment pressure on the domestic banks transmits to a lower repayment pressure on domestic borrowers. In the bad state, this propagation causes a low repayment on domestic loans and a rise in NPLs. Therefore, domestic borrowing costs rise and stifle goods trade. Moreover, the rise in domestic borrowing costs hamper domestic credit extension, leading to a contraction in the price levels of goods. This result suggests that in Regime A, the union-wide monetary policy should not follow the secular trend and perhaps it may even set the secular trend.

We can also observe, however, when the currency union is in Regime C (the solid line), the effect of monetary policy is more conventional. A lower policy rate decreases borrowing costs and improve allocation welfare, although the social cost of default decreases.

5 National Currencies

with Floating Exchange Rates

So far, the model has shown the possibility of welfare improvement as a result of bankruptcy leniency in the cross-border capital markets in a currency union, but the model has not explicitly explained why the cross-border bankruptcy leniency is *particularly* vital in a currency union. To understand this question, one needs to know what benefits has a currency union given up and whether cross-border bankruptcy leniency can recoup these benefits. As such, it is natural and unavoidable to ask what if the currency union were to dissolve, a question largely unaddressed by existing literature.

Therefore, in this section I assume the seemingly provocative scenario of currency union dissolution and consider national currencies priced by floating exchange rates, which I call Regime D. Not to complicate the model exceedingly, I do not consider the transition from a currency union to national currencies but only conduct comparative statics of equilibria of a currency union and of national currencies. I also do not consider the scenario in which national governments intervene in the foreign exchange market, nor do I assume heterogeneous bankruptcy codes between countries. Consequently, the floating exchange rate movements in this section are purely competitive due to income shocks, rather than due to different policy or institutional stances.

Regime D considers a frictionless foreign exchange market that opens at $t = 0$ and $t = 1$. As in Regime A, Regime D assumes a harsh bankruptcy code for cross-country borrowing and no cross-country fiscal transfers. It only considers the friction of possible domestic credit risks stemming from idiosyncratic income shocks. Rather than having a union-wide central bank, each country now has its own national central bank that issues its own national currency. Moreover, each country issues its own *currency-specific Arrow securities* whose payoffs are in the respective national currencies. Outside money and accordingly seigniorage in each country are also in the respective national currencies. Linking the countries are asset markets, goods markets, and particularly frictionless foreign exchange markets. The model structure and agents' interactions are depicted in Fig (11) in Appendix B.1.

Therefore, compared with a currency union, this regime has an additional market, i.e. the foreign exchange market, and for each state, there are two types of Arrow securities issued by country I and country J respectively. Rather than having a common interbank loan and money market, this regime has two interbank loan and money markets, one in each country. The foreign exchange market is assumed to meet twice. At $t = 0$, it opens immediately after the loan markets, and at $t = 1$. it opens before import purchases. The other features remain the same as in the benchmark regime, particularly financial assets are *not* defaultable, i.e. $\lambda > \lambda^h$, and the fiscal union is ruled out, i.e. $\delta^h_{\boldsymbol{s}} = \delta^{bH}_{\boldsymbol{s}} = 0$, $h \in \{i, j\}, H \in \{I, J\}, \boldsymbol{s} \in S$. The timeline is illustrated in Figure (12) in Appendix B.2.

5.1 Country I

Country I's modelling is described in detail. The modelling of country$J$ is exactly symmetric to that of country I (see country J in Appendix B.3).

Households i

Households' preference is the same as in the currency union except that the price deflator $p_{\boldsymbol{s}}^I$ indexing the default cost is now a country-wide price index rather than a union-wide price index. Household i's choice variables include the amount of domestic loans to borrow, foreign exchange to trade, domestic and foreign assets to trade, quantities of consumption goods to consume, the quantity of domestic goods to sell, the amount of money to spend on foreign goods, and the loan repayment rate. In particular, let $f_{IJ\boldsymbol{s}^*}^i (\forall \boldsymbol{s}^* \in S^*)$ be the amount of domestic currency I the households spend on purchasing foreign currency J. Let $\chi_{\boldsymbol{s}^*} (\forall \boldsymbol{s}^* \in S^*)$ be the floating exchange rate such that one unit of currency J is worth $\chi_{\boldsymbol{s}^*}$ units of domestic currency I. An increase in $\chi_{\boldsymbol{s}^*}$ means a depreciation of currency I and an appreciation of currency J. Let $\nu_1^i, \nu_2^i, \nu_{3\boldsymbol{s}}^i, \nu_{4\boldsymbol{s}}^i, \nu_{5\boldsymbol{s}}^i$ be the shadow price of the respective flow of funds constraint. Households i are subject to the following budget sets and flow of funds constraints.

At $t = 0$,

$$f_{IJ}^i \leq \frac{\mu_I^i}{1 + r^I} + m^i, \qquad \nu_1^i, \quad (11)$$

$$b_{J0}^i + \sum_{l=1}^{S} \pi_{Jl} \theta_{Jl}^i \leq \frac{f_{IJ}^i}{\chi}, \qquad \nu_2^i \quad (12)$$

at $t = 1$ in state $\boldsymbol{s}$,

$$f_{IJ\boldsymbol{s}}^i + \phi_{I\boldsymbol{s}}^i (1 + \tau_{I\boldsymbol{s}}) \leq \sum_{l=1}^{S} \pi_{Il} \phi_{Il}^i + p_{I0} q_{I0}^i + p_{I\boldsymbol{s}} q_{I\boldsymbol{s}}^i + \Delta(11), \qquad \nu_{3\boldsymbol{s}}^i \quad (13)$$

$$b_{J\boldsymbol{s}}^i (1 + \tau_{I\boldsymbol{s}}) \leq \theta_{J\boldsymbol{s}}^i + \frac{f_{IJ\boldsymbol{s}}^i}{\chi_{\boldsymbol{s}}} + \Delta(12), \qquad \nu_{4\boldsymbol{s}}^i \quad (14)$$

at $t = 2$,

$$v_{\boldsymbol{s}}^i \mu_I^i \leq \Delta(13)_{\boldsymbol{s}}. \qquad \nu_{5\boldsymbol{s}}^i \quad (15)$$

And the feasibility constraints are satisfied, i.e. $c_{I\boldsymbol{s}^*}^i \leq e_{I\boldsymbol{s}^*}^i - q_{I\boldsymbol{s}^*}^i$ and $c_{J\boldsymbol{s}^*}^i \leq \frac{b_{J\boldsymbol{s}^*}^i}{p_{J\boldsymbol{s}^*}}$, $\boldsymbol{s}^* \in S^*$. At $t = 0$, househols i borrow from the domestic commercial banking sector and obtain local currency to buy foreign currency (11). The households then use foreign currency to buy foreign assets (12) and imports. At the same time households sell domestic assets and exports, and carry the monetary proceeds to the next period.

At $t = 1$ when the state of nature is realised, the households use existing monetary proceeds in the local currency plus this period's export income to deliver the payoffs of domestic assets sold (13) and to purchase foreign currency. The households use the foreign currency at hand and foreign asset payoffs to pay for foreign goods (14).

At $t = 2$, the households use any unused local currency from (13) to pay back loans, subject to their default choice (15).

Domestic Commercial Banking Sector i

Bank i's actions are the same as in the benchmark model except that bank i now borrows interbank loans from the national central banking sector rather than the union-wide central bank. Therefore, the ultimately fiat money in country I is issued by the national central bank and is the national currency. Let μ^i_{CBI} be the interbank loans bank i borrows from the national central bank and ρ_I as the policy rate set by the national central bank. Bank i's maximisation problem is specified as follows.

$$\underbrace{Max}_{\mu^i_{CBI},\mu^i_I,L^i,\omega^i} \sum_{\boldsymbol{s}}^{S} z_{\boldsymbol{s}}\omega^i_{\boldsymbol{s}}.$$

Bank i is subject to the following flow of funds constraints:

$$L^i \leq \frac{\mu^i_{CBI}}{1+\rho_I}, \tag{16}$$

$$\frac{\mu^i_I}{1+r_I} \leq L^i, \tag{17}$$

$$\omega^i_{\boldsymbol{s}} = \Delta(16) + \Delta(17) + R^i_s\mu^i_{I\boldsymbol{s}} - \mu^i_{CBI}. \tag{18}$$

National Government i

National government i collects taxes from domestic households to build a state-contingent bailout fund of $T_{I\boldsymbol{s}}$.

$$T_{I\boldsymbol{s}} = p_{J\boldsymbol{s}}c^i_{J\boldsymbol{s}}\tau_{I\boldsymbol{s}} + \phi^i_{\boldsymbol{s}}\tau_{I\boldsymbol{s}}. \tag{19}$$

National government i uses the bailout funds to rescue the domestic commercial bank whenever the bank's nominal profits would drop to negative, i.e. when the bank would fail. In short, in the bad state the national government makes a state-contingent transfer to ensure $\omega^i_{\boldsymbol{s}} + T_{I\boldsymbol{s}} = 0$.

National Central Bank i

The national central bank lends interbank loans μ^i_{CBI} to the domestic commercial banking sector and provides fiat money in the national currency. The national central bank sets the interbank rate ρ_I, and collects seigniorage in a non-Ricardian way.

5.2 Equilibrium

I define the equilibrium of this floating exchange rate model as Regime D equilibrium. It is an allocation $(c_I, c_J, f_{IJ}, f_{JI}, b_I, b_J, q_I, q_J, \theta_I, \theta_J, \phi_I, \phi_J, \mu_I, \mu_J, \mu_{CBI}, \mu_{CBJ})$ with prices $(p_I, p_J, \pi_I, \pi_J, \chi, r_I, r_J, v, R)$, given policy instruments and regime parameters $(\rho_I, \rho_J,\ \lambda^i,\ \lambda^j,\ \lambda)$ and government action τ such that agents maximise subject to liquidity-in-advance constraints and budget constraints, markets clear, and expectations are rational.

- Goods markets:

$$p_{I\boldsymbol{s}^*} q^i_{I\boldsymbol{s}^*} = b^j_{I\boldsymbol{s}^*},$$

$$p_{J\boldsymbol{s}^*} q^j_{J\boldsymbol{s}^*} = b^i_{J\boldsymbol{s}^*},$$

- Foreign exchange market:

$$f^i_{IJs^*} = \chi_{s*} f^j_{JI^*},$$

- Asset markets:

$$\phi^i_{Il} = \theta^j_{Il},$$

$$\phi^j_{Jl} = \theta^i_{Jl},$$

- Domestic loan markets:

$$\frac{\mu^i_I}{1+r_I} = L^i,$$

$$\frac{\mu^j_J}{1+r_J} = L^j,$$

- Interbank loans and money markets:

$$1+\rho_I = \frac{\mu^i_{CBI}}{M_I},$$

$$1+\rho_J = \frac{\mu^j_{CBJ}}{M_J},$$

- Rational expectation:

$$R^i_{\boldsymbol{s}} = \left\{ \begin{array}{cc} v^i_{\boldsymbol{s}} & \text{if } 1 - v^i_{\boldsymbol{s}} > 0 \\ \text{arbitrary} & \text{if } 1 - v^i_{\boldsymbol{s}} = 0 \end{array} \right\},$$

$$R^j_{\boldsymbol{s}} = \left\{ \begin{array}{cc} v^j_{\boldsymbol{s}} & \text{if } 1 - v^j_{\boldsymbol{s}} > 0 \\ \text{arbitrary} & \text{if } 1 - v^j_{\boldsymbol{s}} = 0 \end{array} \right\}.$$

5.3 Equilibrium Analysis

Lemma 7: Liquidity-in-advance binds and no worthless money at end.

If $r_I > 0, \forall \boldsymbol{s} \in S$, then $\Delta(11) = 0$.

If $r_I > 0$, then $\Delta_{\boldsymbol{s}}(15) = 0$.

If $\rho_I > 0$, then $\Delta(16), \Delta(17)$, and $\Delta_{\boldsymbol{s}}(18) = 0$.

Proof. See Appendix D.9.

Lemma 8. (PPP and UIP):

- **Purchasing Power Parity**: At any Regime D equilibrium, for $h \in \{i, j\}$, suppose $\tau^h_{\boldsymbol{s}} = 0, \forall \boldsymbol{s} \in S$,
  - at $t = 0$ if household h chooses $b^h_{I0} > 0, b^h_J > 0$ and if household h does not spend all his domestic currency on foreign currency, then

$$\frac{U^h_{c^h_{I0}}}{U^h_{c^h_{J0}}} = \frac{p_{I0}}{\chi p_{J0}},$$

  - at $t = 1$ $\forall \boldsymbol{s} \in S$,

$$\frac{U^h_{c^h_{I\boldsymbol{s}}}}{U^h_{c^h_{J\boldsymbol{s}}}} = \frac{p_{I\boldsymbol{s}}}{\chi_{\boldsymbol{s}} p_{J\boldsymbol{s}}}.$$

- **Uncovered Interest Rate Parity**: Suppose at $t = 0$ household h ($h \in \{i, j\}$) can obtain foreign currency by either trading in the FX market or borrowing from the foreign commercial bank. Then if we are in Regime D equilibrium, we must have that

$$\frac{1 + r_I}{1 + r_J} = \frac{E_0 \nu^i_{3\boldsymbol{s}} \chi_{\boldsymbol{s}}}{E_0 \nu^i_{3\boldsymbol{s}} \chi} = \frac{E_0 \nu^j_{3\boldsymbol{s}} / \chi}{E_0 \nu^j_{3\boldsymbol{s}} / \chi_{\boldsymbol{s}}}.$$

Proof. See Appendix D.10.

The purchasing power parity tells us that if households are purchasing a consumption good from country I and another good from country J that give the same marginal utility, then these two goods must have the same real price. Note in equilibrium, at $t = 0$ household i could be buying consumption good J and selling consumption good I, and household j could be buying consumption good I and selling consumption good J. In this case, due to the cost of liquidity we could have $U^i_{c^i_{I0}} / U^i_{c^i_{J0}} = p_{I0} / [\chi p_{J0}(1 + r_I)]$ and $U^j_{c^j_{I0}} / U^j_{c^j_{J0}} = [(1 + r_J) p_{I0}] / (\chi p_{J0})$. The uncovered interest parity is simply a non-arbitrage result on asset prices. It tells us that if the nominal rate of a country is lowered, say, due to a lower credit risk premium, then its currency is expected to appreciate, and vice versa.

Lemma 8 shows that competitive floating exchange rates affect the relative price between goods and link the nominal rates of the two countries. As the nominal rates incorporate credit risk premia of domestic loans, naturally, these credit risk premia are linked by floating exchange rates as well. Proposition 6 examines this relationship between exchange rates and credit risks formally.

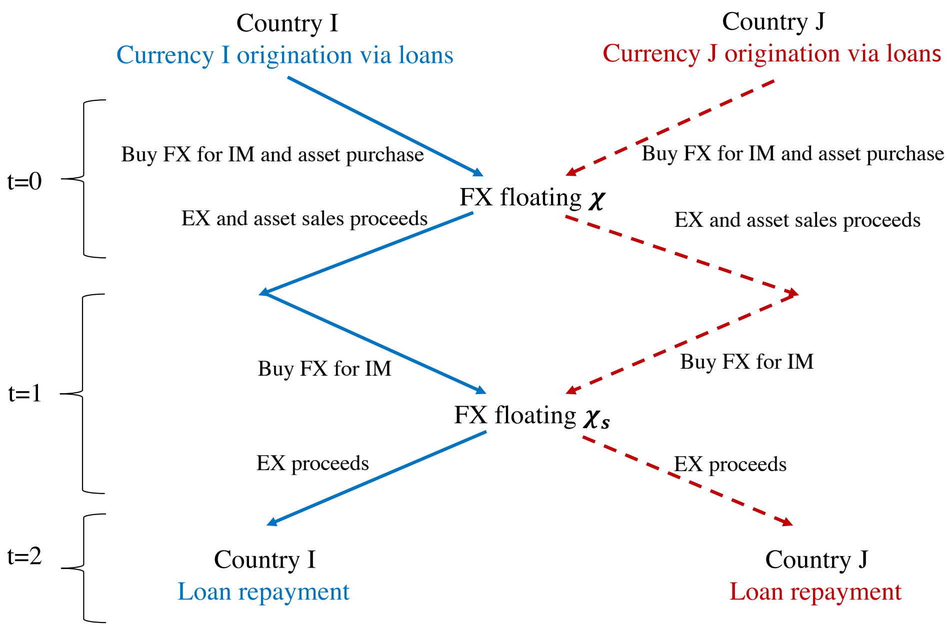
Proposition 6 (credit risk neutralisation): With competitive floating exchange $\chi_{\boldsymbol{s}}$, set the bailout tax to zero, the domestic credit risk turns out state invariant, i.e., $v^h_{\boldsymbol{s}} = v^h, \forall \boldsymbol{s} \in S, h \in \{i, j, \}, H \in \{I, J\}$.

Proof. See Appendix D.11.

The proof of Proposition 6 D.11 shows that at $t = 2$, household i's loan settlement equation can be rearranged as $v^i_{\boldsymbol{s}}\mu^i_I = \frac{\mu^i_I}{1+r_I} + m^i$, and household j's loan settlement equation can be rearranged as $v^j_{\boldsymbol{s}}\mu^j_J = \frac{\mu^j_J}{1+r_J} + m^j$. We can observe that currency I, which is issued against bank loan μ^i_I in country I at $t = 0$, ends up repaying the very same loan against which the currency itself is issued. Similarly, currency J ends up repaying the very same loan against which currency J is issued. Because the face value of the loans are state invariant, the loan repayments must also be state invariant. Therefore, currency exits the system exactly from its own originating country. This is made possible by the FX markets and the floating exchange rates.

To see why, I use Figure (10) as a graphic representation of the proof of Proposition 6. The solid arrows represent the flows of currency I and the dashed arrows represent the flows of currency J. At $t = 0$, currency I is originated against bank credit μ^i_I. Together with outside money, it is used to purchase currency J via the FX market. Country I then uses this amount of currency J to buy imports and foreign assets from country J. Meanwhile, country J spends currency J purchasing currency I via the FX market. This amount of currency I is then spent on buying country I's exports and assets. Thus, currency I flows back to country I and currency J flows back to country J. Similarly at $t = 1$, country I spends some amount of currency I on purchasing some amount of currency J via the FX market. Country I then uses this amount of currency J to buy imports from country J. Country J uses this amount of currency I to buy imports from country I. Thus, this amount of currency I flows back to country I due to its export sales, and this amount of currency J flows back to country J due to country J's export sales. Therefore, currency flows back to its own originating country due to the swap at the FX markets. Whenever the two flows of these currencies meet at the FX markets, the relative strength of these two flows determine the floating exchange rate.

Figure 10: FX flows



Therefore, the floating exchange rate neutralises the domestic credit risks such that the domestic NPL rate turns out the same across states. To see exactly how the floating exchange rate adjusts, take country I as an example and $v_s^i = v_{s'}^i$. Let us now suppose household I wants to repay ϵ_s more in state s to achieve $v_s^i > v_{s'}^i$ instead. I now demonstrate that due to the exchange rate adjustment, $v_s^i > v_{s'}^i$ will turn out a contradiction.

Invoking Lemma 7 and combining (13) and (15), it follows that

$$v_s^i \mu_I^i = \sum_{l=1}^{S} \pi_{Il} \phi_{Il}^i + p_{I0} q_{I0}^i + p_{Is} q_{Is}^i - \phi_{Is}^i - f_{IJs}^i. \tag{20}$$

Household i can increase the loan repayment by ϵ through either decreasing f_{IJs}^i or increasing $p_{Is} q_{Is}^i$ at $t = 1$. Since ϕ_{Is}^i is the position of asset sales of the Arrow security $l = s$ at $t = 0$, it cannot be adjusted at $t = 1$. Suppose household i sells ϵ less f_{IJs}^i, this leads currency I to appreciate, χ_s decreases and $\chi_s f_{JIs}^j$ decreases by ϵ. Thus, country J ends up having ϵ less currency I to buy imports from country I, and $p_{Is} q_{Is}^i$ therefore decreases by ϵ. Overall, the right-hand side of (20) remains unchanged. This is a contradiction. If household i increases $p_{Is} q_{Is}^i$ by ϵ instead, that means the amount of currency I used to purchase household i's exports has to increase by ϵ and χ_s increases, leading to a depreciation. The same contradiction arises. Similar logic obtains if household i wants to achieve $v_s^i < v_{s'}^i$, which also causes contradiction.

Corollary 6.1. (capital flow and banks' survival): Competitive floating exchange rates χ and $= \chi_{\boldsymbol{s}}$ prevents domestic banking insolvency, $\omega_{\boldsymbol{s}}^h = 0$, $\forall \boldsymbol{s} \in S$, $h \in \{i, j\}$, and there is no need for national bailout tax.

Proof. *See Appendix D.12.*

Corollary 6.2. Under the conditions of Proposition 5, if currency union is the regime *a priori*, cross-border default due to bankruptcy leniency obviates the need for floating exchange rates to alleviate domestic banking stress.

Proof. *See Appendix D.13.*

Corollary 6.1 holds under very general conditions. It states that when there are domestic credit risks, floating exchange rates can help absorb the pressure of international capital flow such that domestic banks survive for all states. However, if member countries are in a currency union *a priori*, cross-country default may obviate such need for floating exchange rates. In an ideal scenario with zero domestic credit risks, banks get loans repaid in full for all states of nature, and given markets are complete, the role of floating exchange rate is trivialised (Lemma 5). In other words, if countries do not have domestic credit risks in all states of nature, even without a fiscal union or default in the cross-border capital markets, sharing a common currency brings no particular harm to the health of the banking sector.

As the floating exchange rate neutralised domestic credit risks, one would expect a lower transaction wedge in this Regime than in Regime A, because there is no need to level national bailout tax. Lemma 9 refines the FX constraints at $t = 0$ and under Lemma 9, Corollaries 6.3-6.6 probe into the BoP dynamics and the welfare implications for allocations, prices, and risk sharing.

Lemma 9: FX-in-advance binds.

If $cov(\frac{1}{\chi_{\boldsymbol{s}}}, \nu^i_{2\boldsymbol{s}}) > 0$ and $cov(\chi_{\boldsymbol{s}}, \nu^j_{2\boldsymbol{s}}) > 0$, focusing on symmetric equilibrium, $r_I = r_J$, then $\Delta(12) = 0$.

Proof. *See Appendix D.14.*

Corollary 6.3. (Balance of Payments): Under competitive floating exchange rates and when FX-in-advance binds, current account and capital account exactly balance, $CA^H_{\boldsymbol{s}} + FA^H_{\boldsymbol{s}} = 0, H \in \{I, J\}, \boldsymbol{s} \in S$.

Proof. *See Appendix D.15.*

Corollary 6.4. (allocation efficiency within state): Under Lemma 7, optimal allocation efficiency at $t = 0$ does not obtain due to domestic credit risks and the cost of liquidity; optimal allocation within state obtains at $t = 1$.

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}} \frac{1}{(1 + r_I)(1 + r_J)},$$

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{U^i_{c^i_{J\boldsymbol{s}}}} = \frac{U^j_{c^j_{I\boldsymbol{s}}}}{U^j_{c^j_{J\boldsymbol{s}}}}.$$

Proof. See Appendix D.17.

Corollary 6.5. (risk sharing): Under Lemma 7, optimal risk sharing does not obtain due to the borrowing cost which commands the credit risk premium of domestic loans.

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{1}{(1+r_J)(1+r_I)},$$

$$\frac{U^i_{c^i_{Js}}}{U^i_{c^i_{Js'}}} = \frac{U^j_{c^j_{Js}}}{U^j_{c^j_{Js'}}} \frac{1}{(1+r_J)(1+r_I)}.$$

Proof. See Appendix D.18.

Corollary 6.6. (state prices): With national currencies and competitive floating exchange rate, state prices are affected by the borrow cost and currency appreciation or depreciation.

$$\pi_{Is} = \gamma_s \frac{U^i_{c^i_{Is}}/p_{Is}}{U^i_{c^i_{I0}}/p_{I0}} = \gamma_s \frac{U^j_{c^j_{Js}}/p_{Js}\chi}{U^j_{c^j_{J0}}/p_{J0}\chi_s(1+r_J)},$$

$$\pi_{Js'} = \gamma_{s'} \frac{U^i_{c^i_{Is'}}/p_{Is'}\chi_{s'}}{U^i_{c^i_{I0}}/p_{I0}(1+r_I)\chi} = \gamma_{s'} \frac{U^j_{c^j_{Js'}}/p_{Js'}}{U^j_{c^j_{J0}}/p_{J0}}.$$

Proof. See Appendix D.19.

5.4 Numerical Analysis

Table (4) displays the numerical result of national currencies and compares it with those of currency unions. Note that the functional forms and exogenous variables are the same as in the baseline regime in a currency union. The unique endogenous variables in the national currency case are the exchange rates at $t = 0$ and $t = 1$.

Table 4: National currencies and currency unions

Currency union with internal devaluation (Regime A)					
$W^I = 3.2921$	$SU^I = 3.2496$	$df^i_{s'} = 13.02\%$	$r^I = 8.03\%$	$\tau_{Is'} = 6.45\%$	$\chi = NA$
Currency union with cross-border default (Regime C)					
$W^I = 3.2947 \uparrow$	$SU^I = 3.2916 \uparrow$	$df_{s'} = 1.63\% \downarrow$	$r^I = 1\% \downarrow$	$\tau_{Is'} = 0\% \downarrow$	$\chi = NA$
National currencies and floating exchange rates (Regime D)					
$W^I = 3.2943 \uparrow$	$SU^I = 3.2713 \uparrow$	$df_{s'} = 3.35\% \downarrow$	$r^I = 1\% \downarrow$	$\tau_{Is'} = 0\% \downarrow$	$\chi_0 = 1$
					$\chi_1 = 1.07$
					$\chi_2 = 0.94$

[a] Arrows indicate the direction of travel compared with Regime A.

The national currency case obtains a superior equilibrium to the internal devaluation regime of a currency union. This is because competitive floating exchange rates neutralise domestic credit risks and banks survive for all states, such that bailout taxes are not required. Exchange rate determination obtains via the FX markets. At country I's good state $s = 1$, currency I depreciates and at country I's bad state $s = 2$, currency I appreciates. Therefore, the domestic liquidities flowing back to country I for domestic loan repayment turn out the same for both states, domestic credit risk is state-invariant, and domestic banks do not encounter insolvency due to the shortage of liquidity causing credit risk volatilities. Nevertheless, the equilibrium of the national currency case is slightly inferior to the cross-border default regime in a currency union, suggesting default in the cross-border capital markets in a currency union under this parameterisation obviates the need for floating exchange rates.

6 Discussions and policy implications

6.1 Discussion of results

The first key result of the model is that adjusting the bankruptcy code to allow for default in the cross-border capital markets can improve a currency union in the absence of a fiscal union. Because cross-border default is strategic, it provides an extra boost of liquidity for the borrowing country in the bad state. This extra liquidity acts like "cross-country liquidity transfers" to adjust for the country heterogeneity in a currency union, and consequently it proves to be a close substitute for a fiscal union. The key difference is that a fiscal union resembles a supranational entity that uses a visible hand to move nominal resources between countries, whereas cross-border default works through the invisible hand of markets: default is a choice and assets are conditionally traded voluntarily.

This key difference can be regarded as the strength of cross-border default compared to a fiscal union. Indeed, the fiscal union modelled in Regime B is assumed to be benevolent. This is quite a leap of faith, because there is no obvious mechanism to prevent the fiscal union from using the visible hand to make preferential transfers to selected member countries. Once the model relaxes the benevolence assumption, it would add further support to Regime C (bankruptcy leniency) as a feasible approach to sustain currency unions.

However, the limitation of Regime C (bankruptcy leniency) is that it relies on the assumption that the institutional qualities or the economic fundamentals of member countries in the currency union do not differ significantly. Suppose the two countries have divergent economic fundamentals to start with, for example, country J in its good state is much poorer than country I in its respective good state. Then in state s', i.e. country I's bad state and country J's good state, such a lenient union-wide bankruptcy code may encourage country J to even fully default in its good state. In this scenario, capital markets union collapses, impeding international risk sharing and leading to welfare loss. In this case, the within-union standard fails, and there is simply no room to adjust for the union-wide bankruptcy code. If the model is extended to incorporate production and capital accumulation for multi-periods, the issue becomes more acute. Adjusting the bankruptcy code would need to be timely, because a delay would lead countries to fall out of the within-union standard endogenously due to the persistent internal devaluation effect. Moreover, even if their fundamentals do not differ too much, if country I has a lower institution quality, i.e. its domestic bankruptcy code λ^i is much weaker than that of country J, Corollary 3.2 suggests that there simply may not be any space left to adjust the union-wide bankruptcy code λ, otherwise the financial markets would unravel.

The second key result is that adjusting the bankruptcy code in a currency union can obviate the need for nominal exchange rates to neutralise domestic credit risks. Strategic default in the capital markets union is essentially a compensation for the lost benefits of nominal exchange rates. However, assuming away potential costs of nominal exchange rates, the conditions for bankruptcy leniency to allow for cross-border default to neutralise domestic credit risks are more exacting than those for nominal exchange rates. The conditions for the former entail a lenient cross-country bankruptcy code, a sufficiently low cost of liquidity and a sufficiently high number of bad states, but the conditions for the latter are more general. However, nominal exchange rates could potentially incur (unmodelled) costs, such as competitive devaluation because of discretionary intervention by national governments. Such considerations would render cross-country default in a currency union a more attractive regime.

Furthermore, this model sets the union-wide bankruptcy code of the capital markets union conditional on domestic bankruptcy codes, which are taken as structural parameters. In contrast, Franks and Sussman (2005) constructed an endogenous evolutionary theory of bankruptcy codes, in which bankruptcy codes can emerge either due to freedom of contracting of market participants or because of state activism in law-making. In this paper, I do not model the endogenous emergence of the bankruptcy codes, but I acknowledge that in practice the softening of the bankruptcy code in the capital markets union could emerge via either market forces or law-making, or a combination of both.

6.2 Policy implications and implementation

The immediate policy recommendation is that a currency union needs a capital markets union that features a slightly softened bankruptcy environment. Softening the cross-border bankruptcy environment can take measures to encourage more reorganisation rather than immediate liquidation of assets when companies become insolvent, to shorten the period of credit market exclusion for the defaulters, or to allow for debt discharges and adjustments. To increase the space to design such a union-wide bankruptcy code to induce benign default in the capital markets union, the domestic bankruptcy code, however, should not be too lenient. Thus, when a fiscal union is absent in a currency union, member countries should strengthen their domestic institutions by toughening the domestic bankruptcy code for domestic borrowing, and at the same time mutually agree on a softened union-wide bankruptcy code for cross-country borrowing. An important caveat remains that the union-wide bankruptcy code cannot be too lenient. If it is sufficiently lenient, no country would ever repay in any state, leading to the collapse of the capital markets union and impeding risk sharing.

This scope for policy is relevant not just for the Eurozone, but also for China which has been experiencing prolonged soaring local government debt. The issue for China is more nuanced because the local government debt is largely collateralised by land and real estate. It is further complicated by the Chinese government's implicit guarantee. Consequently, the talk of default of the local government financing vehicle in China has sparked off heated debate among policy makers and academics. This real-world example and the associated macro-financial questions could be modelled in the framework of this paper.

As for the implementation of setting the union-wide bankruptcy code that induces default in the capital markets union, the union-wide bankruptcy code should be common knowledge to all market participants and should be strictly enforced. In this environment, market participants would know that default in cross-border capital markets is a possibility and *ex ante* they know exactly the severity of the default punishment conditional on the size of default. Accordingly, default risks are priced in when assets are traded prior to the realisation of uncertainty. For example, suppose the union-wide bankruptcy code takes the form of credit exclusion as the default punishment and for simplicity suppose borrowers are countries in the currency union. Market participants should know how many periods the defaulter is to be excluded given the size of default. Once a country announces default, it should be excluded from credit to the extent that market participants have come to expect. This kind of implementation, therefore, ensures orderly default. It is important to note the distinction between this type of orderly default and the unanticipated default whose risk is not correctly priced in.

The model is kept stylised such that the results and transmission mechanisms are clear; therefore, the model does not specify the exact form of default punishment. A fully-fledged and more realistic quantitative model could certainly provide further micro-foundation for default punishment, such as credit exclusion or sanctions. Nevertheless, no matter what the exact form of default punishment is, as long as it feeds into the marginal rate of substitution across consumptions, the essence of the current stylised model remains.

6.3 Institutional Details

6.3.1 The TARGET2 System

The model abstracted away the national central banks; thus, it is difficult to find an exact mapping between the model and the TARGET2 system, which is the Eurozone payment system connecting national central banks (NCB)) and the European Central Bank (ECB). However, the two key results of this model can still shed light on the ongoing debate on the imbalance of the TARGET2 system (see Sinn 2011; Sinn and Wollmershäuser 2012; Whelan 2014).

TARGET is the acronym for *Trans-European Automated Real-time Gross Settlement Express Transfer System*. In the TARGET2 system, all Eurosystem banks maintain reserve accounts with their national central banks, which are ultimately connected by ECB as the lender of last resort. The TARGET2 system equips the national central banks with some degree of money creation power such that the creation of euro via ECB becomes less rigid. Therefore, the TARGET2 system allows some national central banks to hold TARGET2 claims and some other national central banks to hold TARGET2 liabilities.

During the Eurozone Crisis, capital flight from the weak periphery largely took the form of depositors moving deposits from weak banks in the periphery into the stronger banks in the core. This transfer of funds would have bankrupted many banks in the periphery, but it was recycled back through the TARGET2 system. In this sense, the TARGET2 system provides some cross-country transfers already, partly resembling the fiscal transfers in Regime B (fiscal union) of my model. Therefore, in reality, the TARGET2 system has likely reduced the bailout costs of the Eurozone Crisis and dampened the internal devaluation effect in Regime A of this model.

Nevertheless, the imbalance of the TARGET2 system is not completely the same as the cross-country fiscal transfers in Regime B. This is because the imbalance of TARGET2 system takes the form of claims and liabilities. These positions are debt positions that eventually need to be settled, whereas Regime B makes fiscal transfers that are free and clear of any debt obligation. If the debt positions on the TARGET2 system need to be repaid in full eventually, it would merely imply the delay of national bailout costs and internal devaluation. If the debt positions on the TARGET2 system were to carry priced-in credit risks and allow for default, a similar welfare-improving benefit could arise as that of Regime C (bankruptcy leniency) in this paper.

6.3.2 Cross-border Insolvency Reforms in Practice

A variety of financial securities are traded in the cross-border capital markets in the Eurozone. These financial securities can be sovereign bonds of member countries, corporate bonds and equities, or structured products of both the public and private debt. The key result of this paper on softening the bankruptcy code in the cross-border capital markets union is a simplification. The insight is the role of a softened bankruptcy environment as a compensation for the removal of flexible nominal exchange rates.

Now let us turn to the practical interpretations of this result in conjunction with the institutional details of both the default punishment for sovereign debt and the cross-border insolvency reforms for private debt in the context of the Eurozone. There was a wide misperception of the sovereign debt default risks before the Eurozone Crisis. The misperception was that member countries in the Eurozone simply would not default, for example, quoting the Economics Commissioner Joaquin Almunia at the time: "No, Greece will not default. Please. In the euro area, the default does not exist." However, the synchronisation of sovereign yields between the core and the periphery before the crisis does not mean the sovereign debt of the periphery was default-free. This synchronisation was largely a result of reduction in the inflation premium in the periphery, and the inflation premium was a proxy for the sovereign's credit risk profile. The reduction in the inflation premium was a natural result of removing the nominal exchange rates, but it does not imply the underlying credit risks suddenly magically vanished.

Therefore, it would be sensible to correct such misperception and to acknowledge and allow for sovereign default in a currency union, particularly when a fiscal union is not present. In this paper I only model the aggregate country-level debt and I do not model the sovereign debt separately; however, the logic of softening the bankruptcy code extends to the sovereign bond markets. In practice, the bankruptcy code for the sovereign debt often takes the form of such default punishment as credit market exclusion. Softening this type of default punishment could mean shorten the period of credit market exclusion per unit of default. In this scenario, when the marginal cost of default is low enough, countries in the Eurozone can declare default and the punishment should be strictly enforced. What is crucial is the correct pricing of default risks in the sovereign yields.

As for private debt, a softened bankruptcy code can be a result of contracting or legal reforms. Regarding the latter, there have been practical steps in the European Union cross-border insolvency reform, albeit not for the specific context of the Eurozone. In 2012, the European Commission proposed to recast the 2000 Insolvency Regulation with the main purpose to help identify the competent jurisdiction and applicable law insolvency proceedings. As the 2000 Insolvency Regulation carries legal uncertainty with some of its key concepts defined in general terms (see Sussman 2006), then the 2015 recast regulation provided further clarification on the "centre of main interest" (COMI). More importantly, in 2016, the European Commission proposed to adopt a directive on business restructuring and giving the troubled businesses a second chance, softening the traditionally punitive stance on default.

In spirit, the 2016 directive is similar to Chapter 11 of the US Bankruptcy Code, and it seems broadly consistent with the policy implication of my model. Nevertheless, in practice, the cross-border insolvency reforms in EU or the Eurozone are extremely complicated because the national-level bankruptcy codes are very diverse (see Davydenko and Franks 2008). My model speaks to this issue because my result on softening the cross-border bankruptcy code is exactly conditional on national-level bankruptcy codes and takes into consideration of their heterogeneity. Thus, my result does not lead to the conclusion of an unconditional top-down harmonisation.

7 Conclusion

This paper has proposed an international finance model specifically for currency unions to address the following question: what alternative arrangements can improve the financial stability and viability of currency unions when a fiscal union is absent? This question is important due to the potential for member countries in a currency union to refuse to establish a fiscal union for political reasons. The model is able to show that one alternative arrangement is a financial regime that features a softened bankruptcy environment in the cross-border capital markets within a currency union.

There are two contributions of this proposed theory. First, the model shows that when domestic credit risks are present and when there is no fiscal union to make cross-country transfers, endogenous default in the capital markets union within a currency union can lead to a Pareto improvement and enhance the viability of a currency union. Second, the model answers the question of why default in the cross-border capital markets is particularly vital for currency unions. That is, under very general conditions competitive floating exchange rates are shown to neutralise domestic credit risks and improve welfare, and sharing a common currency loses such benefits of exchange rates. However, default in the cross-border capital markets under certain conditions recoups the lost benefits of exchange rates. Therefore, when countries join a currency union with an incomplete fiscal union or banking union, the bankruptcy code needs to adjust.

I acknowledge that the endogenous determination of default punishment and the endogenous adoption of currency unions should be the subject of further research. The bankruptcy code λ is the key parameter of default punishment, and it is essentially the price of default. Suppose countries take the price of default as given and "demand" default, then the system would need the "supply" of default to endogenise the price. This sits right at the intersection between law, finance, and political economy. Moreover, this paper takes currency unions as given and the model does not explicitly explain why countries choose to adopt a currency union. One possible explanation is that some countries do not have the institutional capacity to implement a credible domestic bankruptcy code and excessively choose inflation or devaluation as "soft default". Again, this argument goes back to the first point on endogenising the price of default. Excessive inflation or devaluation incurs the cost of persistent capital outflow and further political costs. Therefore, countries adopt a currency union by giving up their monetary autonomy as a self-commitment device.

In conclusion, a broader role of this paper is an initial attempt to bridge the gap between the value of money, exchange rate determination and bankruptcy codes in international finance. This opens new avenues for the research on the endogenous emergence of international dominant currencies and global financial cycles.

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Appendices

A Currency Union - country J

A.1 Country J

Household j

Household j consumes both the domestic goods and foreign goods at $t = 0$ and $t = 1$ and suffers a non-pecuniary penalty from default. Household j's maximisation problem is outlined as follows:

$$\underbrace{Max}_{\mu_J^j,\theta^j,\phi^j,c_J^j,c_I^j,q_J^j,b_I^j,v^j,D^j} \quad E_0\Big\{U^j\Big(c_{J0}^j, c_{I0}^j, c_{J\boldsymbol{s}}^j, c_{I\boldsymbol{s}}^j\Big) - \lambda^j[d_{\boldsymbol{s}}^j]^+ - \lambda[f_{\boldsymbol{s}}^j]^+\Big\},$$

where $\lambda^j[d_{\boldsymbol{s}}^j]^+ = \lambda^j max[(1 - v_{\boldsymbol{s}}^j)\mu_J^j/p_{\boldsymbol{s}}, 0]$ is the non-pecuniary cost of domestic loan default, and $\lambda[f_{\boldsymbol{s}}^j]^+ = \lambda max[(\phi_{\boldsymbol{s}}^j - D_{\boldsymbol{s}}^j)/p_{\boldsymbol{s}}, 0]$ is the non-pecuniary cost of cross-country default.

Subject to

$$b_{I0}^j + \sum_{l=1}^{S} \pi_l \theta_l^j \leq \frac{\mu_J^j}{1 + r_J} + m^j, \tag{21}$$

$$b_{I\boldsymbol{s}}^j(1 + \tau_{J\boldsymbol{s}}) + (D_{\boldsymbol{s}}^j - K_{\boldsymbol{s}}\theta_{\boldsymbol{s}}^j) + \phi_{\boldsymbol{s}}^j \tau_{J\boldsymbol{s}} \leq \Delta(21) + \sum_{l=1}^{S} \pi_l \phi_l^j + p_{I0} q_{I0}^i + p_{J\boldsymbol{s}} q_{J\boldsymbol{s}}^j + \delta_{\boldsymbol{s}}^j, \tag{22}$$

$$v_{\boldsymbol{s}}^j \mu_J^j \leq \Delta(22)_{\boldsymbol{s}}. \tag{23}$$

And for $\boldsymbol{s}^* \in S^*$, the feasibility constraints are satisfied, i.e. $c_{J\boldsymbol{s}^*}^j \leq e_{J\boldsymbol{s}^*}^j - q_{J\boldsymbol{s}^*}^j$ and $c_{I\boldsymbol{s}^*}^j \leq \frac{b_{I\boldsymbol{s}^*}^j}{p_{I\boldsymbol{s}^*}}$. At $t = 0$, household j borrows from the domestic commercial bank j to get money to buy nominal Arrow securities and imports. Household j also gets monetary income by selling nominal Arrow securities and exports that gets carried over to the next period. At $t = 1$, household j sells exports and uses existing money to purchase imports and deliver (net) asset payoffs subject to domestic tax levy. At $t = 2$, household j uses unused money to settle outstanding loans and chooses how much to repay or default.

Domestic Commercial Bank j

Bank j extends loans and provides liquidity for household j. To ensure the liquidity bank j provides has a one-to-one convertibility with the common currency the union-wide central bank issues, bank j needs to borrow interbank loans from the union-wide central bank. Similar to bank i, bank j's maximisation problem is outline as follows,

$$\underbrace{Max}_{\mu^j_{CB},\mu^j_J,L^j,\omega^j} \sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}}\omega^j_{\boldsymbol{s}},$$

subject to

$$L^j \leq \frac{\mu^j_{CB}}{1+\rho}, \tag{24}$$

$$\frac{\mu^j_J}{1+r_J} \leq L^j, \tag{25}$$

$$\omega^j_{\boldsymbol{s}} = \Delta(24) + \Delta(25) + R^j_{\boldsymbol{s}}\mu^j_J - \mu^j_{CB}. \tag{26}$$

At the beginning of $t = 0$, bank j borrows from the union-wide central bank and obtain fiat money in the common currency, ready to be extended as interbank liquidity L^j. Then bank j extends loans to the domestic households but has to ensure the liquidity bank j provides against the bank loans has a one-to-one convertibility with the fiat money issued by the central bank. At $t = 2$, bank j uses the households' loan repayments to pay back the interbank loans to the union-wide central bank.

National Government j

National government j collects taxes from the domestic household based on import expenditures and cross-country borrowing to build a state-contingent bailout fund of $T_{J\boldsymbol{s}}$.

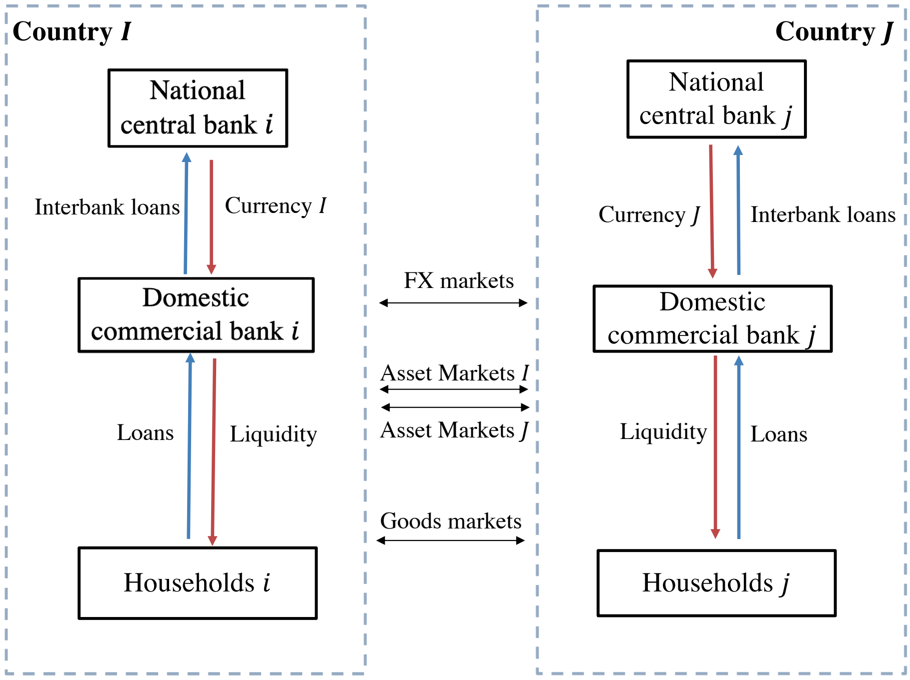
$$T_{J\boldsymbol{s}} = p_{I\boldsymbol{s}}c^j_{I\boldsymbol{s}}\tau_{J\boldsymbol{s}} + \phi^j_{\boldsymbol{s}}\tau_{J\boldsymbol{s}}.$$

National government j uses the bailout funds to rescue the national commercial bank whenever the bank's nominal profits would drop to negative. This is to ensure $\omega^j_{\boldsymbol{s}} + T_{J\boldsymbol{s}} = 0$ at the bad state.

B Regime D - Country J

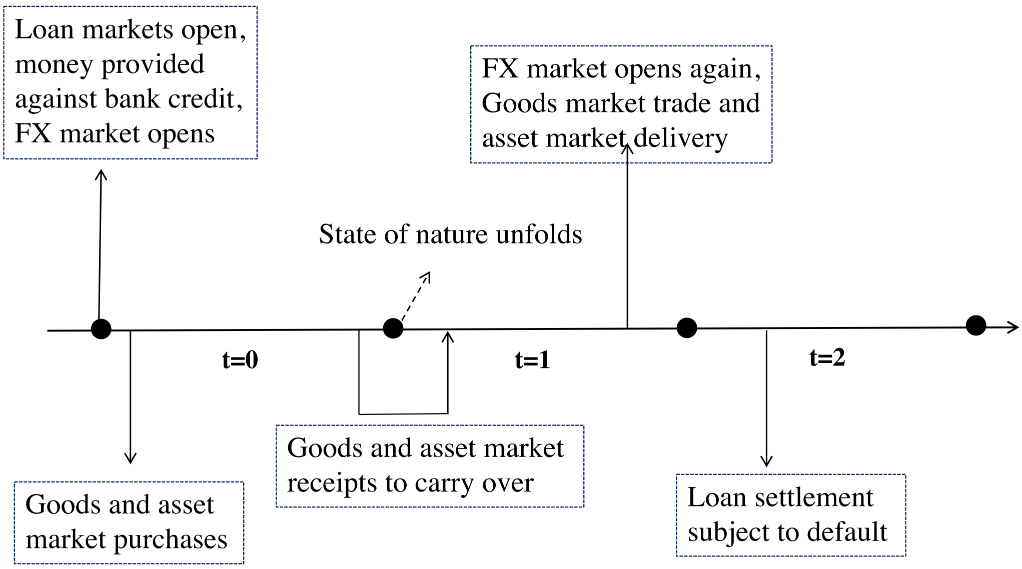
B.1 Structure and Flows

Figure 11: Nominal flows of the economy



B.2 Regime D Timeline

Figure 12: Timeline



B.3 Country J

The modelling of country$J$ is exactly symmetric to that of country I. The brief model outline and description are provided below for completeness.

Household j

Household j's preference is the same as in the benchmark model except that the price deflator p^J_s is a country-wide price index rather than a union-wide price index. The household's maximisation problem is as follows,

$$\underbrace{Max}_{\mu^j_J, f^j_{JI}, \theta^j, \phi^j, c^j_J, c^j_I, q^j_J, b^j_I, v^j} \quad E_0\Big\{U^j\Big(c^j_{J0}, c^j_{I0}, c^j_{J\boldsymbol{s}}, c^j_{I\boldsymbol{s}}\Big) - \lambda^j \frac{max[(1-v^j_{\boldsymbol{s}})\mu^j_J, 0]}{p^J_{\boldsymbol{s}}}\Big\},$$

subject to

at $t = 0$:

$$f^j_{JI} \leq \frac{\mu^j_J}{1+r_J} + m^j, \tag{27}$$

$$b^j_{I0} + \sum_{l=1}^{S} \pi_{Il}\theta^j_{Il} \leq \chi f^j_{JI}, \tag{28}$$

at $t = 1$ in state s:

$$f^j_{JI\boldsymbol{s}} + \phi^j_{J\boldsymbol{s}}(1+\tau_{J\boldsymbol{s}}) \leq \sum_{l=1}^{S} \pi_{Jl}\phi^j_{Jl} + p_{J0}q^j_{J0} + p_{J\boldsymbol{s}}q^j_{J\boldsymbol{s}} + \Delta(27), \tag{29}$$

$$b^j_{I\boldsymbol{s}}(1+\tau_{J\boldsymbol{s}}) \leq \theta^j_{I\boldsymbol{s}} + \chi_{\boldsymbol{s}} f^j_{JI\boldsymbol{s}} + \Delta(28), \tag{30}$$

$$v^j_{\boldsymbol{s}}\mu^j_J \leq \Delta(29)_{\boldsymbol{s}}, \tag{31}$$

$$c^j_{J\boldsymbol{s}^*} \leq e^j_{J\boldsymbol{s}^*} - q^j_{J\boldsymbol{s}^*}, \tag{32}$$

$$c^j_{I\boldsymbol{s}^*} \leq \frac{b^j_{I\boldsymbol{s}^*}}{p_{I\boldsymbol{s}^*}}. \tag{33}$$

Domestic Commercial Bank j

Bank j's actions are the same as in the benchmark model except that bank j now borrows interbank loans from the national central bank rather than a union-wide central bank. Therefore, ultimately fiat money in country J is issued by the national central bank and is in the national currency. Denote μ^j_{CBJ} as the REPO loans bank j borrows from the national central bank and ρ^J as the policy rate set by the national central bank. Bank j's maximisation problem is specified as follows,

$$\underbrace{Max}_{\mu^j_{CBJ},\mu^j_J,L^j,\omega^j} \quad \sum_{\boldsymbol{s}}^{S} z_{\boldsymbol{s}}\omega^j_{\boldsymbol{s}},$$

subject to

$$L^j \leq \frac{\mu^j_{CBJ}}{1+\rho_J}, \tag{34}$$

$$\frac{\mu^j_J}{1+r_J} \leq L^j, \tag{35}$$

$$\omega^j_{\boldsymbol{s}} = \Delta(34) + \Delta(35) + R^j_{\boldsymbol{s}}\mu^j_{J\boldsymbol{s}} - \mu^j_{CBJ}. \tag{36}$$

National Government j

National government j collects taxes from domestic households to build a state-contingent bailout fund of $T_{J\boldsymbol{s}}$.

$$T_{J\boldsymbol{s}} = p_{I\boldsymbol{s}}c^j_{I\boldsymbol{s}}\tau_{J\boldsymbol{s}} + \phi^j_{\boldsymbol{s}}\tau_{J\boldsymbol{s}}. \tag{37}$$

National government j uses the bailout funds to rescue the domestic commercial bank whenever the bank's nominal profits would drop to negative, i.e. the bank fails. In short, at the bad state the national government makes a state-contingent transfer to ensure $\omega^j_{\boldsymbol{s}} + T_{J\boldsymbol{s}} = 0$.

National Central Bank j

The national central bank lends REPO loans μ^J_{CBJ} to the domestic commercial bank and provides fiat money in the national currency. The national central bank sets the REPO loan rate ρ_J.

C Equilibrium and Regime Characterisation

C.1 Proof of Lemma 1.

Proof. First, I identify the condition for household i's liquidity-in-advance constraint to bind. Suppose $r_I > 0$ and suppose further household i does not spend on all the money at hand, i.e. $\Delta(1) > 0$. The household can borrow ϵ less loan, but it only leads to $\epsilon/(1+r_I)$ reduction on the money at hand from (1).

Moreover, from (3), the reduction of loan needing to repay is $v^i_{\boldsymbol{s}}\epsilon$, and the pecuniary benefit due to the reduction of default cost amounts to $(1-v^i_{\boldsymbol{s}})\epsilon$. Thus, there is extra money $(v^i_{\boldsymbol{s}} - 1/(1+r_I) + 1 - v^i_{\boldsymbol{s}})/\epsilon = r_I\epsilon/(1+r_I)$ at $t=2$. Household i can either use this extra money to consume more or increase loan repayment, and both lead to welfare improvement. This is a contradiction; hence, $\Delta(1) = 0$.

Second, I prove the binding condition for bank i's liquidity-in-advance constraint. Suppose bank i does not extend all the money at hand as interbank liquidity to household i, i.e. $\Delta(4) > 0$. Bank i can borrow ϵ less interbank loan, but it only leads to $\epsilon/(1+\rho)$ reduction on the money at hand in (6). The reduction of loan needing to repay is ϵ, which means there is an extra money of $\rho\epsilon/(1+\rho)$ in (6) that adds to bank i's cash flow, increasing its utility. This is a contradiction. Hence, $\Delta(4) = 0$. $\square$

C.2 Proof of Lemma 2

Proof. Suppose $\Delta(5) > 0$, bank i can reduce the interbank liquidity L^i by ϵ, given (4) binds, it means bank i borrows $(1+\rho)\epsilon$ less interbank loans. From (6), the money at hand is reduced by ϵ, but the interbank loan needing to repay is reduced by $(1+\rho)\epsilon$. This means there is an extra money of $\rho\epsilon$ that adds to bank i's cash flow at $t = 2$, increasing its utility. This is a contradiction. Hence, $\Delta(5) = 0$. $\square$

C.3 Proof of Lemma 3.

Proof. Suppose $\Delta_{\boldsymbol{s}}(3) > 0$, then household i can either increase repayment rate $v^i_{\boldsymbol{s}}$ or borrow ϵ more at the interest rate of r_I. If household i increases $v^i_{\boldsymbol{s}}$, it leads to an immediate increase in utility because the cost of default decreases This improvement of welfare means a contradiction. If household i borrows ϵ more, then household i has an extra money of $\epsilon/(1+r_I)$ and uses it to buy more imports or sell less exports, leaving all other actions unchanged, without violating the inequality (3), because the household has enough money at hand to repay the extra loan. The improvement of welfare means a contradiction.

Similarly contradiction arises if $\Delta_{\boldsymbol{s}}(6) > 0$. Therefore, $\Delta_{\boldsymbol{s}}(3) = 0$ and $\Delta_{\boldsymbol{s}}(6) = 0$. $\square$

C.4 Proof of Lemma 4.

Proof. Suppose $\phi^i_s > 0, \theta^i_{s'} > 0$. From household i's perspective, the FOC for ϕ^i_s leads to $\pi_s \frac{U^i_{c^i_{I0}}}{p_{I0}} = (1+\tau_{Is})\gamma_s\eta^I_{1s}$. FOC for $\theta^i_{s'}$ leads to $\pi_{s'}\frac{U^i_{c^i_{J0}}}{p_{J0}} = \gamma_{s'}\eta^i_{1s'}$. Suppose $\eta^i_{1s} = \eta^i_{1s'}$, and also the FOCs for c^i_{I0}, c^i_{J0}, and μ^i_I lead to $\frac{U^i_{c^i_{J0}}}{p_{J0}} = (1+r_I)\frac{U^i_{c^i_{I0}}}{p_{I0}}$, it follows $\frac{\pi_s}{(1+\tau_{Is})\gamma_s} = \frac{\pi_{s'}(1+r_I)}{\gamma_{s'}}$. Hence,

$$\pi_s\gamma_{s'} > \pi_{s'}\gamma_s. \tag{38}$$

On the other hand, from household j's perspective, household j's FOC for θ^j_s leads to $\pi_s\frac{U^j_{c^j_{I0}}}{p_{I0}} = \gamma_s\eta^j_{1s}$, FOC for $\phi^j_{s'}$ gives $\pi_{s'}\frac{U^j_{c^j_{J0}}}{p_{J0}} = \gamma_{s'}(1+\tau_{Js'})\eta^j_{1s'}$. Given $\eta^j_{1s} = \eta^j_{1s'}$, and also the FOCs for $c^j_{J0}, c^j_{I0}, \mu^j_J$ lead to $\frac{U^j_{c^j_{I0}}}{p_{I0}} = (1+r_J)\frac{U^j_{c^j_{J0}}}{p_{J0}}$, it follows that $\frac{\pi_s(1+r_J)}{\gamma_s} = \frac{\pi_{s'}}{(1+\tau_{Js'})\gamma_{s'}}$. Hence,

$$\pi_1\gamma_2 < \pi_2\gamma_1. \tag{39}$$

We can see that (39) contradicts (38).

Suppose $\theta^i_s > 0, \phi^i_{s'} > 0$, by the same logic, a contradiction also arises. Therefore, it is not possible that $\eta^i_{1s} = \eta^i_{1s'}$ and $\eta^j_{1s} = \eta^j_{1s'}$. $\square$

C.5 Proof of Lemma 5

Proof. It follows from the banks' FOC that $\forall \boldsymbol{s} \in S$, $h \in \{i,j\}$, $\omega^h_{\boldsymbol{s}} = (\frac{v^h_{\boldsymbol{s}}}{\sum_{\boldsymbol{s}=1}^{S} \eta_{\boldsymbol{s}} v^h_{\boldsymbol{s}}} - 1)\mu^h_{CB}$. Given the assumption that $v^h_{\boldsymbol{s}} = 1$, $\forall \boldsymbol{s} \in S$, $\omega^h_{\boldsymbol{s}} = 0$. □

C.6 Proof of Proposition 1

- The Fisher effect

  Denote η^i_0, $\eta^i_{\boldsymbol{s}1}$, and $\eta^i_{\boldsymbol{s}2}$ as the shadow prices of the three flow of funds constraints of household i. Suppose that household i has some money left over the moment the inter-period domestic loan comes due. From household i's FOCs for c^i_{J0} and μ^i_I, it follows that $\eta^i_0 = (1+r_I)E_0\eta^i_{\boldsymbol{s}2}$, namely, $\frac{U^i_{c^i_{J0}}}{p_{J0}} = (1+r_I)E_0\frac{U^i_{c^i_{I\boldsymbol{s}}}}{p_{I\boldsymbol{s}}}$. And also household i's FOCs for $c^i_{I\boldsymbol{s}}$ and $c^i_{J\boldsymbol{s}}$ give $\frac{U^i_{c^i_{I\boldsymbol{s}}}}{p_{I\boldsymbol{s}}} = \frac{U^i_{c^i_{J\boldsymbol{s}}}}{p_{J\boldsymbol{s}}(1+\tau_{I\boldsymbol{s}})}$. It follows that

$$1 + r_I = \Big(E_0(\frac{U^i_{c^i_{J\boldsymbol{s}}}}{U^i_{c^i_{J0}}})(\frac{p_{J0}}{p_{J\boldsymbol{s}}})\frac{1}{(1+\tau_{I\boldsymbol{s}})(1+\iota_{I\boldsymbol{s}})}\Big)^{-1}.$$

- Quantity Theory of Money

  Invoking Lemma 1, summing up the flow of funds constraints of both households at $t = 0$, and using market clearing conditions for goods market, domestic loan markets and interbank loan and money market, it follows that

$$p_{I0}q^i_{I0} + p_{J0}q^j_{J0} = M_0 - \sum_{h\in\{i,j\}}\sum_{m=1}^{S}\pi_m\theta^h_m + \sum_{h\in\{i,j\}} m^h.$$

  For $t = 1$, summing up the flow of funds constraints of both households at $t = 1$, and using market clearing conditions for goods market, domestic loan markets, interbank loan and money market, and asset markets, it follows that

$$p_{I\boldsymbol{s}}q^i_{I\boldsymbol{s}} + p_{J\boldsymbol{s}}q^j_{J\boldsymbol{s}} = M + \sum_{h\in\{i,j\}}\epsilon^h - \sum_{H\in\{I,J\}} T_{H\boldsymbol{s}} - \sum_{h\in\{i,j\}}\Delta^h_{\boldsymbol{s}}.$$

- Money non-neutrality

  For the general method of proof on money non-neutrality, please see Tsomocos (2001, 2003). Here a specific proof by contradiction is provided. Combine household i's FOCs for c^i_{I0}, c^i_{J0}, and μ^i_I, we obtain

$$\frac{U^i_{c^i_{J0}}}{p_{J0}} = (1+r_I)\frac{U^i_{c^i_{I0}}}{p_{I0}}, \tag{40}$$

  combine household j's FOCs for c^j_{J0}, c^j_{I0}, and μ^j_J, we obtain

$$\frac{U^j_{c^j_{I0}}}{p_{I0}} = (1+r_J)\frac{U^j_{c^j_{J0}}}{p_{J0}}. \tag{41}$$

Suppose the union-wide central bank increases the interbank rate ρ. Suppose consumption and default remain unchanged. Based on banks' FOCs that $1+r_I = \frac{1+\rho}{\sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^i}$ and $1+r_J = \frac{1+\rho}{\sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^j}$, we can see that r_I and r_J both increase. Eq(40) implies $\frac{p_{I0}}{p_{J0}}$ has to increase, whereas Eq(41) implies that $\frac{p_{J0}}{p_{I0}}$ has to increase. This is a contradiction. □

C.7 Proof of Corollary 1.1

Proof. Let $z_{\boldsymbol{s}}$ be the risk-neutral probabilities $\forall \boldsymbol{s} \in S$. Bank i's FOC has $1 + r_I = \frac{1+\rho}{\sum_{s=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^i}$, $\omega_{\boldsymbol{s}}^i = v_{\boldsymbol{s}}^i \mu_I^i - \mu_{CB}^i = (\frac{v_{\boldsymbol{s}}^i}{\sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^i} - 1)\mu_{CB}^i$. Given the assumption $v_s^i > \sum_{s=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^i$, and $v_{s'}^i < \sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^i$, then $\omega_s^i > 0$, and $\omega_{s'}^i < 0$. Similarly, for bank j, $\omega_{s'}^j > 0$ and $\omega_s^j < 0$.

At state s, the union-wide central bank's profits $\omega_s^{CB} = \mu_{CB}^i - \frac{\mu_{CB}^i}{1+\rho} + v_s^j \mu_s^j - \frac{\mu_{CB}^j}{1+\rho} + T_s^j$.

Rearranging the algebra and plugging in the market clearing condition for interbank loans and money, as well as bank j's FOCs,

$$
\begin{aligned}
\omega_s^{CB} =& \mu_{CB}^i - \frac{\mu_{CB}^i}{1+\rho} + v_s^j \mu_J^j - \frac{\mu_{CB}^j}{1+\rho} + T_s^j \\
=& \mu_{CB}^i - \frac{\mu_{CB}^i}{1+\rho} + v_s^j \mu_J^j - \frac{1+\rho}{1+r_J}\mu_J^j + \frac{1+\rho}{1+r_J}\mu_J^j - \frac{\mu_{CB}^j}{1+\rho} + T_s^j \\
=& \frac{\rho}{1+\rho}(\mu_{CB}^i + \mu_{CB}^j) + (v_s^j - \frac{(1+\rho)\sum_{s=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^j}{1+\rho})\mu_J^j + T_s^j \\
=& \frac{\rho}{1+\rho}(\mu_{CB}^i + \mu_{CB}^j) + (v_s^j - \frac{(1+\rho)\sum_{s=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^j}{1+\rho})\frac{(1+\rho)}{\sum_{s=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^j}\frac{\mu_{CB}^j}{1+\rho} + T_s^j \\
=& \rho M + (\frac{v_s^j}{\sum_{s=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^j} - 1)\mu_{CB}^j + T_s^j,
\end{aligned}
$$

Moreover, via the flow of funds and budget constraints of the households and commercial banks, total outside money that flows into the union-wide central bank equals $\sum_{h\in\{i,j\}} m^h - \omega_s^i$. Thus,

$$
\rho M + (\frac{v_s^j}{\sum_{s=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^j} - 1)\mu_{CB}^j + T_s^j = \sum_{h\in\{i,j\}} m^h - \omega_s^i.
$$

Similar proof follows for state s',

$$
\rho M + (\frac{v_{s'}^i}{\sum_{s=1}^{S} z_{\boldsymbol{s}} v_{\boldsymbol{s}}^i} - 1)\mu_{CB}^i + T_{s'}^i = \sum_{h\in\{i,j\}} m^h - \omega_{s'}^j.
$$

□

C.8 Proof of Proposition 2

Proof. At country I's bad state s' when credit risks are present, suppose $\lambda > \lambda^i$, it means the marginal cost of default on financial securities is larger than the marginal cost of default on domestic loans. At the bad state s', household i's FOC for $v^i_{s'}$ gives $\frac{\lambda^i}{p_{s'}} = \frac{U^i_{c^i_{Is'}}}{p_{Is'}}$, it follows that

$$\frac{\lambda}{p_{s'}} > \frac{U^i_{c^i_{Is'}}}{p_{Is'}}. \tag{42}$$

The left-hand side of (42) is the marginal cost of default on financial securities and the right-hand side of (42) is the marginal benefit. Since the marginal cost of default on financial securities is larger than the marginal benefit, household i fully delivers on cross-country borrowing, i.e. $D^i_{s'} = \phi^i_{s'}$.

At the good state household i's FOC for v^i_s gives $\frac{\lambda^i}{p_s} \geq \frac{U^i_{c^i_{Is}}}{p_{Is}}$. Since $\lambda > \lambda^i$, it follows that $\frac{\lambda}{p_s} > \frac{U^i_{c^i_{Is}}}{p_{Is}}$. Hence, $D^i_s = \phi^i_s$.

Overall, $\forall \boldsymbol{s} \in S$, $D^i_{\boldsymbol{s}} = \phi^i_{\boldsymbol{s}}$.

Now suppose the cross-country bankruptcy code is more lenient, i.e. $\lambda < \lambda^i$. It means the marginal cost of default on financial securities is smaller than the marginal cost of default on domestic loans. At the bad state, $\lambda \leq \frac{U^i_{c^i_{Is'}}}{p_{Is'}}$, and $\frac{\lambda^i}{p_{s'}} \geq \frac{U^i_{c^i_{Is'}}}{p_{Is'}}$. It is less costly to default on financial securities, so household would default on financial securities first instead of domestic loans, i.e. $D^i_{s'} < \phi^i_{s'}$.

At the good state household i's FOC for v^i_s gives $\frac{\lambda^i}{p_{s'}} \geq \frac{U^i_{c^i_{Is'}}}{p_{Is'}}$, since $\lambda < \lambda^i$, the relationship between $\frac{\lambda}{p_{s'}}$ and $\frac{U^i_{c^i_{Is'}}}{p_{Is'}}$ is ambiguous. The marginal cost of default on financial securities can be larger than, or equal to, or smaller than the marginal benefit. Therefore, $D^i_s \leq \phi^i_s$.

Overall, $\forall \boldsymbol{s} \in S$, $D^i_{\boldsymbol{s}} \leq \phi^i_{\boldsymbol{s}}$. The same logic follows for the case of country J. □

D Equilibrium Analysis

D.1 Proof of Proposition 3

Proof. Since $\frac{\bar{\lambda}}{p_2} = \eta^i_{11}$, $\frac{a_2\bar{\lambda}}{p_2} = \eta^i_{22}$ holds. Given $\eta^j_{12} < \frac{\lambda}{p_2}$ and $a_2\lambda < a_2\bar{\lambda} < \lambda^i$, then $\eta^j_{12} < \frac{\lambda}{p_2} < \eta^i_{12}$. These conditions are the on-the-verge conditions between the households' marginal benefit of default and their marginal cost of default on Arrow securities. Thus, if holding short positions, household i defaults on Arrow security $l = 2$ fully while household j fully delivers the payoff of Arrow security $l = 2$.

Due to risk-aversion, it is straightforward to see that household i buys Arrow security

$l = 2$ because $e_1^i > e_2^i$, $\eta_{11}^i < \eta_{12}^i$, and household j sells Arrow security $l = 2$ because $e_1^j < e_2^j$, $\eta_{11}^j > \eta_{12}^j$. However, to see why household i also sells Arrow security $l = s'$ at the same time whenever $\Big(K_2 - \pi_2\big(v_2^i(1 + r_I) - 1\big)\Big)/p_{I2} > \pi_2 r^l/p_{I1}$ holds, I show via proof by contradiction.

Suppose $\phi_2^i = 0$ is the equilibrium outcome. Suppose now household i sells ϵ amount of Arrow security $l = 2$. According to the market clearing condition of Asset markets, household i would need to buy ϵ amount of Arrow security $l = 2$. This means household i needs to borrow $\pi_2\epsilon$ more money at $t = 0$ while needing to pay back $\pi_2\epsilon v_2^i(1 + r_I)$ at state 2, so the extra monetary cost is $\pi_2\epsilon\big(v_2^i(1+r_I) - 1\big)$. However, because household i can default fully on the ϵ amount of Arrow security sold, the extra money inflow due to default amounts to $K_2\epsilon$, where $K_2 = 1 - \frac{\epsilon}{\sum_{h\in i,j}\phi_2^h}$. Thus, the total money inflow $K_2\epsilon - \pi_2\epsilon\big(v_2^i(1+r_I) - 1\big)$ is positive. For the good state 1, $\eta_{21}^i < \frac{\lambda^i}{p_1}$ holds, $v_1^i = 1$, so the extra monetary cost is $\pi_2\epsilon r_l$. If $\Big(K_2 - \pi_2\big(v_2^i(1 + r_I) - 1\big)\Big)/p_{I2} > \pi_2 r^l/p_{I1}$ holds, then $\nabla U^i(\cdot)(K_2\epsilon - \pi_2\epsilon\big(v_2^i(1+r_I) - 1\big)) > \nabla U^i(\cdot)(\pi_2\epsilon r^l)$ holds, leading to an overall increase in household i's expected utility. This is inconsistent with $\phi_2^i = 0$ as an equilibrium outcome. □

D.2 First-order conditions of Currency Unions

D.2.1 FOCs of Regimes A & B

:

$$\frac{U^i_{c^i_{J0}}}{p_{J0}} = (1 + r_I)E_0\frac{U^i_{c^i_{Is}}}{p_{Is}}, \tag{43}$$

$$\frac{U^i_{c^i_{I0}}}{p_{I0}} = E_0\frac{U^i_{c^i_{Js}}}{p_{Js}(1 + \tau_{Is})}, \tag{44}$$

$$\frac{U^j_{c^j_{I0}}}{p_{I0}} = (1 + r_J)E_0\frac{U^j_{c^j_{Js}}}{p_{Js}}, \tag{45}$$

$$\frac{U^j_{c^j_{J0}}}{p_{J0}} = E_0\frac{U^j_{c^j_{Is}}}{p_{Is}(1 + \tau_{Js})}, \tag{46}$$

$$1 + r_I = \frac{1 + \rho}{\sum_{s=1}^S z_s v_s^i}, \tag{47}$$

$$1 + r_J = \frac{1 + \rho}{\sum_{s=1}^S z_s v_s^j}, \tag{48}$$

for state s:

$$\frac{U^i_{c^i_{Is}}}{p_{Is}} = \frac{U^i_{c^i_{Js}}}{p_{Js}(1+\tau_{Is})}, \tag{49}$$

$$\pi_s \frac{U^i_{c^i_{I0}}}{p_{I0}} = \gamma_s \frac{U^i_{c^i_{Js}}}{p_{Js}}, \tag{50}$$

$$\frac{\lambda^i}{p_s} \geq \frac{U^i_{c^i_{Is}}}{p_{Is}}, \tag{51}$$

$$\frac{\lambda}{p_s} > \frac{U^i_{c^i_{Is}}}{p_{Is}}, \tag{52}$$

$$\pi_s \frac{U^j_{c^j_{I0}}}{p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}}{p_{Is}(1+\tau_{Js})}, \tag{53}$$

$$\frac{U^j_{c^j_{Js}}}{p_{Js}} = \frac{U^j_{c^j_{Is}}}{p_{Is}(1+\tau_{Js})}, \tag{54}$$

$$\frac{\lambda^j}{p_s} \geq \frac{U^j_{c^j_{Js}}}{p_{Js}}, \tag{55}$$

$$\frac{\lambda}{p_s} > \frac{U^j_{c^j_{Js}}}{p_{Js}}, \tag{56}$$

for state s':

$$\pi_{s'} \frac{U^i_{c^i_{J0}}}{p_{J0}} = \gamma_{s'} \frac{U^i_{c^i_{Js'}}}{p_{Js'}(1+\tau_{Is'})}, \tag{57}$$

$$\frac{U^i_{c^i_{Is'}}}{p_{Is'}} = \frac{U^i_{c^i_{Js'}}}{p_{Js'}(1+\tau_{Is'})}, \tag{58}$$

$$\frac{\lambda^i}{p_{s'}} \geq \frac{U^i_{c^i_{Is'}}}{p_{Is'}}, \tag{59}$$

$$\frac{\lambda}{p_{s'}} > \frac{U^i_{c^i_{Is'}}}{p_{Is'}}, \tag{60}$$

$$\pi_{s'} \frac{U^j_{c^j_{J0}}}{p_{J0}} = \gamma_{s'} \frac{U^j_{c^j_{Is'}}}{p_{Is'}}, \tag{61}$$

$$\frac{U^j_{c^j_{Js'}}}{p_{Js'}} = \frac{U^j_{c^j_{Is'}}}{p_{Is'}(1+\tau_{Js'})}, \tag{62}$$

$$\frac{\lambda^j}{p_{s'}} \geq \frac{U^j_{c^j_{Js'}}}{p_{Js'}}, \tag{63}$$

$$\frac{\lambda}{p_{s'}} > \frac{U^j_{c^j_{Js'}}}{p_{Js'}}. \tag{64}$$

D.2.2 FOCs of Regime C

The following FOCs are derived under the conditions of Proposition 5. Under the conditions of Proposition 5, $\phi^i_{s'}, \phi^j_{s'}, \theta^i_{s'} > 0$, $\theta^j_{s'} = 0, D^i_{s'} = 0, D^j_{s'} = \phi^j_{s'}$, $\phi^j_s, \phi^i_s, \theta^j_s > 0, \theta^i_s = 0, D^j_s = 0, D^i_s = \phi^i_s$, and $\tau^i_s, \tau^j_s, \tau^i_{s'}, \tau^j_{s'} = 0$.

$$\frac{U^i_{c^i_{J0}}}{p_{J0}} = (1+r_I)E_0\frac{U^i_{c^i_{Is}}}{p_{Is}}, \tag{65}$$

$$\frac{U^i_{c^i_{I0}}}{p_{I0}} = E_0\frac{U^i_{c^i_{Js}}}{p_{Js}}, \tag{66}$$

$$\frac{U^j_{c^j_{J0}}}{p_{J0}} = E_0\frac{U^j_{c^j_{Is}}}{p_{Is}}, \tag{67}$$

$$\frac{U^j_{c^j_{I0}}}{p_{I0}} = (1+r_J)E_0\frac{U^j_{c^j_{Js}}}{p_{Js}}, \tag{68}$$

$$1+r_I = \frac{1+\rho}{\sum_{s=1}^S z_s v^i_s}, \tag{69}$$

$$1+r_J = \frac{1+\rho}{\sum_{s=1}^S z_s v^j_s}, \tag{70}$$

for state s

$$\pi_s\frac{U^i_{c^i_{I0}}}{p_{I0}} = \gamma_s\frac{U^i_{c^i_{Js}}}{p_{Js}}, \tag{71}$$

$$\frac{\lambda}{p_s} > \frac{U^i_{c^i_{Is}}}{p_{Is}}, \tag{72}$$

$$\frac{\lambda^i}{p_s} > \frac{U^i_{c^i_{Is}}}{p_{Is}}, \tag{73}$$

$$\frac{U^i_{c^i_{Is}}}{p_{Is}} = \frac{U^i_{c^i_{Js}}}{p_{Js}}, \tag{74}$$

$$\pi_s \frac{U^j_{c^j_{I0}}}{p_{I0}} = \gamma_s K_s \frac{U^j_{c^j_{Is}}}{p_{Is}}, \tag{75}$$

$$\pi_s \frac{U^j_{c^j_{J0}}}{p_{J0}} = \gamma_s \frac{\lambda_s}{p_s}, \tag{76}$$

$$\frac{\lambda}{p_s} < \frac{U^j_{c^j_{Js}}}{p_{Js}}, \tag{77}$$

$$\frac{U^j_{c^j_{Is}}}{p_{Is}} = \frac{U^j_{c^j_{Js}}}{p_{Js}}, \tag{78}$$

$$\frac{\lambda^j}{p_s} > \frac{U^j_{c^j_{Js}}}{p_{Js}}, \tag{79}$$

for state s':

$$\frac{\lambda}{p_{s'}} < \frac{U^i_{c^i_{Is'}}}{p_{Is'}}, \tag{80}$$

$$\pi_{s'} \frac{U^i_{c^i_{I0}}}{p_{I0}} = \gamma_{s'} \frac{\lambda_{s'}}{p_{s'}}, \tag{81}$$

$$\pi_{s'} \frac{U^i_{c^i_{J0}}}{p_{J0}} = \gamma_{s'} K_{s'} \frac{U^i_{c^i_{Js'}}}{p_{Js'}}, \tag{82}$$

$$\frac{U^i_{c^i_{Is'}}}{p_{Is'}} = \frac{U^i_{c^i_{Js'}}}{p_{Js'}}, \tag{83}$$

$$\frac{\lambda^i}{p_{s'}} > \frac{U^i_{c^i_{Is'}}}{p_{Is'}}, \tag{84}$$

$$\pi_{s'} \frac{U^j_{c^j_{J0}}}{p_{J0}} = \gamma_{s'} \frac{U^j_{c^j_{Is'}}}{p_{Is'}}, \tag{85}$$

$$\frac{U^j_{c^j_{Is'}}}{p_{Is'}} = \frac{U^j_{c^j_{Js'}}}{p_{Js'}}, \tag{86}$$

$$\frac{\lambda^j}{p_{s'}} > \frac{U^j_{c^j_{Js'}}}{p_{Js'}}, \tag{87}$$

$$\frac{\lambda}{p_{s'}} > \frac{U^j_{c^j_{Js'}}}{p_{Js'}}. \tag{88}$$

D.3 Proof of Lemma 6

Proof: Given $\lambda > \lambda^h$, $h \in \{i, j\}$. at state s, $FA^I_s = -\phi^i_s$, then $\delta^i_s = \phi^i_s - p_{Is}q^i_{Is} + p_{Js}c^i_{Js}$, and $\delta^j_s = -\theta^j_s - p_{Js}q^j_{Js} + p_{Is}c^j_{Is}$. Because $\phi^i_s = \theta^j_s$, $p_{Is}q^i_{Is} = p_{Is}c^j_{Is}$, and $p_{Js}c^i_{Js} = p_{Js}q^j_{Js}$, it follows that $\delta^i_s + \delta^j_s = 0$. Moreover, invoking Lemma 1, $v^i_s\mu^i_I = \frac{\mu^j_J}{1+r_J} + m^j + p_{Is}q^i_{Is} - p_{Js}c^i_{Js} - \phi^i_s + \delta^i_s = \frac{\mu^j_J}{1+r_J} + m^j$, and $v^j_s\mu^j_J = \frac{\mu^i_I}{1+r_I} + m^i$.

Similarly, for state s', $\delta^i_{s'} + \delta^j_{s'} = 0$, $v^i_{s'}\mu^i_I = \frac{\mu^j_J}{1+r_J} + m^j$, $v^j_{s'}\mu^j_J = \frac{\mu^i_I}{1+r_I} + m^i$.

Therefore, $v^i_s = v^i_{s'} = v^i$, and $v^j_s = v^j_{s'} = v^j$, i.e. $var(1 - v^{h,B.a}_{\boldsymbol{s}}) = 0$.

□

D.4 Proof of Proposition 4

Proof: Commercial bank h's FOC gives $\omega^h_{\boldsymbol{s}} = (v^h_{\boldsymbol{s}} - \sum^S_{\boldsymbol{s}=1} z_{\boldsymbol{s}}v^h_{\boldsymbol{s}})\mu^h_H$. In Regime A, $\lambda > \lambda^h$ and $\delta^h_{\boldsymbol{s}} = 0$ for $h \in \{i, j\}$, $\forall \boldsymbol{s} \in S$. Given $v^h_{\boldsymbol{s}} < \sum^S_{\boldsymbol{s}=1} z_{\boldsymbol{s}}v^h_{\boldsymbol{s}}$, $\omega^h_{\boldsymbol{s}} < 0$. In Regime B.a, $\lambda > \lambda^h$, and $\delta^h_{\boldsymbol{s}} = -FA^H_{\boldsymbol{s}} - CA^h_{\boldsymbol{s}}$, as shown in Proof of Lemma 6 $v^h_{\boldsymbol{s}} = v^h$. It follows $v^h_{\boldsymbol{s}} - \sum^S_{\boldsymbol{s}=1} z_{\boldsymbol{s}}v^h_{\boldsymbol{s}} = 0$ and $\omega^h_{\boldsymbol{s}} = 0$.

For the case of Regime C, the conditions are more exacting. Given the conditions in Proposition 3, i.e. consider the case where $S = \{1, 2\}$, let $\gamma_1 = \gamma_2$, $e^i_1 > e^i_2$, and $e^j_1 < e^j_2$. Suppose that in equilibrium $\lambda < p_2\eta^i_{12} < \lambda^i$, $\eta^j_{12} < \frac{\lambda}{p_2}$ and $\eta^i_{11} < \eta^i_{12}$ holds. Suppose $\Big(K_2 - \pi_2\big(v^i_2(1 + r_I) - 1\big)\Big)/p_{I2} > \pi_2 r_I/p_{I1}$ holds, then, $\phi^i_2, \phi^j_2, \theta^i_2 > 0$, $\theta^j_2 = 0$, $D^i_2 = 0, D^j_2 = \phi^j_2$, and $0 < K_2 < 1$. Assume similar conditions for the other state. Thus, households purchase and sell the Arrow security of their respective bad state in order to default fully on the Arrow security, without defaulting on domestic loans. It follows that $v^h_{\boldsymbol{s}} = 1$ and $\omega^h_{\boldsymbol{s}} = 0$, where $\boldsymbol{s} \in \{1, 2\}$.

□

D.5 Proof of Corollary 4.2

Proof: Suppose the conditions of Proposition 4 are met. For the allocation at $t = 0$, combine households i's FOCs (43), (44), (49), (58), we have

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{p_{I0}}{p_{J0}}\frac{1}{1 + r_I}. \tag{89}$$

Likewise, combining household j's FOCs (45), (46), (54), (62) leads to

$$\frac{U^j_{c^j_{J0}}}{U^j_{c^i_{I0}}} = \frac{p_{J0}}{p_{I0}}\frac{1}{1+r_J}. \tag{90}$$

Combine (89) and (90), we have

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}}\frac{1}{(1+r_I)(1+r_J)}. \tag{91}$$

For the allocation at $t = 1$ at state $\boldsymbol{s}$, $\boldsymbol{s} \in S$, household i's FOCs for $c^i_{I\boldsymbol{s}}$ and $c^i_{J\boldsymbol{s}}$ give

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{p_{I\boldsymbol{s}}} = \frac{U^i_{c^i_{J\boldsymbol{s}}}}{p_{J\boldsymbol{s}}(1+\tau_{I\boldsymbol{s}})}. \tag{92}$$

Household j's FOCs for $c^j_{J\boldsymbol{s}}$ and $c^j_{I\boldsymbol{s}}$ give

$$\frac{U^j_{c^j_{J\boldsymbol{s}}}}{p_{J\boldsymbol{s}}} = \frac{U^j_{c^j_{I\boldsymbol{s}}}}{p_{I\boldsymbol{s}}(1+\tau_{J\boldsymbol{s}})}. \tag{93}$$

Combine (92) and (93), we have

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{U^i_{c^i_{J\boldsymbol{s}}}} = \frac{U^j_{c^j_{I\boldsymbol{s}}}}{U^j_{c^j_{J\boldsymbol{s}}}}\frac{1}{(1+\tau_{I\boldsymbol{s}})(1+\tau_{J\boldsymbol{s}})}. \tag{94}$$

In Particular, for Regime B and Regime C, $\tau_{H\boldsymbol{s}} = 0$, and therefore,

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{U^i_{c^i_{J\boldsymbol{s}}}} = \frac{U^j_{c^j_{I\boldsymbol{s}}}}{U^j_{c^j_{J\boldsymbol{s}}}}, \tag{95}$$

□

D.6 Proof of Corollary 4.3

Proof: Under the conditions of Proposition 4. For Regime A, combine (49) and (50), we obtain

$$\frac{U^i_{c^i_{Is}}}{p_{Is}} = \frac{\pi_s U^i_{c^i_{I0}}}{\gamma_s p_{I0}(1+\tau_{Is})}. \tag{96}$$

Combine (57) and (58) we obtain

$$\frac{U^i_{c^i_{Is'}}}{p_{Is'}} = \frac{\pi_{s'} U^i_{c^i_{J0}}}{p_{J0}\gamma_{s'}}. \tag{97}$$

Combine (96), (97) and (89), we have

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{\pi_s \gamma_{s'} p_{Is}}{\pi_{s'} \gamma_s p_{Is'}(1+\tau_{Is})(1+r_I)}. \tag{98}$$

Combine (53), (61) and (90), we have

$$\frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} = \frac{\pi_s(1+\tau_{Js})\gamma_{s'} p_{Is}(1+r_J)}{\pi_{s'} \gamma_s p_{Is'}}. \tag{99}$$

Combine (98) and (99), we have

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{1}{(1+\tau_{Is})(1+\tau_{Js})(1+r_I)(1+r_J)}. \tag{100}$$

The same rearrangement of FOCs in Regime B gives us exactly the same risk sharing conditions as in (102) except that $\tau_{H\boldsymbol{s}} = 0$ in Regime B, i.e.,

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{1}{(1+r_I)(1+r_J)}. \tag{101}$$

For Regime C, combine (71) and (74):

$$\frac{U^i_{c^i_{Is}}}{p_{Is}} = \frac{\pi_s U^i_{c^i_{I0}}}{\gamma_s p_{I0}}. \tag{102}$$

Rearrange (75):

$$\frac{U^j_{c^j_{Is}}}{p_{Is}} = \frac{\pi_s U^j_{c^j_{I0}}}{\gamma_s K_s p_{I0}}. \tag{103}$$

Combine (82) and (83):

$$\frac{U^i_{c^i_{Is'}}}{p_{Is'}} = \frac{\pi_{s'} U^i_{c^i_{J0}}}{\gamma_{s'} K_{s'} p_{J0}}. \tag{104}$$

Rearrange (85):

$$\frac{U^j_{c^j_{Is'}}}{p_{Is'}} = \frac{\pi_{s'} U^j_{c^j_{J0}}}{\gamma_{s'} p_{J0}}. \tag{105}$$

Note that as Proposition 4 shows that $v^h_{\boldsymbol{s}} = 1$ in Regime C; thus, $r^h_{\boldsymbol{s}} = \rho$. Combining (102), (103), (104) and (105), we have

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{K_s K_{s'}}{(1+\rho)^2}.$$

Similar expressions obtain for risk sharing with respect to consumption good J. □

D.7 Proof of Corollary 4.4

Proof: Under the conditions of Proposition 4. in Regime A, for state s household i' sells the Arrow security of that state. Rearrange (50), i.e. household i's FOC for ϕ^i_s, we obtain

$$\pi_s = \gamma_s \frac{U^i_{c^i_{Js}}/p_{Js}}{U^i_{c^i_{I0}}/p_{I0}}. \tag{106}$$

Household j' buys the Arrow security of that state. Combine (53) and (90), we obtain

$$\pi_s = \gamma_s \frac{U^j_{c^j_{Is}}/p_{Is}}{U^j_{c^j_{J0}}/p_{J0}(1+\tau_{Js})(1+r_J)}. \tag{107}$$

For state s', household i's buys the Arrow security of that state. Combine (57) and (89) we obtain

$$\pi_{s'} = \gamma_{s'} \frac{U^i_{c^i_{Js'}}/p_{Js'}}{U^i_{c^i_{I0}}/p_{I0}(1+\tau_{Is'})(1+r_I)}. \tag{108}$$

Household j sells the Arrow security of that state. Rearrange (61), i.e. household j's FOC for $\phi^j_{s'}$, we obtain

$$\pi_{s'} = \gamma_{s'} \frac{U^j_{c^j_{Is'}}/p_{Is'}}{U^j_{c^j_{J0}}/p_{J0}}. \tag{109}$$

It follows that

$$\pi_s = \gamma_s \frac{U^i_{c^i_{Js}}/p_{Js}}{U^i_{c^i_{I0}}/p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}/p_{Is}}{U^j_{c^j_{J0}}/p_{J0}(1+\tau_{Js})(1+r_J)},$$

$$\pi_{s'} = \gamma_{s'} \frac{U^i_{c^i_{Js'}}/p_{Js'}}{U^i_{c^i_{I0}}/p_{I0}(1+\tau_{Is'})(1+r_I)} = \gamma_{s'} \frac{U^j_{c^j_{Is'}}/p_{Is'}}{U^j_{c^j_{J0}}/p_{J0}}.$$

For Regime B, similar rearrangements of FOCs give

$$\pi_s = \gamma_s \frac{U^i_{c^i_{Js}}/p_{Js}}{U^i_{c^i_{I0}}/p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}/p_{Is}}{U^j_{c^j_{J0}}/p_{J0}(1+r_J)},$$

$$\pi_{s'} = \gamma_{s'} \frac{U^i_{c^i_{Js'}}/p_{Js'}}{U^i_{c^i_{I0}}/p_{I0}(1+r_I)} = \gamma_{s'} \frac{U^j_{c^j_{Is'}}/p_{Is'}}{U^j_{c^j_{J0}}/p_{J0}}.$$

For Regime C, for state s let us rearrange (71):

$$\pi_s \frac{U^i_{c^i_{I0}}}{p_{I0}} = \gamma_s \frac{U^i_{c^i_{Js}}}{p_{Js}}. \tag{110}$$

Combine (68), (75), (78), and (86), we obtain

$$\pi_s \frac{U^j_{c^j_{I0}}}{p_{I0}} = \gamma_s K_s \frac{U^j_{c^j_{Is}}}{p_{Is}}. \tag{111}$$

Moreover, as shown in Proposition 4 and Corollary 4.3, $r_h = \rho$ in Regime C. It follows that

$$\pi_s = \gamma_s \frac{U^i_{c^i_{Js}}/p_{Js}}{U^i_{c^i_{I0}}/p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}/p_{Is}K_s}{U^j_{c^j_{J0}}/p_{J0}(1+\rho)}.$$

Similarly for state s' we obtain

$$\pi_{s'} = \gamma_{s'} \frac{U^i_{c^i_{Js'}}/p_{Js'}K_{s'}}{U^i_{c^i_{I0}}/p_{I0}(1+\rho)} = \gamma_{s'} \frac{U^j_{c^j_{Is'}}/p_{Is'}}{U^j_{c^j_{J0}}/p_{J0}}.$$

□

D.8 Proof of Proposition 5

Proof. First I show decreasing λ to λ' leads to utility increase for both household i and household j on the margin.

In the Regime A equilibrium, $p_2\eta^i_{12} = \lambda^i < \lambda$, it follows that

$$\frac{\lambda}{p_2} > \eta^i_{12}. \tag{112}$$

The marginal cost of default on financial securities is larger than the marginal benefit, so household i fully delivers on assets. Given $\eta^i_{11} < \eta^i_{12}$, $\phi^i_2 = 0$ and $\theta^i_2 > 0$.

Now let us decrease λ to λ', and $\lambda' < p_2\eta^i_{12} = \lambda^i$. It follows that

$$\frac{\lambda'}{p_2} < \eta^i_{12}. \tag{113}$$

Equation (113) states the marginal cost of default on financial securities is larger than the marginal benefit, so household i would fully default if holding short positions.

Suppose now household i sells an infinitesimal ϵ of Arrow security $l = 2$, i.e. $\phi^i_2 = \epsilon$. According to the market clearing condition of asset markets, household i would need to buy ϵ amount of Arrow security $l = 2$. This means household i needs to borrow $\pi_2\epsilon$ more money at $t = 0$ while needing to pay back $\pi_s\epsilon v^i_2(1 + r_I)$ at $t = 2$, so the extra monetary cost is $\pi_2\epsilon(v^i_2(1 + r_I) - 1)$. However, because household i fully defaults on the ϵ amount of Arrow security sold, so the extra money inflow due to default amounts to $K_2\epsilon$, where $K_2 = 1 - \frac{\epsilon}{\sum_{h\in\{i,j\}}\phi^h_s}$. Thus the total money inflow at state 2 is $K_s\epsilon - \pi_2\epsilon(v^i_2(1 + r_I) - 1)$. For the good state 1, the monetary cost is $\pi_s\epsilon r_I$. Since $\Big(K_2 - \pi_2\big(v^i_2(1 + r_I) - 1\big)\Big)/p_{I2} > \pi_2 r_I/p_{I1}$ holds, then on the margin $\nabla U^i(\cdot)(K_2\epsilon - \pi_2\epsilon\big(v^i_2(1+r_I) - 1\big)) > \nabla U^i(\cdot)(\pi_2\epsilon r_I)$ holds, leading to an overall increase in household i's expected utility.

Similarly, setting $\lambda = \lambda^C$ leads to an overall increase in household j's expected utility on the margin.

Second I show that the expected utilities of commercial banks are zero and remain unchanged. From Proposition 4, $\omega^h_{\boldsymbol{s}} = (v^h_{\boldsymbol{s}} - \sum^S_{\boldsymbol{s}=1} z_{\boldsymbol{s}} v^h_{\boldsymbol{s}})\mu^h_H$, $h \in \{i, j\}, H \in \{I, J\}$. It follows that $U^h(\cdot) = \sum^S_{\boldsymbol{s}=1} z_{\boldsymbol{s}}\omega^h_{\boldsymbol{s}} = \sum^S_{\boldsymbol{s}=1}(z_{\boldsymbol{s}}(v^h_{\boldsymbol{s}} - \sum^S_{\boldsymbol{s}=1} z_{\boldsymbol{s}} v^h_{\boldsymbol{s}})\mu^h_H) = (\sum^S_{\boldsymbol{s}=1} z_{\boldsymbol{s}} v^h_{\boldsymbol{s}} - \sum^S_{\boldsymbol{s}=1} z_{\boldsymbol{s}} \sum^S_{\boldsymbol{s}=1} z_{\boldsymbol{s}} v^h_{\boldsymbol{s}})\mu^h_H = 0$. □

D.9 Proof of Lemma 7

Proof. See the the proofs in C.1, C.2, and C.3. □

D.10 Proof of Lemma 8

Proof. Set $\tau^h_{\boldsymbol{s}} = 0\ \forall \boldsymbol{s} \in S$, $h \in \{i, j\}$.

- Purchasing Power Parity

  At $t = 0$, suppose household i purchases consumption good I and good J. FOC for c^i_{I0} gives

$$U^i_{c^i_{I0}} = p_{I0}\nu^i_1. \tag{114}$$

  FOC for c^i_{J0} gives

$$U^i_{c^i_{J0}} = p_{J0}\nu^i_2. \tag{115}$$

  FOC for f^i_{IJ} gives

$$\nu_1^i = \frac{1}{\chi}\nu_2^i. \tag{116}$$

Combine (114), (115), and (116):

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{p_{I0}}{\chi p_{J0}}. \tag{117}$$

If household i only purchases good J and sells consumption good I, then FOC for c^i_{I0} becomes

$$U^i_{c^i_{I0}} = p_{I0} E_0 \nu^i_{3\boldsymbol{s}}. \tag{118}$$

FOC for μ^i_I gives

$$\nu_1^i = (1 + r_I) E_0 \nu^i_{3\boldsymbol{s}}. \tag{119}$$

Combine (115), (116), (118), and (119) :

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{p_{I0}}{\chi p_{J0}(1 + r_I)}. \tag{120}$$

Similarly, if household j buys both good I and good J, then $\frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}} = \frac{p_{I0}}{\chi p_{J0}}$, and if household only buys good I and sells good J, then $\frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}} = \frac{(1+r_J)p_{I0}}{\chi p_{J0}}$.

At $t = 1$, household h's FOCs for $c^h_{I\boldsymbol{s}}$, $c^h_{J\boldsymbol{s}}$, and the FOC for FX lead to

$$\frac{U^h_{c^h_{I\boldsymbol{s}}}}{U^h_{c^h_{J\boldsymbol{s}}}} = \frac{p_{I\boldsymbol{s}}}{\chi_{\boldsymbol{s}} p_{J\boldsymbol{s}}}. \tag{121}$$

- Uncovered Interest Rate Parity

  Suppose household i can obtain currency J via FX market and foreign bank j. If household i obtains currency J via FX market, then household i's FOC for $\theta^i_{s'}$ gives

$$\pi_{Js'} \nu_2^i = \gamma_{s'} \nu^i_{4s'}. \tag{122}$$

  Household i's FOC for f^i_{IJ} gives

$$\nu_1^i = \frac{1}{\chi}\nu_2^i. \tag{123}$$

Household i's FOCs for μ_I^i and $\Delta_{\boldsymbol{s}}^i$(13) give

$$\nu_1^i = (1 + r_I) E_0 \nu_{3\boldsymbol{s}}^i. \tag{124}$$

Household i's FOC for $f_{IJ\boldsymbol{s}}^i$ gives

$$\nu_{3\boldsymbol{s}}^i \chi_{\boldsymbol{s}} = \nu_{4\boldsymbol{s}}^i. \tag{125}$$

Combine (122), (123), (124), and (125):

$$\pi_{Js'} \chi (1 + r_I) E_0 \nu_{3\boldsymbol{s}}^i = \gamma_{s'} \nu_{4s'}^i. \tag{126}$$

If household i obtains currency J by borrowing μ_J^i from foreign bank j. Let $\nu_0^{i'}$ be the shadow price of the liquidity-in-advance constraint, then household i's FOC for $\theta_{s'}^i$ gives

$$\pi'_{Js'} \nu_0^{i'} = \gamma_{s'} \nu_{4s'}^i. \tag{127}$$

Household i's FOC for μ_J^i gives

$$\nu_0^{i'} = (1 + r_J) E_0 \nu_{4\boldsymbol{s}}^i. \tag{128}$$

Combine (125), (127) and (128):

$$\pi'_{Js'} (1 + r_J) E_0 \chi_{\boldsymbol{s}} \nu_{3\boldsymbol{s}}^i = \gamma_{s'} \nu_{4s'}^i. \tag{129}$$

To exclude arbitrage, it must be $\pi_{Js'} = \pi'_{Js'}$, and combine (126) and (129):

$$\frac{1 + r_I}{1 + r_J} = \frac{E_0 \nu_{3\boldsymbol{s}}^i \chi_{\boldsymbol{s}}}{E_0 \nu_{3\boldsymbol{s}}^i \chi}. \tag{130}$$

Now suppose household j can obtain currency I via FX market and foreign bank i. If household j obtains currency I via FX market, then household j's FOC for θ_s^i gives

$$\pi_{Is} \nu_2^j = \gamma_s \nu_{4s}^j. \tag{131}$$

Household j's FOC for f_{JI}^j gives

$$\nu_1^j = \chi \nu_2^j. \tag{132}$$

Household j's FOC for μ_J^j gives

$$\nu_1^j = (1 + r_J) E_0 \nu_{3\boldsymbol{s}}^j. \tag{133}$$

Household j's FOC for $f_{JI\boldsymbol{s}}^j$ gives

$$\nu_{3s}^j = \chi_s \nu_{4s}^j. \tag{134}$$

Combine (131), (132), (133) and (134):

$$\pi_{Is} \frac{1}{\chi}(1 + r_J) E_0 \nu_{3s}^j = \gamma_s \nu_{4s}^j. \tag{135}$$

If household j obtains currency I by borrowing μ_I^j from bank i. Let $\nu_0^{j'}$ be the shadow price of the liquidity-in-advance constraint, then household j's FOC for θ_s^j gives

$$\pi'_{Is} \nu_0^{j'} = \gamma_s \nu_{4s}^j. \tag{136}$$

Household j's FOC for μ_I^j gives

$$\nu_0^{j'} = (1 + r_I) E_0 \nu_{4s}^j. \tag{137}$$

Combine (136), (137) and (134):

$$\pi'_{Is}(1 + r_J) E_0 \frac{\nu_{3s}^j}{\chi_s} = \gamma_s \nu_{4s}^j. \tag{138}$$

To rule out arbitrage, it must be $\pi_{Is} = \pi'_{Is}$, so it follows that

$$\frac{1 + r_I}{1 + r_J} = \frac{E_0 \nu_{3s}^j / \chi}{E_0 \nu_{3s}^j / \chi_s}. \tag{139}$$

□

D.11 Proof of Proposition 6

Proof. Invoking Lemma 4, substitute (13) into (15), given $\tau_{Hs} = 0$, we have

$$v_s^i \mu_I^i = \sum_{l=1}^{S} \pi_{Il} \phi_{Il}^i + p_{I0} q_{I0}^i + p_{Is} q_{Is}^i - f_{IJs}^i - \phi_{Is}^i. \tag{140}$$

Since $\sum_{l=1}^{S} \pi_{Il} \phi_{Il}^i + p_{I0} q_{I0}^i = \sum_{l=1}^{S} \pi_{Il} \theta_{Il}^j + b_{I0}^j = \chi f_{JI}^j - \Delta_2^j = f_{IJ}^i - \Delta_2^j$. Substitute it in (140), we have

$$v_s^i \mu_I^i = f_{IJ}^i - \Delta_2^j + p_{Is} q_{Is}^i - f_{IJs}^i - \phi_{Is}^i. \tag{141}$$

With market clearing conditions $\theta_{Is}^j = \phi_{Is}^i$, $f_{IJs}^i = f_{JIs}^j \chi_s$ and $p_{Is} q_{Is}^i = b_{Is}^j$, and also $\theta_{Is}^j + f_{JIs}^j \chi_s + \Delta_2^j = b_{Is}^j$, (141) becomes

$$v_s^i \mu_I^i = f_{IJ}^i. \tag{142}$$

Given Δ(11), (142) becomes

$$v_{\boldsymbol{s}}^i \mu_I^i = \frac{\mu_I^i}{1+r_I} + m^i. \tag{143}$$

Similarly for country J

$$v_{\boldsymbol{s}}^j \mu_J^j = \frac{\mu_J^j}{1+r_J} + m^j. \tag{144}$$

Thus, $v_{\boldsymbol{s}}^i = v^i$ and $v_{\boldsymbol{s}}^j = v^j$. □

D.12 Proof of Corollary 6.1

Proof. As shown in the proof of Corollary 1.1, bank h's FOC gives $\omega_{\boldsymbol{s}}^h = (\frac{v_{\boldsymbol{s}}^h}{\sum_{\boldsymbol{s}=1}^S \eta_{\boldsymbol{s}} v_{\boldsymbol{s}}^h} - 1)\mu_{CBH}^h$, $\boldsymbol{s} \in S, h \in \{i,j\}, H \in \{I,J\}$. As shown in the proof of Proposition 6, $v_{\boldsymbol{s}}^h = v^h$, thus $\frac{v_{\boldsymbol{s}}^h}{\sum_{\boldsymbol{s}=1}^S \eta_{\boldsymbol{s}} v_{\boldsymbol{s}}^h} - 1 = 0$, $\omega_{\boldsymbol{s}}^h = 0$, and there is no need for national bailout tax. □

D.13 Proof of Corollary 6.2

Proof. Under the conditions of Proposition 5, $\omega_{\boldsymbol{s}}^h = 0$, $\boldsymbol{s} \in S, h \in \{i,j\}$. □

D.14 Proof of Lemma 9

Proof. Let $\Delta_{02}^i = \Delta$(12), and denote Δ_{02}^j for the equivalent of household j. Suppose $\Delta_{02}^i \neq 0$ and $\Delta_{02}^j \neq 0$. Household i's FOC for Δ_{02}^i and household j's FOC for Δ_{02}^j lead to

$$\nu_2^i = E_0 \nu_{4\boldsymbol{s}}^i, \tag{145}$$

$$\nu_2^j = E_0 \nu_{4\boldsymbol{s}}^j. \tag{146}$$

Household i and household j's FOCs for FX give $\nu_1^i = \nu_2^i/\chi$ and $\nu_1^j = \chi\nu_2^j$. Combine these two equations with (145) and (146), we have

$$\chi \nu_1^i = E_0 \nu_{4\boldsymbol{s}}^i, \tag{147}$$

$$1/\chi \nu_2^j = E_0 \nu_{4\boldsymbol{s}}^j. \tag{148}$$

Moreover, household i's FOC for μ_I^i gives $\nu_1^i = (1+r_I)E_0\nu_{5\boldsymbol{s}}^i$. Substitute it in (147), we have $\chi(1+r_I)E_0\nu_{5\boldsymbol{s}}^i = E_0\nu_{4\boldsymbol{s}}^i$. Also, FOC for $f_{IJ\boldsymbol{s}}^i$ gives $\nu_{3\boldsymbol{s}}^i = 1/\chi_{\boldsymbol{s}}\nu_{4\boldsymbol{s}}^i$, and $\nu_{3\boldsymbol{s}}^i = \nu_{5\boldsymbol{s}}^i$, it follows that

$$E_0 \frac{(1+r_I)\chi}{\chi_{\boldsymbol{s}}} \nu_{4\boldsymbol{s}}^i = E_0 \nu_{4\boldsymbol{s}}^i. \tag{149}$$

Similarly for household j

$$E_0 \frac{(1+r_J)\chi_s}{\chi} \nu^j_{4s} = E_0 \nu^j_{4s}, \tag{150}$$

which are equivalent to

$$cov(\frac{(1+r_I)\chi}{\chi_s}, \nu^i_{4s}) + E_0 \frac{(1+r_I)\chi}{\chi_s} E_0 \nu^i_{4s} = E_0 \nu^i_{4s}, \tag{151}$$

$$cov(\frac{(1+r_J)\chi_s}{\chi}, \nu^i_{4s}) + E_0 \frac{(1+r_J)\chi_s}{\chi} E_0 \nu^j_{4s} = E_0 \nu^j_{4s}. \tag{152}$$

Given $cov(\frac{1}{\chi_s}, \nu^i_{4s}) > 0$ and $cov(\chi_s, \nu^j_{4s}) > 0$, from (151) and (152) it follows that

$$E_0 \frac{(1+r_I)\chi}{\chi_s} < 1, \tag{153}$$

and

$$E_0 \frac{(1+r_J)\chi_s}{\chi} < 1. \tag{154}$$

Given $r_I = r_J$, (153) and (154) contradict each other. □

D.15 Proof of Corollary 6.3

Proof. When FX-in-advance binds, the current account net inflow of country I is

$$CA^I_s = p_{Is} q^i_{Is} - \chi_s b^i_{Js} = p_{Is} q^i_{Is} - \chi_s \theta^i_{Js} - f^i_{IJs}. \tag{155}$$

The capital account net inflow of country I is

$$FA^I_s = -\phi^i_{Is} + \chi_s \theta^i_{Js}. \tag{156}$$

The net position of the BoP is

$$CA^I_s + FA^I_s = p_{Is} q^i_{Is} - f^i_{IJs} - \phi^i_{Is}. \tag{157}$$

From (13) we know

$$p_{Is} q^i_{Is} = \Delta(13) + f^i_{IJs} + \phi^i_{Is} - (\sum_{l=1}^{S} \pi_{Il} \phi^i_{Il} + p_{I0} q^i_{I0}). \tag{158}$$

Combine (157) and (158) we have

$$CA^I_{\boldsymbol{s}} + FA^I_{\boldsymbol{s}} = \Delta(13) - (\sum_{l=1}^{S} \pi_{Il}\phi^i_{Il} + p_{I0}q^i_{I0}). \tag{159}$$

As shown in Lemma 6, $\Delta(13) = \frac{\mu^i_I}{1+r_I} + m^i$. Also $\sum_{l=1}^{S} \pi_{Il}\phi^i_{Il} + p_{I0}q^i_{I0} = f^j_{JI}\chi = f^i_{IJ} = \frac{\mu^i_I}{1+r_I} + m^i$. Therefore, $CA^I_{\boldsymbol{s}} + FA^I_{\boldsymbol{s}} = 0$. Likewise for country J $CA^J_{\boldsymbol{s}} + FA^J_{\boldsymbol{s}} = 0$. □

D.16 First-order conditions of Regime D

The following FOCs are derived under Lemma 7.

$$(1+r_I)E_0\frac{U^i_{c^i_{Is}}}{p_{Is}} = \frac{1}{\chi}\frac{U^i_{c^i_{J0}}}{p_{J0}}, \tag{160}$$

$$\frac{U^i_{c^i_{I0}}}{p_{I0}} = E_0\frac{U^i_{c^i_{Is}}}{p_{Is}}, \tag{161}$$

$$(1+r^J)E_0\frac{U^j_{c^j_{Js}}}{p_{Js}} = \chi\frac{U^j_{c^j_{I0}}}{p_{I0}}, \tag{162}$$

$$\frac{U^j_{c^j_{J0}}}{p_{J0}} = E_0\frac{U^j_{c^j_{Js}}}{p_{J\boldsymbol{s}}}, \tag{163}$$

$$1 + r_I = \frac{1+\rho_I}{\sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}}v^i_{\boldsymbol{s}}}, \tag{164}$$

$$1 + r_J = \frac{1+\rho_J}{\sum_{\boldsymbol{s}=1}^{S} z_{\boldsymbol{s}}v^j_{\boldsymbol{s}}}, \tag{165}$$

For state s:

$$\frac{U^i_{c^i_{Is}}}{p_{Is}} = \frac{1}{\chi_s}\frac{U^i_{c^i_{Js}}}{p_{Js}}, \tag{166}$$

$$\pi_{Is}\frac{U^i_{c^i_{I0}}}{p_{I0}} = \gamma_s\frac{U^i_{c^i_{Is}}}{p_{Is}}, \tag{167}$$

$$\frac{\lambda^i}{p^I_s} \geq \frac{U^i_{c^i_{Is}}}{p_{Is}}, \tag{168}$$

$$\frac{\lambda}{p^I_s} > \frac{U^i_{c^i_{Is}}}{p_{Is}}, \tag{169}$$

$$\frac{U^j_{c^j_{Js}}}{p_{Js}} = \chi_s \frac{U^j_{c^j_{Is}}}{p_{Is}}, \tag{170}$$

$$\pi_{Is} \frac{U^j_{c^j_{I0}}}{p_{I0}} = \gamma_s \frac{U^j_{c^j_{Is}}}{p_{Is}}, \tag{171}$$

$$\frac{\lambda^j}{p^J_s} \geq \frac{U^j_{c^j_{Js}}}{p_{Js}}, \tag{172}$$

$$\frac{\lambda}{p^J_s} > \frac{U^j_{c^j_{Js}}}{p_{Js}}, \tag{173}$$

For state s':

$$\frac{U^i_{c^i_{Is'}}}{p_{Is'}} = \frac{1}{\chi_{s'}} \frac{U^i_{c^i_{Js'}}}{p_{Js'}}, \tag{174}$$

$$\frac{U^i_{c^i_{J0}}}{p_{J0}} \pi_{Js'} = \gamma_{s'} \frac{U^i_{c^i_{Js'}}}{p_{Js'}}, \tag{175}$$

$$\frac{\lambda^i}{p^I_{s'}} \geq \frac{U^i_{c^i_{Is'}}}{p_{Is'}}, \tag{176}$$

$$\frac{\lambda}{p^I_{s'}} > \frac{U^i_{c^i_{Is'}}}{p_{Is'}}, \tag{177}$$

$$\frac{U^j_{c^j_{Js'}}}{p_{Js'}} = \chi_{s'} \frac{U^j_{c^j_{Is'}}}{p_{Is'}}, \tag{178}$$

$$\pi_{Js'} \frac{U^j_{c^j_{J0}}}{p_{J0}} = \gamma_{s'} \frac{U^j_{c^j_{Js'}}}{p_{Js'}}, \tag{179}$$

$$\frac{\lambda^j}{p^J_{s'}} \geq \frac{U^j_{c^j_{Js'}}}{p_{Js'}}, \tag{180}$$

$$\frac{\lambda}{p^J_{s'}} > \frac{U^j_{c^j_{Js'}}}{p_{Js'}}. \tag{181}$$

□

D.17 Proof of Corollary 6.4

Proof. To obtain the MRS between goods at $t = 0$, let us combine (160) and (161):

$$(1 + r_I)\frac{U^i_{c^i_{I0}}}{p_{I0}} = \frac{1}{\chi}\frac{U^i_{c^i_{J0}}}{p_{J0}}. \tag{182}$$

Combine (162) and (163):

$$(1 + r_J)\frac{U^j_{c^j_{J0}}}{p_{J0}} = \chi\frac{U^j_{c^j_{I0}}}{p_{I0}}. \tag{183}$$

Combine (182) and (183):

$$\frac{U^i_{c^i_{I0}}}{U^i_{c^i_{J0}}} = \frac{U^j_{c^j_{I0}}}{U^j_{c^j_{J0}}}\frac{1}{(1 + r_I)(1 + r_J)}. \tag{184}$$

To obtain the MRS between goods at $t = 1$, combine (166) and (174), and combine (170) and (178), we obtain $\forall \boldsymbol{s} \in S$:

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{p_{I\boldsymbol{s}}} = \frac{1}{\chi_{\boldsymbol{s}}}\frac{U^i_{c^i_{J\boldsymbol{s}}}}{p_{J\boldsymbol{s}}}, \tag{185}$$

$$\frac{U^j_{c^j_{J\boldsymbol{s}}}}{p_{J\boldsymbol{s}}} = \chi_{\boldsymbol{s}}\frac{U^j_{c^j_{I\boldsymbol{s}}}}{p_{I\boldsymbol{s}}}. \tag{186}$$

Combine (185) and (186):

$$\frac{U^i_{c^i_{I\boldsymbol{s}}}}{U^i_{c^i_{J\boldsymbol{s}}}} = \frac{U^j_{c^j_{I\boldsymbol{s}}}}{U^j_{c^j_{J\boldsymbol{s}}}}. \tag{187}$$

□

D.18 Proof of Corollary 6.5

Proof. To derive the risk sharing condition between households with respect to consumption good I, let us rearrange (167):

$$U^i_{c^i_{Is}} = \frac{\pi_{Is} p_{Is} U^i_{c^i_{I0}}}{p_{I0}\gamma_s}. \tag{188}$$

Combine (160), (161), (174), and (175):

$$U^i_{c^i_{Is'}} = \frac{\pi_{Js'}\chi(1 + r_I)p_{Is'}U^i_{c^i_{I0}}}{p_{I0}\chi_{s'}\gamma_{s'}}. \tag{189}$$

Combine (188) and (189):

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{\pi_{Is} p_{Is} \chi_{s'} \gamma_{s'}}{\gamma_s \pi_{Js'} \chi (1 + r_I) p_{Is'}}. \tag{190}$$

Rearrange (171):

$$U^j_{c^j_{Is}} = \frac{p_{Is} \pi_{Is} U^j_{c^j_{I0}}}{p_{I0} \gamma_s}. \tag{191}$$

Combine (162), (163), (178), and (179):

$$U^j_{c^j_{Is'}} = \frac{\chi U^j_{c^j_{I0}} \pi_{Js'} p_{Is'}}{p_{I0}(1 + r_J) \gamma_{s'} \chi_{s'}}. \tag{192}$$

Combine (191) and (192):

$$\frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} = \frac{p_{Is} \pi_{Is} (1 + r_J) \gamma_{s'} \chi_{s'}}{\gamma_s \chi \pi_{Js'} p_{Is'}}. \tag{193}$$

Combine (190) and (193)

$$\frac{U^i_{c^i_{Is}}}{U^i_{c^i_{Is'}}} = \frac{U^j_{c^j_{Is}}}{U^j_{c^j_{Is'}}} \frac{1}{(1 + r_J)(1 + r_I)}. \tag{194}$$

Similar rearrangements of algebra lead to a similar expression as follows for the risk sharing condition with respect to consumption good J:

$$\frac{U^i_{c^i_{Js}}}{U^i_{c^i_{Js'}}} = \frac{U^j_{c^j_{Js}}}{U^j_{c^j_{Js'}}} \frac{1}{(1 + r_J)(1 + r_I)}. \tag{195}$$

□

D.19 Proof of Corollary 6.6

Proof. For the state price of state s, combine (167) (170), (171), and (183):

$$\pi_{Is} = \gamma_s \frac{U^i_{c^i_{Is}} / p_{Is}}{U^i_{c^i_{I0}} / p_{I0}} = \gamma_s \frac{U^j_{c^j_{Js}} / p_{Js} \chi}{U^j_{c^j_{J0}} / p_{J0} \chi_s (1 + r_J)}. \tag{196}$$

For the state price of state s', combine (174), (175), (179), and (182):

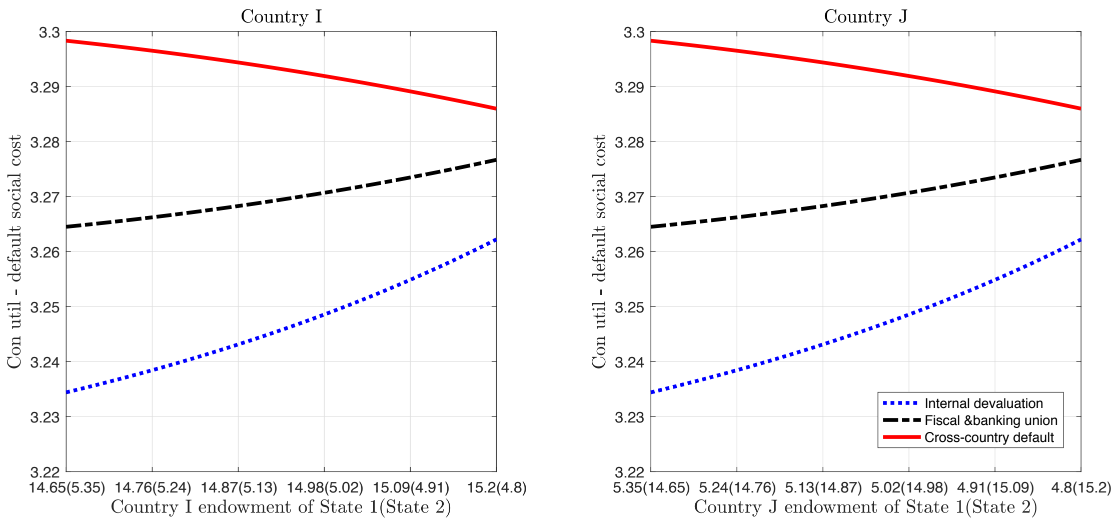
$$\pi_{Js'} = \gamma_{s'} \frac{U^i_{c^i_{Is'}}/p_{Is'}\chi_{s'}}{U^i_{c^i_{I0}}/p_{I0}(1+r_I)\chi} = \gamma_{s'} \frac{U^j_{c^j_{Js'}}/p_{Js'}}{U^j_{c^j_{J0}}/p_{J0}}. \tag{197}$$

□

E Numerical Illustrations

E.1

Figure 13: Social utility (shock variance)



E.2

Figure 14: Social utility (e^i_{I2})