Does Household Finance Matter? Small Financial Errors with Large Social Costs*

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Abstract

Households with familiarity bias tilt their portfolios towards a few risky assets. Consequently, household portfolios are underdiversified and excessively volatile. To understand the implications of underdiversification for social welfare, we solve in closed form a model of a stochastic, dynamic, general-equilibrium economy with a large number of heterogeneous firms and households, who bias their investments toward a few familiar assets. We find that the direct mean-variance loss from holding an underdiversified portfolio that is excessively risky is a modest 1.66% per annum, consistent with the estimates in Calvet, Campbell, and Sodini (2007). However, we show that in a more general model with *intertemporal* consumption, underdiversified portfolios increase consumption-growth volatility, amplifying the mean-variance losses by a factor of four. Moreover, in general equilibrium where growth is endogenous, underdiversified portfolios distort also aggregate investment and growth, so that the overall effect on social welfare is about six times as large as the direct mean-variance loss. We demonstrate that even when forcing the familiarity biases in portfolios to cancel out across households, their implications for consumption and investment choices do not cancel—individual household biases can have significant aggregate effects. Our results illustrate that financial markets are not a mere sideshow to the real economy and that financial literacy, regulation, and innovation that improve the financial decisions of households can have a significant positive impact on social welfare.

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1 Introduction and Motivation

One of the fundamental insights of standard portfolio theory (Markowitz (1952, 1959)) is to hold diversified portfolios. However, evidence from natural experiments (Huberman (2001)) and empirical work (Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2014)) shows households invest in underdiversified portfolios that are biased toward a few familiar assets.[1] Familiarity biases may be a result of geographical proximity, employment relationships, language, social networks, and culture (Grinblatt and Keloharju (2001)). Holding portfolios biased toward a few familiar assets forces households to bear more financial risk than is optimal. Calvet, Campbell, and Sodini (2007) study empirically the importance of household portfolio return volatility for welfare.[2] They find that, within a static mean-variance framework, the welfare costs for individual households arising from underdiversified portfolios are modest. We extend the static framework to a dynamic, general-equilibrium, production-economy setting to examine how underdiversification in household portfolios impacts intertemporal consumption choices of individual households, and upon aggregation, real investment, aggregate growth, and social welfare.

In our paper, we address the following questions. How large are the welfare costs of underdiversification for individual households? These costs are an example of what is often referred to as an "internality" in the public-economics literature.[3] Do the consequences of household-level portfolio errors cancel out, or does aggregation amplify their effects, thereby distorting growth and imposing significant social costs? How large are the negative macroeconomic effects for the aggregate economy because households invest in underdiversified portfolios? Are pathologies such as familiarity biases in financial markets merely a sideshow or do they impact the real economy?[4] In short, does household finance matter?

Our paper makes two contributions. First, we show that even if the welfare loss to a household from investing in an underdiversified portfolio is modest, once we incorporate the effect of an underdiversified portfolio on the household's *intertemporal* consumption choice, the internality to the household is amplified by a factor of four. Second, household-level

[1] For surveys of the portfolio behavior of households, see Guiso, Haliassos, and Jappelli (2002), Haliassos (2002), Vissing-Jorgensen (2003), Campbell (2006), and Guiso and Sodini (2013).

[2] The analogous question at the macroeconomic level has been studied by Lucas (1987, 2003).

[3] Herrnstein, Loewenstein, Prelec, and Vaughan Jr. (1993, p. 150) use internality to refer to a "within-person externality," which occurs when a person ignores a consequence of her own behavior for herself.

[4] For a review of the literature on the interaction between financial markets and the real economy, see Bond, Edmans, and Goldstein (2012).

distortions to individual consumption stemming from excessive financial risk taking are amplified further by aggregation and have a substantial effect on aggregate growth and social welfare. Overall, combining the impact of underdiversification on intertemporal consumption and aggregate growth amplifies social welfare losses by a factor of six. Thus, financial markets are not a sideshow—internalities at the micro-level arising from underdiversified household portfolios can create a macro-level general-equilibrium effect in the form of reduced economic growth. These results suggest that financial literacy, financial regulations, and financial innovations that lead households to make better financial decisions can lead to large benefits, not just for individual households, but also for society.

To analyze the effects of underdiversification in household portfolios on the aggregate economy, we construct a model of a production economy that builds on the framework developed in Cox, Ingersoll, and Ross (1985). As in Cox, Ingersoll, and Ross, there are a finite number of firms whose physical capital is subject to exogenous shocks. But, in contrast with Cox, Ingersoll, and Ross, we have heterogeneous households with Epstein and Zin (1989) and Weil (1990) preferences and familiarity bias. Each household is more familiar with a small subset of firms. Familiarity bias creates a desire to concentrate investments in a few familiar firms rather than holding a portfolio that is well-diversified across all firms. Importantly, we specify the model so that households are symmetric in their familiarity biases. The symmetry assumption ensures that the familiarity biases cancel out—that is, each firm's expected share of aggregate investment is the same as when there are no biases.

We conceptualize the idea of greater familiarity with particular assets, introduced in Huberman (2001), via *ambiguity* in the sense of Knight (1921). The lower the level of ambiguity about an asset, the more "familiar" is the asset. To allow for *differences* in familiarity across assets, we start with the modeling approach in Uppal and Wang (2003) and extend it along three dimensions: one, we distinguish between risk across states of nature and over time by giving households Epstein-Zin-Weil preferences, as opposed to time-separable preferences; two, we consider a production economy instead of an endowment economy; three, we consider a general-equilibrium rather than a partial-equilibrium framework.

Following Kahneman, Wakker, and Sarin (1997), we distinguish between a household's *experienced* utility—the household's actual well-being as a function of its choices—and its *decision* utility—the objective it seeks to maximize when making its portfolio and consumption choices.[5] In our context, the decision utility exhibits familiarity bias, while experienced utility does not. We determine the optimal portfolio decision of each household in the presence of familiarity bias using the household's decision utility. Because of the familiarity-induced tilt, the portfolio return is excessively risky relative to the return of the optimally-diversified portfolio without familiarity bias. This extra financial risk also changes the intertemporal consumption-saving decision of a household. The resulting consumption decisions of a household are much more volatile than in the absence of familiarity bias. Upon aggregation, the excessively volatile consumption of individual households distorts aggregate growth and reduces social welfare.

The welfare of each individual household, and of society as a whole, is measured using experienced utility. The inefficient risk-return tradeoff from the underdiversified portfolio reduces the experienced mean-variance utility of the individual household; calibrating the model to the empirical findings in Calvet, Campbell, and Sodini (2007) suggests that the resulting internality is modest. However, when we allow for intermediate consumption, the underdiversified portfolio increases intertemporal consumption volatility, which magnifies the internality from portfolio underdiversification by a factor of four. Upon aggregation, the excessively volatile consumption of individual households distorts aggregate growth and leads to an even larger loss in social welfare compared to the direct loss to individual households from underdiversified portfolios. The overall effect on social welfare of allowing underdiversification to impact intertemporal consumption and aggregate growth is to multiply the individual welfare losses of around 1.66% per annum by a factor of six.

Our results suggest that financial literacy, financial regulation, and financial innovation designed to mitigate familiarity bias could reduce investment mistakes by households and hence have a substantial impact on social welfare. Our work thereby provides an example of how improving the decisions made by households in financial markets can generate positive benefits for society; Thaler and Sunstein (2003) provide other examples of how public policy can be used to reduce the investment mistakes of households.

We now describe the related literature. There is a great deal of evidence showing that households hold poorly-diversified portfolios. Guiso, Haliassos, and Jappelli (2002), Haliassos (2002), Campbell (2006), and Guiso and Sodini (2013) highlight underdiversification in their surveys of household portfolios. Polkovnichenko (2005), using data from the Survey

[5] See Chetty (2015) for an excellent exposition of behavioral economics in the context of public policy in general, and of the distinction between decision utility and experienced utility in particular.

of Consumer Finances, finds that for households that invest in individual stocks directly, the median number of stocks held was two from 1983 until 2001, when it increased to three, and that poor diversification is often attributable to investments in employer stock, which is a significant part of equity portfolios. Barber and Odean (2000) and Goetzman and Kumar (2008) report similar findings of underdiversification based on data for individual investors at a U.S. brokerage firm. In an influential paper, Calvet, Campbell, and Sodini (2007) examine detailed government records covering the entire Swedish population. They find that of the investors who participate in equity markets, many are poorly diversified and bear significant idiosyncratic risk. Campbell, Ramadorai, and Ranish (2012) report that for their data on Indian households, "the average number of stocks held across all accounts and time periods is almost 7, but the median account holds only 3.4 stocks on average over its life." They also estimate that mutual fund holdings are between 8% and 16% of household direct equity holdings over the sample period.[6]

Typically, the few risky assets that households hold are ones with which they are "familiar." Huberman (2001) introduces the idea that households invest in familiar assets and provides evidence of this in a multitude of contexts; for example, households in the United States prefer to hold the stock of their local telephone company. Grinblatt and Keloharju (2001), based on data on Finnish investors, find that investors are more likely to hold stocks of Finnish firms that are located close to the investor, communicate in the investor's native language, and have a chief executive of the same cultural background. Massa and Simonov (2006) also find that investors tilt their portfolios away from the market portfolio and toward stocks that are geographically and professionally close to the investor. French and Poterba (1990) and Cooper and Kaplanis (1994) document that investors bias their portfolios toward "home equity" rather than diversifying internationally. Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2014) test the relation between familiarity bias and several household portfolio-choice puzzles. Based on a survey of U.S. households, they find that familiarity bias is related to stock-market participation, the fraction of financial assets in stocks, foreign-stock ownership, own-company-stock ownership, and underdiversification. They also show that these results cannot be explained by risk aversion.

[6]Lack of diversification is a phenomenon that is present not just in a few countries, but across the world. Countries for which there is evidence of lack of diversification include: Australia (Worthington (2009)), France (Arrondel and Lefebvre (2001)), Germany (Börsch-Supan and Eymann (2002) and Barasinska, Schäfer, and Stephan (2008)), India (Campbell, Ramadorai, and Ranish (2012)), Italy (Guiso and Jappelli (2002)), Netherlands (Alessie and Van Soest (2002)), and the United Kingdom (Banks and Smith (2002)).

The most striking example of investing in familiar assets is the investment in "own-company stock," that is, stock of the company where the person is employed. Haliassos (2002) reports extensive evidence of limited diversification based on the tendency of households to hold stock in the employer's firm. Mitchell and Utkus (2004) report that five million Americans have over sixty percent of their retirement savings invested in own-company stock and that about eleven million participants in 401(k) plans invest more than twenty percent of their retirement savings in their employer's stock. Benartzi, Thaler, Utkus, and Sunstein (2007) find that only thirty-three percent of the investors who own company stock realize that it is riskier than a diversified fund with many different stocks. Remarkably, a survey of 401(k) participants by the Boston Research Group (2002) found that half of the respondents said that their company stock had the same or less risk than a money market fund, even though there was a high level of awareness amongst the respondents about the experience of Enron's employees, who lost a substantial part of their retirement funds that were invested in Enron stock.[7]

The rest of this paper is organized as follows. We describe the main features of our model in Section 2. The choice problem of a household that exhibits a bias toward familiar assets is solved in Section 3, and the general-equilibrium implications of aggregating these choices across all households are described in Section 4. We evaluate the quantitative implications of the model in Section 5. We conclude in Section 6. Proofs for all results are collected in the appendix.

2 The Model

In this section, we develop a parsimonious model of a stochastic dynamic general equilibrium economy with a finite number of production sectors and household types. Growth occurs endogenously in this model via capital accumulation. When defining the decision utility of households, we show how to extend Epstein and Zin (1989) and Weil (1990) preferences to allow for familiarity biases, where the level of the bias differs from one risky asset to another.

[7] At the end of 2000, 62 percent of Enron employees' 401(k) assets were invested in company stock; between January 2001 and January 2002, the value of Enron stock fell from over \$80 per share to less than \$0.70 per share.

2.1 Firms

There are N firms indexed by $n \in \{1, \ldots, N\}$. The value of the capital stock in each firm at date t is denoted by $K_{n,t}$ and the output flow by

$$Y_{n,t} = \alpha K_{n,t}, \tag{1}$$

for some constant technology level $\alpha > 0$. The level of a firm's capital stock can be increased by investment at the rate $I_{n,t}$. We thus have the following capital accumulation equation for an individual firm:

$$dK_{n,t} = I_{n,t}\, dt + \sigma\, K_{n,t}\, dZ_{n,t},$$

where σ, the volatility of the exogenous shock to a firm's capital stock, is constant over time and across firms. The term $dZ_{n,t}$ is the increment in a standard Brownian motion and is firm-specific; the correlation between $dZ_{n,t}$ and $dZ_{m,t}$ for $n \neq m$ is denoted by ρ, which is also assumed to be constant over time and the same for all pairs $n \neq m$. Firm-specific shocks create ex-post heterogeneity across firms. The $N \times N$ correlation matrix of returns on firms' capital stocks is given by $\Omega = [\Omega_{nm}]$, where the elements of the matrix are

$$\Omega_{nm} = \begin{cases} 1, & n = m, \\ \rho, & n \neq m. \end{cases}$$

Firm-level heterogeneity gives rise to benefits from diversifying investments across firms. We assume that the expected rate of return is the same across the N firms. Thus, diversification benefits manifest themselves solely through a reduction in risk—expected returns do not change with the level of diversification.

A firm's output flow is divided between its investment flow and dividend flow:

$$Y_{n,t} = I_{n,t} + D_{n,t}.$$

We can therefore rewrite the capital accumulation equation as

$$dK_{n,t} = \big(\alpha K_{n,t} - D_{n,t}\big) dt + \sigma\, K_{n,t}\, dZ_{n,t}. \tag{2}$$

2.2 The Investment Opportunities of Households

There are H households indexed by $h \in \{1, \ldots, H\}$. Housholds can invest their wealth in two classes of assets. The first is a risk-free asset, which has an interest rate i that we assume for now is constant over time—and we show below, in Section 4.2, that this is indeed the case in equilibrium. Let $B_{h,t}$ denote the stock of wealth invested by household h in the risk-free asset at date t:

$$\frac{dB_{h,t}}{B_{h,t}} = i\,dt.$$

Additionally, households can invest in N risky firms, or equivalently, the equity of these N firms. We denote by $K_{hn,t}$ the stock of household h's wealth invested in the n'th risky firm. Given that the household's wealth, $W_{h,t}$, is held in either the risk-free asset or invested in the risky firms, we have that:

$$W_{h,t} = B_{h,t} + \sum_{n=1}^{N} K_{hn,t}.$$

The proportion of a household's wealth invested in firm n is denoted by ω_{hn}, and so

$$K_{hn,t} = \omega_{hn} W_{h,t},$$

implying that the wealth invested in the risk-free asset is

$$B_{h,t} = \Big(1 - \sum_{h=1}^{N} \omega_{hn}\Big) W_{h,t}.$$

Household h consumes the dividends distributed by firm n to it:

$$C_{hn,t} = D_{hn,t} = \frac{K_{hn,t}}{K_{n,t}} D_{n,t},$$

where $C_{hn,t}$ is the consumption rate of household h from the dividend flow of firm n. Hence, the dynamic budget constraint for household h is given by

$$\frac{dW_{h,t}}{W_{h,t}} = \Big(1 - \sum_{n=1}^{N} \omega_{hn,t}\Big)\, idt + \sum_{n=1}^{N} \omega_{hn,t}\Big(\alpha dt + \sigma dZ_{n,t}\Big) - \frac{C_{h,t}}{W_{h,t}} dt,$$

where $C_{h,t}$ is the consumption rate of household h and $C_{h,t} = \sum_{n=1}^{N} C_{hn,t}$.

2.3 Preferences and Familiarity Biases of Households

Each household has a decision-utility function, which it maximizes when making its consumption and portfolio choices, and an experienced-utility function which measures its welfare. The decision utility of a household is subject to familiarity biases, whereas its experienced utility is not. The experienced utility of a household is measured using consumption and portfolio choices obtained from optimizing its decision utility. We explain experienced and decision utility below.

2.3.1 Experienced Utilities of Households

A household's experienced utility is modeled by standard Epstein-Zin preferences and it is not subject to familiarity biases. More formally, a household's date-t utility level, $U_{h,t}$, is defined as in Epstein and Zin (1989) by an intertemporal aggregation of date-t consumption flow, $C_{h,t}$, and the date-t certainty-equivalent of date $t + dt$ utility:

$$U_{h,t} = \mathcal{A}(C_{h,t}, \mu_t[U_{h,t+dt}]),$$

where $\mathcal{A}(\cdot, \cdot)$ is the time aggregator, defined by

$$\mathcal{A}(x, y) = \left[(1 - e^{-\delta dt})x^{1-\frac{1}{\psi}} + e^{-\delta dt}y^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{3}$$

in which $\delta > 0$ is the rate of time preference, $\psi > 0$ is the elasticity of intertemporal substitution, and $\mu_t[U_{h,t+dt}]$ is the date-t certainty equivalent of $U_{h,t+dt}$.[8]

The standard definition of a certainty equivalent amount of a risky quantity is the equivalent risk-free amount in static utility terms, and so the certainty equivalent $\mu_t\left[U_{h,t+dt}\right]$ satisfies

$$u_\gamma\left(\mu_t\left[U_{h,t+dt}\right]\right) = E_t[u_\gamma(U_{h,t+dt})], \tag{4}$$

where $u_\gamma(\cdot)$ is the static utility index defined by the power utility function[9]

$$u_\gamma(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}, & \gamma > 0, \gamma \neq 1 \\ \ln x, & \gamma = 1, \end{cases} \tag{5}$$

and the conditional expectation $E_t[\cdot]$ is defined relative to a reference probability measure $\mathbb{P}$, which we discuss below.

[8] The only difference with Epstein and Zin (1989) is that we work in continuous time, whereas they work in discrete time. The continuous-time version of recursive preferences is known as Stochastic Differential Utility (SDU), and is derived formally in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a proof of existence and uniqueness.

[9] In continuous time the more usual representation for utility is given by $J_{h,t}$, where $J_{h,t} = u_\gamma(U_{h,t})$, with the function u_γ defined in (5).

We can exploit our continuous-time formulation to write the certainty equivalent of household utility an instant from now in a more intuitive fashion:

$$\mu_t[U_{h,t+dt}] = E_t[U_{h,t+dt}] - \frac{1}{2}\,\gamma\, U_{h,t}\, E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]. \tag{6}$$

The above expression[10] reveals that the certainty equivalent of utility an instant from now is just the expected value of utility an instant from now adjusted downward for risk. Naturally, the size of the risk adjustment depends on the risk aversion of the household, γ. The risk adjustment depends also on the volatility of the proportional change in household utility, given by $E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]$. Additionally, the risk adjustment is scaled by the current utility of the household, $U_{h,t}$.[11]

2.3.2 Decision Utilities of Households

In contrast to experienced utility, decision utility is subject to familiarity biases, which impact the decisions made by households. The biased decisions made by households enter their experienced utilities, thereby making them worse off.

We now describe the motivation for the way we model the decision utility of a household. Typically, standard models of portfolio choice assume that households know the true expected return α on the value of each capital stock. Such perfect knowledge would make each household fully familiar with every firm and the probability measure $\mathbb{P}$ would then be the true objective probability measure.[12] However, in practice households do not know the true expected returns, so they do not view $\mathbb{P}$ as the true objective probability measure—they treat it merely as a common reference measure. The name "reference measure" is chosen to capture the idea that even though households do not observe true expected returns, they do observe the same data and use it to obtain identical point estimates for expected returns.

We assume households are averse to their lack of knowledge about the true expected return and respond by reducing their point estimates. For example, household h will change

[10] The derivation of this result, and the ones that follow, is given in the appendix.

[11] The scaling ensures that if the expected proportional change in household utility and its volatility are kept fixed, doubling current household utility also doubles the certainty equivalent. For a further discussion, see Skiadas (2009, p. 213).

[12] In continuous time when the source of uncertainty is a Brownian motion, one can always determine the true volatility of the return on the capital stock by observing its value for a finite amount of time; therefore, a household can be uncertain only about the expected return.

the empirically estimated return on capital for firm n from α to $\alpha + \nu_{hn,t}$, thereby *reducing* the magnitude of the firm's expected risk premium ($\nu_{hn,t} \leq 0$ if $\alpha > i$ and $\nu_{hn,t} \geq 0$ if $\alpha < i$). The size of the reduction depends on each household's familiarity with a particular firm—the reduction is smaller for firms with which the household is more familiar. Differences in familiarity across households lead them to use different estimates of expected returns in their decision making, despite having observed the same data. We can see this explicitly by observing that in the presence of familiarity, the contribution of risky portfolio investment to a household's expected return on wealth changes from $\sum_{n=1}^{N} \omega_{hn,t}\alpha dt$ to $\sum_{n=1}^{N} \omega_{hn,t}(\alpha + \nu_{hn,t})dt$. The adjustment to the expected return on a household's wealth stemming from familiarity bias is thus

$$\sum_{n=1}^{N} \omega_{hn,t}\nu_{hn,t}dt. \tag{7}$$

Without familiarity bias, the decision of a household on how much to invest in a particular firm depends solely on the certainty equivalent. Therefore, to allow for familiarity bias it is natural to generalize the concept of the certainty equivalent. For date $t + dt$ decision utility in the presence of familiarity bias, we extend Uppal and Wang (2003) and define the *familiarity-biased certainty equivalent* by

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \mu_t[U_{h,t+dt}] + U_{h,t} \times \left(\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^{\top}\boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma}\frac{\boldsymbol{\nu}_{h,t}^{\top}\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2} \right) dt, \tag{8}$$

where $U_{W_{h,t}} = \frac{\partial U_{h,t}}{\partial W_{h,t}}$, $\boldsymbol{\omega}_{h,t} = (\omega_{h1,t}, \ldots, \omega_{hN,t})^{\top}$ is the column vector of portfolio weights, $\boldsymbol{\nu}_{h,t} = (\nu_{h1,t}, \ldots, \nu_{hN,t})^{\top}$, and $\Gamma_h = [\Gamma_{h,nm}]$ is the $N \times N$ diagonal matrix defined by

$$\Gamma_{h,nm} = \begin{cases} \frac{1-f_{hn}}{f_{hn}}, & n = m, \\ 0, & n \neq m, \end{cases}$$

where $f_{hn} \in [0, 1]$ is a measure of how familiar the household is with firm, n. A larger value for f_{hn} indicates more familiarity, with $f_{hn} = 1$ implying perfect familiarity, and $f_{hn} = 0$ indicating no familiarity at all.

The first term in (8), the pure certainty equivalent $\mu_t[U_{h,t+dt}]$, does not depend directly on the familiarity-bias adjustments. As before, we introduce the scaling factor $U_{h,t}$ (see footnote 11 for the role of the scaling factor). The next term, $\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\boldsymbol{\nu}_{h,t}^{\top}\boldsymbol{\omega}_{h,t}$, is the adjustment to the expected change in household utility. It is the product of the elasticity of household utility with respect to wealth, $\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}$, and the change in the expected return on household wealth arising from the adjustment made to returns, which is given in (7).

The tendency to make adjustments to expected returns is tempered by a penalty term, $\frac{1}{2}\frac{1}{\gamma}\frac{\boldsymbol{\nu}_{h,t}^{\top}\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2}$, which captures two distinct features of household decision making. The first pertains to the idea that when a household has more accurate estimates of expected returns, she will be less willing to adjust them. The accuracy of household's expected return estimates is measured by their standard errors, which are proportional to σ.[13] With smaller standard errors, there is a stiffer penalty for adjusting returns away from their empirical estimates. The second feature pertains to familiarity, reflected by Γ_h: when a household is more familiar with a particular firm, the penalty for adjusting its return away from its estimated value is again larger.

3 Portfolio Decision and Welfare of an Individual Household

We solve the model described above in two steps. First, we solve in partial equilibrium for decisions of an individual household that suffers from familiarity bias. To solve the individual household's intertemporal decision problem, we show that the portfolio-choice problem can be interpreted as the problem of a mean-variance household, where the familiarity bias in her decision utility is captured by adjusting expected returns. We then show how the mean-variance portfolio decision impacts the intertemporal consumption decision of the household. Comparing the experienced utility to the decision utility allows us to measure the internality (welfare loss) at the individual-household level resulting from familiarity bias. Then, in the next section, we aggregate over all households to get the general-equilibrium effect on social welfare resulting from familiarity bias in the decision utilities of individual households.

3.1 The Intertemporal Decision Problem of an Individual Household

If a household did not suffer from familiarity biases, her experienced and decision utility functions would coincide, and so she would choose her consumption rate and portfolio policy

[13] In our continuous-time framework, an infinite number of observations are possible in finite time, so standard errors equal the volatility of proportional changes in the capital stock, σ, divided by the square root of the length of the observation window.

as follows:

$$\sup_{C_{h,t}} \mathcal{A}\Big(C_{h,t}, \sup_{\boldsymbol{\omega}_{h,t}} \mu_{h,t}[U_{h,t+dt}]\Big). \tag{9}$$

However, in the presence of familiarity biases, she decides on her consumption rate and portfolio policy by optimizing her decision utility. Familiarity bias drives a wedge between her experienced and decision utilities. Therefore, in her decision utility, the time aggregator in (3) is unchanged—all that one needs to do is to replace the maximization of the certainty-equivalent $\sup_{\boldsymbol{\omega}_{h,t}} \mu_t[U_{h,t+dt}]$, with the combined maximization and minimization of the familiarity-based certainty equivalent, $\sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_t} \mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ to obtain

$$\sup_{C_{h,t}} \mathcal{A}\Big(C_{h,t}, \sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} \mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\Big). \tag{10}$$

A household, because of its familiarity bias, chooses $\boldsymbol{\nu}_{h,t}$ to *minimize* its familiarity-biased certainty equivalent; that is, the household adjusts expected returns more for firms with which it is less familiar, which acts to reduce the familiarity-biased certainty equivalent.[14] By comparing (9) and (10), we can see that once a household has chosen the vector $\boldsymbol{\nu}_{h,t}$ to adjust the expected returns of each firm for familiarity bias, the household makes consumption and portfolio choices in the standard way.

Given any portfolio decision $\boldsymbol{\omega}_{h,t}$ for a household, finding the adjustments to firm-level expected returns is a simple matter of minimizing the familiarity-biased certainty equivalent in (8). For a given portfolio $\omega_{h,t}$, the adjustment $\nu_{hn,t}$ to firm n's expected return is given by:

$$\nu_{hn,t} = -\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\left(\frac{1}{f_{hn}} - 1\right)\sigma^2\gamma\,\omega_{hn,t}, n \in \{1,\dots,n\}. \tag{11}$$

The above expression shows that if a household is fully familiar with firm n, $f_{hn} = 1$, then she makes no adjustment to the firm's expected return. If she is less than fully familiar, $f_{hn} \in [0,1)$, one can see that $\nu_{hn,t}$ is negative (positive) when $\omega_{hn,t}$ is positive (negative), reflecting the idea that lack of familiarity leads a household to moderate its portfolio choices, shrinking both long and short positions toward zero.

To solve a household's consumption-portfolio choice problem using her decision utility, we use Ito's Lemma to derive the continuous-time limit of (10), which leads to the following

[14] In the language of decision theory, households are averse to ambiguity and so they minimize their familiarity-biased certainty equivalents.

Hamilton-Jacobi-Bellman equation:

$$0 = \sup_{C_{h,t}} \left(\delta\, u_\psi \left(\frac{C_{ht}}{U_{ht}} \right) + \sup_{\omega_t} \inf_{\nu_{h,t}} \frac{1}{U_{h,t}} \mu^{\boldsymbol{\nu}}_{h,t} \left[\frac{dU_{h,t}}{dt} \right] \right), \tag{12}$$

where the function $u_\psi(\cdot)$ is given by

$$u_\psi(x) = \frac{x^{1-\frac{1}{\psi}} - 1}{1 - \frac{1}{\psi}}, \psi > 0,$$

and

$$\mu^{\boldsymbol{\nu}}_{h,t} \left[dU_{h,t} \right] = \mu^{\boldsymbol{\nu}}_{h,t} \left[U_{h,t+dt} - U_{h,t} \right] = \mu^{\boldsymbol{\nu}}_{h,t} \left[U_{h,t+dt} \right] - U_{h,t},$$

with $\mu^{\boldsymbol{\nu}}_{h,t} \left[U_{h,t+dt} \right]$ given in (8).

Assuming a constant risk-free rate, homotheticity of preferences combined with constant returns to scale for production leads to an investment opportunity set that is constant over time, and hence, implies that maximized household utility is a constant multiple of household wealth. In this case, the Hamilton-Jacobi-Bellman equation can be decomposed into two parts: an *intertemporal* consumption-choice problem and a *mean-variance* optimization problem for a household with familiarity bias:

$$0 = \sup_{C_{h,t}} \left(\delta\, u_\psi \left(\frac{C_{ht}}{U_{ht}} \right) - \frac{C_{ht}}{W_{ht}} + \sup_{\boldsymbol{\omega}_t} \inf_{\boldsymbol{\nu}_{h,t}} MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) \right). \tag{13}$$

In the above expression, $MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t})$ is the objective function of a mean-variance household with familiarity bias:

$$MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) = i + \big(\alpha - i\big) \mathbf{1}^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2} \gamma \sigma^2 \boldsymbol{\omega}_{h,t}^\top \Omega \boldsymbol{\omega}_{h,t} + \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma} \frac{\boldsymbol{\nu}_{h,t}^\top \Gamma_h^{-1} \boldsymbol{\nu}_{h,t}}{\sigma^2}, \tag{14}$$

where $\mathbf{1}$ denotes the $N \times 1$ unit vector, $i + \big(\alpha - i\big) \mathbf{1}^\top \boldsymbol{\omega}_{h,t}$ is the expected portfolio return, $-\frac{1}{2} \gamma \sigma^2 \boldsymbol{\omega}_{h,t}^\top \Omega \boldsymbol{\omega}_{h,t}$ is the penalty for portfolio variance, $\boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t}$ is the adjustment to the portfolio's expected return arising from familiarity bias, and $\frac{1}{2\gamma} \frac{\boldsymbol{\nu}_{h,t}^\top \Gamma_h^{-1} \boldsymbol{\nu}_{h,t}}{\sigma^2}$ is the penalty for adjusting expected returns.[15]

In the first part of the mean-variance problem with familiarity bias, the firm-level expected returns are optimally adjusted downward because of lack of familiarity. Because

[15] The familiarity-bias adjustment is obtained from a minimization problem, so the associated penalty is positive, in contrast with the penalty for return variance.

household utility is a constant multiple of wealth, the expression for the optimal adjustment to expected returns in (11) simplifies to:

$$\boldsymbol{\nu}_{h,t} = -\gamma\sigma^2 \, \Gamma_h \, \boldsymbol{\omega}_{h,t}. \tag{15}$$

Substituting the above expression into (14), we see that each household faces the following mean-variance portfolio problem:

$$\sup_{\boldsymbol{\omega}_{h,t}} MV(\boldsymbol{\omega}_{h,t}) = \left(i + \left(\alpha\mathbf{1} + \frac{1}{2}\boldsymbol{\nu}_{h,t} - i\mathbf{1}\right)^\top \boldsymbol{\omega}_{h,t} \right) - \frac{1}{2}\gamma\sigma^2 \boldsymbol{\omega}_{h,t}^\top \Omega \boldsymbol{\omega}_{h,t}, \tag{16}$$

in which $\boldsymbol{\nu}_{h,t}$ is given by (15). If the household were fully familiar with all firms, then Γ_h is the zero matrix, and from (15) we can see the adjustment to expected returns is zero and the portfolio weights are exactly the standard mean-variance portfolio weights. For the case where the household is completely unfamiliar with all firms, then each $\Gamma_{h,nn}$ becomes infinitely large and $\boldsymbol{\omega}_h = \mathbf{0}$: complete unfamiliarity leads the household to avoid any investment in risky firms, in which case we get non-participation in the stock market in this partial-equilibrium setting.

3.2 Optimal Portfolio of an Individual Household

In this section, we derive the portfolio of an individual household that maximizes the household's decision utility. We then show the relation between the portfolio chosen and the welfare of a household with a mean-variance objective function, as in Campbell (2006, p. 1574) and Calvet, Campbell, and Sodini (2006).

Solving from (16) the first-order condition for the vector of optimal portfolio weights, $\boldsymbol{\omega}_{h,t}$, and substituting the resulting optimal weights into (15), we see that the optimal adjustment to expected returns is:

$$\boldsymbol{\nu}_h = (\alpha - i)[(I + \Gamma_h \Omega^{-1})^{-1} - I]. \tag{17}$$

If $\rho = 0$, the above expression becomes particularly simple to interpret:

$$\boldsymbol{\nu}_h = -(\alpha - i)(\mathbf{1} - \boldsymbol{f}_h),$$

where $\boldsymbol{f}_h$ is the household-specific vector of familiarity coefficients

$$\boldsymbol{f}_h = (f_{h1}, \ldots, f_{hN})^\top.$$

In this case it is easy to see that the size of a household's adjustment to a particular firm's return is smaller when the level of familiarity, f_{hn}, is larger; if $f_{hn} = 1$, then the adjustment vanishes altogether.

The vector of optimal portfolio weights is

$$\boldsymbol{\omega}_h = \left[\frac{1}{\gamma} \frac{\alpha - i}{\sigma^2} \right] \boldsymbol{q}_h, \tag{18}$$

where the term in square brackets is the standard expression for the portfolio weight in the absence of familiarity bias and $\boldsymbol{q}_h$ is the correlation-adjusted familiarity vector

$$\boldsymbol{q}_h = (\Omega + \Gamma_h)^{-1}\mathbf{1}. \tag{19}$$

For the special case of $\rho = 0$, the vector of portfolio weights reduces to

$$\boldsymbol{\omega}_h = \left[\frac{1}{\gamma} \frac{\alpha - i}{\sigma^2} \right] \boldsymbol{f}_h.$$

As a household's level of familiarity with a particular firm n decreases, f_{hn} decreases, and therefore, the proportion of her wealth that she chooses to invest in firm n also decreases.

We can also write a household's portfolio decision in terms of her capital-allocation decision, i.e. the proportion of wealth allocated to risky assets, denoted by $\pi_h = \sum_{n=1}^{N} \omega_{hn}$, and her portfolio of only risky assets, denoted by

$$\boldsymbol{x}_h = \frac{\boldsymbol{\omega}_h}{\pi_h}.$$

We find that

$$\pi_h = \frac{1}{\gamma} \frac{SR_{\boldsymbol{x}_h}}{\sigma_{\boldsymbol{x}_h}} \frac{1}{1 + b_h}, \tag{20}$$

where $SR_{\boldsymbol{x}_h}$ is the Sharpe ratio of the portfolio of risky assets, i.e.

$$SR_{\boldsymbol{x}_h} = \frac{\alpha - i}{\sigma_{\boldsymbol{x}_h}},$$

in which

$$\sigma^2_{\boldsymbol{x}_h} = \sigma^2 \boldsymbol{x}_h^\top \Omega \boldsymbol{x}_h$$

is the variance of a household's portfolio of risky assets and the distortion in a household's capital allocation decision stemming from familiarity bias is given by

$$b_h = \frac{\mathbf{1}^\top \boldsymbol{q}_h}{\boldsymbol{q}_h^\top \Omega \boldsymbol{q}_h} - 1.$$

With no familiarity bias, a household minimum-variance portfolio is given by $x_{hn} = \frac{1}{N}$, because all risky assets have the same volatility and correlation. The variance of this equal-weighted portfolio of only risky assets is

$$\sigma^2_{1/N} = \sigma^2 \left(\frac{1}{N} + \left(1 - \frac{1}{N}\right) \rho \right),$$

and the Sharpe ratio of the portfolio is denoted by $SR_{1/N}$, where

$$SR_{1/N} = \frac{\alpha - i}{\sigma_{1/N}}.$$

The familiarity-biased portfolio of only risky assets, $\boldsymbol{x}_h$, is the minimum-variance portfolio with a familiarity-biased adjustment. Familiarity bias tilts the portfolio with only risky assets away from $\frac{1}{N}$, creating an underdiversified portfolio with higher variance, so that $\sigma_{\boldsymbol{x}_h} > \sigma_{1/N}$. Therefore, the Sharpe ratio is reduced: $SR_{\boldsymbol{x}_h} < SR_{1/N}$. This leads the household to reduce the proportion of her overall wealth held in risky assets.

We now compute the welfare loss from familiarity bias experienced by a household that maximizes just the mean-variance objective function in (14); this result will be useful in comparing the welfare gain in the absence of intertemporal consumption to that where the investor desires to smooth intertemporal consumption.

Using the choices based on her decision utility, a household's mean-variance experienced-utility can be expressed as

$$MV^e_{\boldsymbol{x}_h} = i + \frac{1}{2\gamma} SR^2_{\boldsymbol{x}_h}(1 - d_h^2), \tag{21}$$

where

$$d_h = \frac{b_h}{1 + b_h}, \quad \text{with} \quad b_h \geq 0 \quad \text{and} \quad 0 \leq d_h \leq 1.$$

On the other hand, the optimized experienced utility in the absence of a familiarity bias is

$$MV^e_{1/N} = i + \frac{1}{2\gamma} SR^2_{1/N}. \tag{22}$$

We can see that for a given interest rate, removing a household's familiarity biases has two beneficial effects. First, a household increases its overall investment in firms—reflected in the fact that $d_h = 0$, when there is no familiarity bias. Second, a household's risky portfolio becomes less volatile, because it is fully diversified—such a portfolio has higher Sharpe ratio: $SR_{1/N} > SR_{\boldsymbol{x}_h}$.

Subtracting (21) from (22), the gain in mean-variance experienced utility is equivalent to an increase in the risk-free interest rate of

$$MV^e_{1/N} - MV^e_{\boldsymbol{x}_h} = \frac{1}{2}\,\tilde{\pi}_h\,\sigma_{\boldsymbol{x}_h}\left(\frac{SR^2_{1/N} - SR^2_{\boldsymbol{x}_h}}{SR_{\boldsymbol{x}_h}}\right) + \frac{1}{2\gamma}SR^2_{\boldsymbol{x}_h}d_h^2, \tag{23}$$

where $\tilde{\pi}_h = \frac{1}{\gamma}\frac{SR_{\boldsymbol{x}_h}}{\sigma_{\boldsymbol{x}_h}}$, is the portfolio choice with the correct capital-allocation decision between the risk-free and risky assets. The first term in (23) represents the gain from diversification, which is the measure of utility gain used in Campbell (2006, p. 1574), while the second term represents the gain from allocating capital optimally between the risk-free asset and risky assets.

3.3 Optimal Consumption of an Individual Household

Having analyzed the portfolio decision of an individual household and the welfare implications of the portfolio decision for mean-variance experienced utility, we now study an individual household's consumption decision.

We first solve for optimal consumption in terms of a household's decision utility, denoted by U^d_{ht}. From the Hamilton-Jacobi-Bellman equation in (12), the first-order condition with respect to consumption is

$$\delta\left(\frac{C_{ht}}{U^d_{ht}}\right)^{-\frac{1}{\psi}} = \frac{U^d_{ht}}{W_{h,t}}.$$

Substituting the above first-order condition into the Hamilton-Jacobi-Bellman equation allows us to solve for household decision utility, and hence, optimal consumption. We find that

$$U^d_{h,t} = \left(\frac{C_{ht}/W_{ht}}{\delta^{\psi}}\right)^{\frac{1}{1-\psi}} W_{h,t},$$

where a household's optimal consumption-to-wealth ratio is:

$$\frac{C_{h,t}}{W_{h,t}} = \psi\delta + (1-\psi)\left(\left[i + \left(\alpha\mathbf{1} + \frac{1}{2}\boldsymbol{\nu}_{h,t} - i\mathbf{1}\right)^{\top}\boldsymbol{\omega}_{h,t}\right] - \frac{1}{2}\gamma\sigma^2\boldsymbol{\omega}^{\top}_{h,t}\Omega\boldsymbol{\omega}_{h,t}\right) \tag{24}$$

$$= \psi\delta + (1-\psi)\left(i + \frac{1}{2\gamma}SR^2_{\boldsymbol{x}_h}\frac{1}{1+b_h}\right). \tag{25}$$

We see from (25) that the optimal consumption-wealth ratio is a weighted average of the impatience parameter δ and the mean-variance objective function based on *decision utility*.[16] The presence of $SR_{\boldsymbol{x}_h}$ and b in the above expression shows that the household's portfolio choice impacts her intertemporal consumption choice.

Finally, an individual household's experienced utility level, $U_{h,t}$, is given by

$$U_{h,t} = \kappa_h W_{h,t},$$

where κ_h is given by

$$\kappa_h = \left[\frac{\psi\delta + (1-\psi)\left(i + \frac{1}{2\gamma} SR^2_{\boldsymbol{x}_h}\right)}{\delta^{\psi}} \right]^{\frac{1}{1-\psi}}. \tag{26}$$

4 Social Welfare and Growth

In this section, we study social welfare and growth in general equilibrium. In contrast with Section 3, where we examined how familiarity bias impacts an individual household, we now focus on aggregate quantities. That is, we investigate the general-equilibrium effects on aggregate growth and social welfare of the distortion in the consumption of individual households when we aggregate over all households and impose market clearing.

4.1 No Aggregate Familiarity Bias Across Households

In this section, we explain how the familiarity bias is specified for each household so that it is "symmetric" across households and "cancels out in aggregate."

By "canceling out in aggregate" we mean that the bias in the cross-sectional average risky portfolio across households is zero. We express this condition formally by first writing household h's risky portfolio weight for firm n as the unbiased weight plus a bias, i.e.

$$x_{hn} = \frac{1}{N} + \epsilon_{hn},$$

where $\frac{1}{N}$ is the unbiased portfolio weight and ϵ_{hn} is the bias of household h's portfolio when investing in firm n. We can now see that "cancelling out in aggregate" is equivalent to the

[16] For experienced utility, the factor $\frac{1}{1+b_h}$ would be set equal to $1 - d_h^2$.

following condition

$$\frac{1}{H}\sum_{h=1}^{H}\epsilon_{hn}=0, \quad \forall n, \tag{27}$$

which applies for each firm.[17] The above condition says that while it is possible for an individual household's portfolio to be biased, that is, to deviate from the unbiased $\frac{1}{N}$ portfolio, this bias must cancel out when forming the average portfolio across all households. We shall refer to (27) as the "no-aggregate-bias condition."

The following symmetry condition implies that the no-aggregate-bias condition holds. For every household $h \in \{1,\ldots,H\}$, define the correlation-adjusted familiarity vector $(q_{h1},\ldots,q_{hN})$. The symmetry condition states the following: (1) given a household $h \in \{1,\ldots,H\}$, for all households $h' \in \{1,\ldots,H\}$, there exists a permutation $\tau_{h'}$ such that $\tau_{h'}(q_{h'1},\ldots,q_{h'N}) = (q_{h1},\ldots,q_{hN})$; and, (2) given a firm $n \in \{1,\ldots,N\}$, for all firms $n' \in \{1,\ldots,N\}$, there exists a permutation $\tau_{n'}$ such that $\tau_{n'}(q_{1n'},\ldots,q_{Hn'}) = (q_{1n},\ldots,q_{Hn})$.

To interpret the symmetry condition further, observe that it implies that

$$\frac{1}{H}\sum_{h=1}^{H}q_{hn}=\frac{1}{N}\sum_{n=1}^{N}q_{hn}, \ \ \forall\, h \text{ and } \forall\, n. \tag{28}$$

Intuitively, the condition in (28) says that the mean correlation-adjusted familiarity of a household across all firms, $\frac{1}{N}\sum_{n=1}^{N}q_{hn}$, is equal to the mean correlation-adjusted familiarity toward a firm across all households, $\frac{1}{H}\sum_{h=1}^{H}q_{hn}$.

Observe also that the condition in equation (28) is equivalent to

$$\forall n,\, \forall h,\, \frac{1}{H}\hat{q}_n=\frac{1}{N}\hat{q}_h, \tag{29}$$

where

$$\hat{q}_n=\sum_{h=1}^{H}q_{hn}, \qquad \text{and} \qquad \hat{q}_h=\sum_{n=1}^{N}q_{hn}.$$

From (29) we can also see that $\hat{q}_n$ and $\hat{q}_h$ must be independent of n and h, respectively.

[17] An equivalent way of expressing equation (27) is that the mean risky portfolio equals the $\frac{1}{N}$ portfolio:

$$\forall n,\, \frac{1}{H}\sum_{h=1}^{H}x_{hn}=\frac{1}{N}.$$

4.2 The Equilibrium Risk-free Interest Rate

We now characterize the equilibrium in the economy we are studying by imposing market clearing in the risk-free bond market. The risk-free bond is in zero net-supply, which implies that the demand for bonds aggregated across all households must be zero:

$$\sum_{h=1}^{H} B_{h,t} = 0.$$

The amount of wealth held in the bond by household h is given by

$$B_{h,t} = (1 - \mathbf{1}^\top \boldsymbol{\omega}_h) W_{h,t},$$

where $\mathbf{1}^\top \boldsymbol{\omega}_h$ is the proportion of household h's wealth invested in all risky assets. Summing the demand for bonds over households gives

$$0 = \sum_{h=1}^{H} B_{h,t} = \sum_{h=1}^{H} (1 - \mathbf{1}^\top \boldsymbol{\omega}_h) W_{h,t} = \sum_{h=1}^{H} \left(1 - \frac{\alpha - i}{\gamma \sigma^2} \sum_{n=1}^{N} q_{hn}\right) W_{h,t}.$$

As a consequence of the symmetry assumption, each household will have the same aggregate familiarity across the N assets:

$$\sum_{n=1}^{N} q_{hn} = \sum_{n=1}^{N} q_{jn} = \widehat{q}.$$

Therefore, the market-clearing condition for the bond simplifies to

$$0 = \left(1 - \frac{\alpha - i}{\gamma \sigma^2} \widehat{q}\right) \sum_{h=1}^{H} W_{h,t}.$$

The equilibrium risk-free interest rate is thus given by the constant

$$i = \alpha - \gamma\, \sigma_p^2, \tag{30}$$

where

$$\sigma_p^2 = \frac{\sigma^2}{\widehat{q}}$$

is the variance of the portfolio held by each household adjusted for familiarity bias. We can see immediately that reducing familiarity (that is, a reduction in $\widehat{q}$) increases the riskiness of each household's portfolios, σ_p, leading to a greater precautionary demand for the risk-free asset, and hence, a decrease in the risk-free interest rate.

4.3 Aggregate Growth and Social Welfare

Substituting the equilibrium interest rate in (30) into the expression for the partial-equilibrium consumption-wealth ratio in (25) gives the consumption-wealth ratio in general equilibrium, which is common across households:

$$\frac{C_{h,t}}{W_{h,t}} = c = \psi\delta + (1-\psi)\left(\alpha - \frac{1}{2}\gamma\sigma_p^2\right).$$

The right-hand side of the above expression is constant. Exploiting the fact that the consumption-wealth ratio is constant across households and also over time allows us to obtain the ratio of aggregate consumption, $C_t^{\mathrm{agg}} = \sum_{h=1}^{H} C_{h,t}$, to aggregate wealth, $W_t^{\mathrm{agg}} = \sum_{h=1}^{H} W_{h,t}$:

$$\frac{C_t^{\mathrm{agg}}}{W_t^{\mathrm{agg}}} = c = \psi\delta + (1-\psi)\left(\alpha - \frac{1}{2}\gamma\sigma_p^2\right). \tag{31}$$

In equilibrium, the aggregate level of the capital stock equals the aggregate wealth of households, because the bond is in zero net supply: $K_t^{\mathrm{agg}} = W_t^{\mathrm{agg}}$, where $K_t^{\mathrm{agg}} = \sum_{n=1}^{N} K_{h,t}$ is the aggregate level of the capital stock. Therefore, we obtain from (31) the aggregate consumption-capital and consumption-output ratios:

$$\frac{C_t^{\mathrm{agg}}}{K_t^{\mathrm{agg}}} = c \quad \text{and} \quad \frac{C_t^{\mathrm{agg}}}{Y_t^{\mathrm{agg}}} = \frac{c}{\alpha},$$

where aggregate output is given by $Y_t^{\mathrm{agg}} = \sum_{n=1}^{N} Y_{n,t} = \alpha \sum_{n=1}^{N} K_{n,t}$.

We now derive the aggregate investment-capital ratio. The aggregate investment flow, I_t^{agg}, is the sum of the investment flows into each firm, $I_t^{\mathrm{agg}} = \sum_{n=1}^{N} I_{n,t}$. The aggregate investment flow must be equal to aggregate output flow less the aggregate consumption flow, i.e.

$$I_t^{\mathrm{agg}} = \alpha K_t^{\mathrm{agg}} - C_t^{\mathrm{agg}}.$$

It follows that the aggregate investment-capital ratio is given by

$$\frac{I_t^{\mathrm{agg}}}{K_t^{\mathrm{agg}}} = \alpha - c = \psi(\alpha-\delta) - \frac{1}{2}(\psi-1)\gamma\sigma_p^2. \tag{32}$$

A decrease in an individual household's average familiarity makes its portfolio riskier, that is, σ_p^2 increases. If the substitution effect dominates ($\psi > 1$), the aggregate investment-capital ratio in (32) falls because households will consume more of their wealth.

We now determine trend output growth, g, defined by

$$g = E_t \left[\frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}} \right].$$

Firms all have constant returns to scale and differ only because of shocks to their capital stocks. Therefore, the aggregate growth rate of the economy is the aggregate investment-capital ratio:

$$g = \frac{I_t^{\text{agg}}}{K_t^{\text{agg}}} = \alpha - c = \psi(\alpha - \delta) - \frac{1}{2}(\psi - 1)\gamma\sigma_p^2. \tag{33}$$

From the expression in (33), we see that a fall in the aggregate investment-capital ratio, $I_t^{\text{agg}}/K_t^{\text{agg}}$, reduces output growth, g.

We now study social welfare, that is the aggregate welfare of all households. An individual household's experienced utility level is given by $U_{h,t} = \kappa_h W_{h,t}$, where κ_h is defined in (26). Our symmetry condition implies that average familiarity is equal across households. Hence, the portfolio held by each household has the same Sharpe, implying that the utility-wealth ratio $\kappa_h = \kappa$. Substituting into κ_h the expression for the market-clearing interest rate, we obtain

$$\kappa = \left[\frac{\psi\delta + (1-\psi)\left(\alpha - \frac{1}{2}\gamma\sigma_p^2\right)}{\delta^{\psi}} \right]^{\frac{1}{1-\psi}}. \tag{34}$$

Thus, experienced social welfare is given by U_t^{social}, where

$$U_t^{\text{social}} = \sum_{h=1}^{H} U_{h,t} = \kappa \sum_{h=1}^{H} W_{h,t} = \kappa\, K_t^{\text{agg}},$$

where in the last equality, we have used the fact that aggregate household wealth $\sum_{h=1}^{H} W_{h,t}$ must equal the level of the aggregate capital stock $K_t^{\text{agg}} = \sum_{n=1}^{N} K_{n,t}$, because the bond is in zero net supply.

From the expression in (34), we can see that for a given level of the aggregate capital stock, familiarity biases at the household level increase the portfolio risk, σ_p^2, and decrease experienced social welfare. The intuition is that familiarity biases induce individual households to hold underdiversified portfolios, which leads them to also reduce their overall investment in risky assets. Higher portfolio risk distorts the intertemporal consumption decisions of households. Consequently, aggregate investment and growth are also distorted, which reduces experienced social welfare.

4.4 The Welfare Gains from Holding Better-Diversified Portfolios

Education in finance theory is not widespread. For instance, the vast majority of high school students receive no education in portfolio choice. Even at the university level, only a minority of students study economics or finance. We know that households benefit from their own individual financial education if it allows them to choose better diversified-portfolios as a consequence of overcoming their familiarity biases; that is, financial education has a positive internality. But how significant would be the gains to society of widespread financial education, financial innovation, and financial regulation that lead households to invest in better-diversified portfolios?

To answer this question, we need to understand that the welfare gains take place via two different channels. One is a micro-level internality, whereby a household's welfare is increased purely from choosing a better diversified set of investments—the return on a household's financial wealth then becomes less risky, which also reduces her consumption-growth volatility. The second is a macro-level general-equilibrium effect, which raises the welfare of all households. From where does this macroeconomic effect arise? Its source lies in the decline of risk in every household's portfolio. If the substitution effect dominates the income effect ($\psi > 1$), households prefer to consume less today and invest more in risky firms; therefore, aggregate investment increases, raising the trend growth rate of the economy, and increasing social welfare. If the income effect dominates ($\psi < 1$), households prefer to consume more today and invest less in risky production, thereby reducing trend growth, but still increasing welfare.

We now show analytically how to disentangle the micro-level internality channel from the macro-level general-equilibrium channel. In equilibrium, the level of experienced social welfare can be written as

$$U_t^{\text{social}} = (\delta^{\psi} p_t^{\text{agg}})^{\frac{1}{\psi-1}} K_t^{\text{agg}}, \tag{35}$$

where p_t^{agg} is the price-dividend ratio of the aggregate capital stock, or equivalently, the aggregate wealth-consumption ratio:

$$p_t^{\text{agg}} = \frac{K_t^{\text{agg}}}{C_t^{\text{agg}}} = \frac{W_t^{\text{agg}}}{C_t^{\text{agg}}}.$$

Importantly, we choose to write the aggregate price-dividend ratio in (36) in terms of the endogenous expected growth rate of aggregate output, g, and the volatility of household portfolios, σ_p, i.e.

$$p_t^{\text{agg}} = \frac{1}{\delta + \left(\frac{1}{\psi} - 1\right)\left(g - \frac{1}{2}\gamma\sigma_p^2\right)}, \tag{36}$$

where we see from the expression for g in equation (33) that g itself is a function of σ_p^2.

The micro-level positive internality stems from a reduction in household portfolio risk, brought about by financial education, innovation, and regulation. The reduction in risk stems from improved diversification:

$$\Delta\sigma_p^2 = -\underbrace{\left(\sigma_{\boldsymbol{x}_h}^2 - \sigma_{1/N}^2\right)}_{\text{diversification}} < 0$$

The macro-level general-equilibrium effect manifests itself via a change in expected aggregate consumption growth, g, which we can write as follows

$$\Delta g = -\frac{1}{2}(\psi - 1)\gamma\Delta\sigma_p^2 = \frac{1}{2}(\psi - 1)\gamma\underbrace{\left(\sigma_{\boldsymbol{x}_h}^2 - \sigma_{1/N}^2\right)}_{\text{diversification}}.$$

The micro-level positive internality and the macro-level general-equilibrium effect combine to give the total impact on the aggregate price-dividend ratio and social welfare as follows

$$\frac{d\ln\left(\frac{U_t^{\text{agg}}}{K_t^{\text{agg}}}\right)}{d\ln(\sigma_p^2)} = \frac{\partial\ln\left(\frac{U_t^{\text{agg}}}{K_t^{\text{agg}}}\right)}{\partial\ln g}\frac{\partial\ln g}{\partial\ln(\sigma_p^2)} + \frac{\partial\ln\left(\frac{U_t^{\text{agg}}}{K_t^{\text{agg}}}\right)}{\partial\ln(\sigma_p^2)}, \tag{37}$$

where the first term on the right-hand side captures the macro-level general-equilibrium effect and the second term gives the micro-level positive internality. Computing the relevant derivatives gives

$$\frac{d\ln\left(\frac{U_t^{\text{agg}}}{K_t^{\text{agg}}}\right)}{d\ln(\sigma_p^2)} = -\frac{1}{2}\gamma\, P_t^{\text{agg}}\,\sigma_p^2\left(\underbrace{1 - \frac{1}{\psi}}_{\substack{\text{macro-level}\\ \text{general-equilibrium effect}}} + \underbrace{\frac{1}{\psi}}_{\substack{\text{micro-level}\\ \text{interality}}}\right). \tag{38}$$

We can see that a decline in the risk of household portfolios always increases social welfare. The relative importance of the micro-internality and macro-level channels is determined by the elasticity of intertemporal substitution, ψ. If ψ is higher, a reduction in risk at the micro-level has a greater impact at the macro-level, because households are more willing to adjust their consumption intertemporally.

5 Implications of Financial Policy for Social Welfare

Our main goal in this section is to make statements about social welfare for a plausibly parameterized general-equilibrium model. Below, we explain our choice of parameter values, then compute their quantitative implications for social welfare and explain the economic intuition for our results.

When households are intertemporal consumers, experienced social welfare is given by

$$U_t^{\text{social}} = \kappa K_t^{\text{agg}},$$

where

$$\kappa = \begin{cases} \left[\frac{\psi\delta + (1-\psi)\left(\delta + \frac{1}{\psi}\left(g - \frac{1}{2}\gamma\sigma_p^2\right)\right)}{\delta^{\psi}} \right]^{\frac{1}{1-\psi}} & \psi \neq 0, \\ MV & \psi = 0, \end{cases} \tag{39}$$

in which MV is given by

$$MV = \delta + \frac{1}{\psi}\left(g - \frac{1}{2}\sigma_p^2\right) = \alpha - \frac{\gamma}{2}\sigma_p^2,$$

and the endogenous aggregate growth rate g is given by (33). From (39), we see that for the special case in which the elasticity of intertemporal substitution $\psi = 0$, the social utility per aggregate capital, which is given by κ, reduces to the mean-variance case studied by Calvet, Campbell, and Sodini (2007) in partial equilibrium.

To compute the experienced social welfare gains if each household were to switch to holding a diversified portfolio, we need an estimate of the portfolio volatility for a household that is underdiversified along with an estimate of portfolio volatility if the household were holding a well-diversified portfolio. For both parameters, we use the estimates in Calvet, Campbell, and Sodini (2006, p. 14), where the portfolio volatility of the median underdiversified household is 20.7% per annum and the volatility of the household's portfolio if it invested in a diversified portfolio would be 14.7% per annum. Note that the estimate of 20.7% per annum accounts for the investment by the median household of about half its wealth in well-diversified mutual funds.

The next parameter we need is the expected rate of return on stocks, α. Calvet, Campbell, and Sodini (2006, p. 14) estimate that the equity risky premium, $\alpha - i$, is 6.7% per annum. They also use an interest rate of 3.7% per annum, which implies that $\alpha = 10.4\%$ per annum. In our model, the equity risk premium is endogenous (see equation (30)). We choose the relative risk aversion in our model to be $\gamma = 1.56363$ so that our model matches the equity risk premium of 6.7% per annum.

The final set of parameters that we need to specify are the preference parameters for the subjective rate of time preference, δ, and the elasticity of intertemporal substitution, ψ. We choose the base-case values of $\delta = 0.03$ and $\psi = 1.25$ and report results for a range of values around these base-case values.

Using the above parameter values, we compute experienced social welfare per unit of capital stock for three settings in general equilibrium, as summarized in the three columns of numbers in Table 1. In the first setting, we consider mean-variance households; in our general model this corresponds to the special case where all households have zero elasticity of intertemporal substitution: $\psi = 0$. If households were to shift from underdiversified portfolios with a volatility of $\sigma_{\boldsymbol{x}} = 20.7\%$ to diversified portfolios with a volatility of only $\sigma_{1/N} = 14.7\%$, social welfare per unit capital stock would increase by:

$$MV^e_{1/N} - MV^e_{\boldsymbol{x}} = \Big(\alpha - \frac{\gamma}{2}\sigma^2_{1/N}\Big) - \Big(\alpha - \frac{\gamma}{2}\sigma^2_{\boldsymbol{x}}\Big) = \frac{\gamma}{2}\Big(\sigma^2_{\boldsymbol{x}} - \sigma^2_{1/N}\Big) = 0.0166,$$

which can be interpreted as an increase of 1.66% in the annualized expected return on the aggregate capital stock (i.e. aggregate wealth).

One could also compute the percentage increase in the initial aggregate capital stock, λ, that is required to raise experienced social welfare with poorly diversified portfolios to that under perfectly diversified portfolios:

$$K^{\text{agg}} \times MV^e_{1/N} = K^{\text{agg}}(1+\lambda) \times MV^e_{\boldsymbol{x}}$$

$$\lambda = \frac{MV^e_{1/N}}{MV^e_{\boldsymbol{x}}} - 1,$$

where K^{agg} is initial level of the capital stock. According to this measure, reported in the second row of the first column of numbers in Table 1, a shift from portfolios with a volatility of 20.7% to portfolios with a volatility of only 14.7% is equivalent to a $\lambda = 23.55\%$ increase in the initial capital stock. Using a back-of-the-envelope calculation, this increase in the initial capital stock can be related to an annualized return via the following simple expression:

$$\frac{23.55\%}{T} = 1.66\%,$$

Table 1: Social Welfare Gains from Household Portfolio Diversification
In this table, we report the potential gains to experienced social welfare if an effective policy solution could be found for households' familiarity biases, thereby allowing households to benefit from portfolio diversification. We report results for three settings. In the first setting, households have mean-variance utility. In the second setting, households are intertemporal consumers, but growth is exogenous. In the third setting, households are intertemporal consumers and growth is endogenous. The gains to social welfare are reported using three measures. The first measure reports the gain in levels for social welfare per unit of capital stock. The second measure reports the percentage gain in social welfare per unit of capital stock. The third measure reports the percentage gain in social welfare per unit of capital stock, expressed as a per annum return over a period of $T = 14.17$ years. The parameter values we have assumed are: $\sigma_{1/N} = 14.7\%$ p.a.; $\sigma_{\boldsymbol{x}} = 20.7\%$ p.a.; $\alpha = 10.4\%$ p.a.; $\gamma = 1.56363$; $\delta = 0.03$ p.a.; $\psi = 1.25$.

	Mean-Variance household	Intertemporal household (exogenous growth)	Intertemporal household (endogenous growth)
Social welfare gain in levels	0.0166	0.16	0.24
Social welfare gain in %	23.55	107.79	155.29
$(1/T)$Social welfare gain in %	1.66	7.60	10.95

implying that an increase in the aggregate capital stock of 23.55% is equivalent to an annualized return of 1.66% per annum over $T = 14.17$ years.[18] This is reported in the last row of the first column of numbers in Table 1.

Next, we look at experienced social welfare when households desire to smooth consumption intertemporally ($\psi > 0$). These results are reported in the column titled "Intertemporal household (exogenous growth)" of Table 1. We find that when $\psi = 1.25$, shifting from portfolios with a volatility 20.7% to portfolios with a volatility of 14.7%, keeping the aggregate growth rate fixed exogenously (at the level where portfolio volatility of each household is equal to 20.7%), raises social welfare per unit capital stock by:

$$\kappa_{1/N} - \kappa_{\boldsymbol{x}}\big|_{g\,\text{fixed}} = 0.1679,$$

rather than the 0.0166 for the mean-variance case that ignores the effect on intertemporal consumption. The experienced social welfare gain is about ten times larger now because the households benefit not just from reducing the volatility of their portfolios, but also from the utility gain driven by intertemporal consumption smoothing. This larger increase in social welfare gain is similar in magnitude if one were to measure the gain in terms of the required increase in the initial aggregate capital stock:

$$K^{\text{agg}} \times \kappa_{1/N}\big|_{g\,\text{fixed}} = K^{\text{agg}}(1+\lambda)\kappa_{\boldsymbol{x}}$$

$$\lambda = \frac{\kappa_{1/N}\big|_{g\,\text{fixed}}}{\kappa_{\boldsymbol{x}}} - 1 = 107.79\%,$$

[18] For the details of this calculation, observe that $1 + \lambda = e^{r\,T}$, where r is the annualized return over T years, and so $r = \frac{1}{T}\ln(1+\lambda) \approx \frac{1}{T}\lambda$.

instead of the 23.55% for the mean-variance case. Similarly, the change in initial capital stock is equivalent to an annualized expected return of 7.60% per annum over $T = 14.17$ years, in comparison to the 1.66% per annum for the mean-variance case.

Finally, we look at the case where aggregate growth is endogenous, and hence, changes as all households shift to a portfolio with less risk. These results are reported in the last column, titled "Intertemporal household (endogenous growth)," of Table 1. We find that if $\psi = 1.25$, shifting from portfolios with a volatility 20.7% to portfolios with a volatility of 14.7%, raises social welfare per unit capital stock by 0.2418 instead of the 0.1679 for the case with exogenous growth, and 0.0166 for the mean-variance case. The social welfare gain is much larger now because households benefit not just from reducing the volatility of their portfolio, and the improvement from smoothing their intertemporal consumption, but also from the increase in aggregate growth.

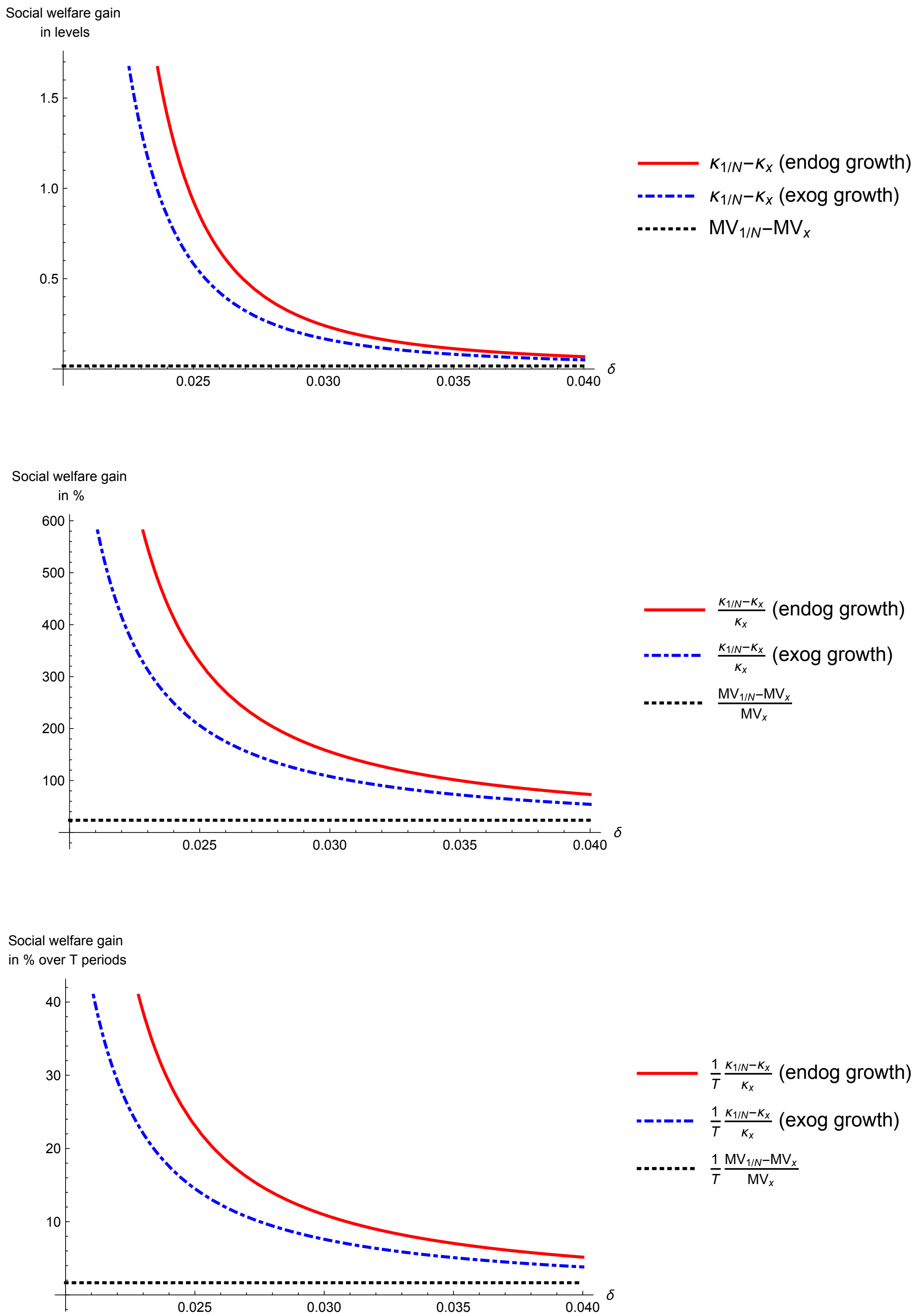
The social welfare gain of 0.2418 compared to 0.1679 is of course similar in magnitude to what one would obtain by measuring the gain in terms of the required increase in initial capital stock: $\lambda = 155.29\%$ compared to 107.79%. Similarly, using the same scaling of $T = 14.17$ years as for the mean-variance case, we find that that the change in initial capital stock is equivalent to an annualized expected return of 10.95% per annum with endogenous growth, in comparison to 7.60% per annum with exogenous growth, and 1.66% per annum for the mean-variance case.

In summary, the above results suggest that the modest increase in social welfare for mean-variance households from holding a better diversified portfolio is about six times larger once we allow for the possibility that households can smooth consumption intertemporally and that the aggregate effects of these changes could lead to an increase in growth.

As in all dynamic models, the effect of the intertemporal allocation of capital becomes less pronounced as households become more impatient. The effect of an increase in δ is illustrated in the figure below, where the three panels correspond to the three different measures

In Figure 1, we plot the social welfare gain for the three settings examined in the three rows of the table above as households' impatience changes. The figure has three panels, where the first panel reports the social welfare gain per unit capital stock in levels, the second panel reports the social welfare gain per unit capital stock in percentage terms, and the

Figure 1: Social welfare as δ changes

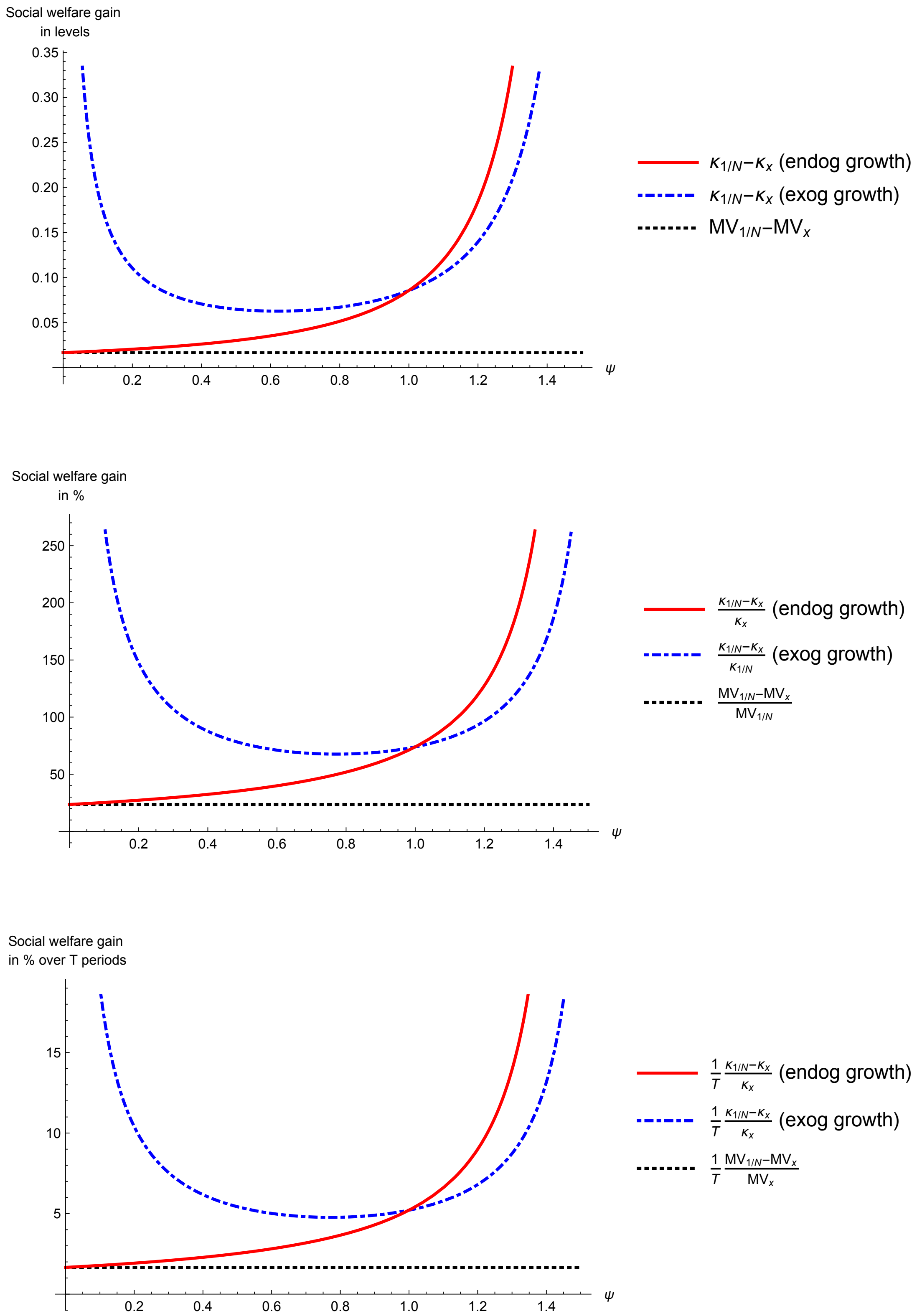


third panel reports the social welfare gain per unit capital stock in percentage terms based on an annualized return over $T = 14.17$ years. The dashed (black) line in the figure shows the welfare gains from diversification for a mean-variance household: this line is flat because the mean-variance utility does not depend on δ. The dotted-dashed (blue) line shows the gains from portfolio diversification for a household with intertemporal consumption when growth is exogenous: this line shows that for patient households (low δ) with a strong willingness to postpone consumption to future dates, the welfare gains can be amplified by an order of magnitude relative to the mean-variance case. The solid (red) line shows that the welfare gain to society from portfolio diversification when growth is endogenous: this line shows that the social welfare gains exceed the private gains to individual households, with the gap between the two increasing as δ decreases. These lines show that a large part of the amplification stems not from the direct internality of a reduction in micro-level volatility from portfolio diversification, but instead from the smoothing of intertemporal consumption and the macro-level effect on aggregate growth.

In Figure 2, we plot the social welfare gain per unit of capital stock for the three settings examined in the three rows of the table above as the household's elasticity of intertemporal substitution changes. Just as in Figure 1, the figure has three panels, where the first panel reports the social welfare gain per unit capital stock in levels, the second panel reports the social welfare gain per unit capital stock in percentage terms, and the third panel reports the social welfare gain per unit capital stock in percentage terms based on an annualized return over $T = 14.17$ years. This figure shows that if improved financial policies were to lead households to diversify their portfolios, thereby reducing the risk that they bear, then there is also a macro-level effect on aggregate growth. This effect increases growth when households are sufficiently willing to substitute consumption over time ($\psi > 1$)—the reason being that greater diversification decreases the price of risk, and so it is optimal for households to consume less today and save more, leading to greater real investment and hence higher aggregate growth. As households become more willing to substitute consumption intertemporally, there is an increase in the impact of financial diversification on social welfare. The dashed (black) line shows the welfare gains from diversification for a mean-variance household: this line is flat because mean-variance utility does not depend on ψ. The dotted-dashed (blue) line shows the gains from portfolio diversification for a household with intertemporal consumption when growth is exogenous: this line shows that the welfare gain from portfolio diversification for an individual household exceeds that for

Figure 2: Social welfare as ψ changes

In this figure, we plot three welfare measures as the elasticity of intertemporal substitution, ψ, varies.



the mean-variance household. The solid (red) line shows that the welfare gain to society from portfolio diversification when growth is endogenous. This line intersects the dotted-dashed (blue) line at $\psi = 1$ because at that point the income and substitution effects offset each other exactly and so society chooses not to adjust aggregate investment at all. For the region where $\psi > 1$, society is willing to consume less today and invest more, leading to positive growth effects; thus, in this region the social gains, given by the solid (red) line, exceed the private gains, given by the dotted-dashed (blue) line. The reverse is true for $\psi < 1$, with the social welfare gains coinciding with the gains for a mean-variance household when $\psi = 0$.

Above, we have assumed that household portfolios consist of investments only in financial assets. However, many households invest a major share of their wealth in real estate (in fact, the investment in real estate is typically levered) and some households invest also in entrepreneurial ventures. These investments would imply that household portfolios are even less well diversified than we have assumed above. Consequently, the social welfare gains from improved diversification would be even larger than we have calculated.

On the other hand, we have assumed that firms can adjust their investment policies instantly and at no cost; if the adjustment of physical capital takes time, then the magnitude of the effects we have identified will be smaller. To study the impact of assuming that investment levels can be adjusted instantaneously, one can use the approach in Obstfeld (1994, p. 1325) where it is assumed that the annual welfare gain converges toward the long-run annual gain at an instantaneous rate of x percent, which is about 2.2% per annum based on the work of Barro, Mankiw, and Sala-i-Martin (1992). Therefore, the actual capitalized social welfare gain, λ^{actual} is related to the reported social welfare gain λ as follows:

$$\lambda^{\text{actual}} = \int_0^\infty i\,\lambda(1 - e^{-x\,t})e^{-i\,t}dt = \lambda\frac{x}{i + x}.$$

If the interest rate is 0.56% per annum, then $\frac{x}{i+x} = 79\%$. This implies that the actual social welfare gains are about 79% of the welfare gains reported in the tables above, indicating that they are still quite large.

6 Conclusion

Our results indicate that the impact on household and social welfare of financial policy, through education, innovation, and regulation, can be substantial—the potential gains are equivalent to an increase in the return on aggregate wealth of around 10%. Most of this gain arises from a multiplier effect applied to the gains for a mean-variance investor, which is driven by the effects of improved portfolio diversification on intertemporal consumption smoothing and aggregate growth. The analysis in our paper suggests that the answer to the question posed in the title is a resounding "yes." Household finance matters a great deal because small improvements in the financial decisions of individual households have the potential to generate large economic gains for society: a small step for households can be a giant leap for society.

Thaler and Sunstein (2003) recommend "nudges" that gently guide people in a direction that increases experienced utility. Below, we consider a variety of such policies that could ameliorate the familiarity biases of households. One policy measure is to introduce default portfolios that are well diversified. There is substantial evidence that the choice of a default option can be important (see Samuelson and Zeckhauser (1988)), because when a particular choice is designated as the default, it attracts a disproportionate market share. For example, households could be offered a small number of portfolios to choose from, with the portfolios having different levels of risk, but all of them being well diversified.[19] Cronqvist and Thaler (2004) describe the experience of Sweden, where the government introduced a private plan for social security savings. Participants in this plan were allowed to form their own portfolios by selecting up to five funds from an approved list, where one fund was chosen (with some care) to be a "default" fund for anyone who, for whatever reason, did not make an active choice. This default fund was diversified internationally—with 65% invested in non-Swedish stock, 17% in Swedish stocks, 10% in inflation indexed-bonds, 4% in hedge funds, and 4% in private equity—and had a very low expense ratio (17 basis points). In the context of our model, the default fund would be one that was diversified across the N risky assets.

[19]Similar to the policy advocated by Benartzi and Thaler (2004), where people commit in advance to allocating a proportion of their future salary increases toward retirement savings, one could design sensible default options that encourage households to invest in portfolios that are diversified across equities and asset classes. Madrian and Shea (2001) study the impact of automatic enrollment on 401(k) savings behavior. They find that participation is significantly higher under automatic enrollment and that a substantial fraction of the participants retain the default contribution rate and fund allocation.

A second policy measure is financial education. For example, households could be educated about the benefits of diversification. Empirical evidence suggests that financial literacy can play an important role in improving decisions made by households. For instance, Bayer, Bernheim, and Scholz (2008) find that both participation in and contributions to voluntary savings plans are significantly higher when employers offer frequent seminars about the benefits of planning for retirement. Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2014) also find that, while general education has only a small effect in reducing familiarity bias, an increase in financial competence does reduce this bias. Financial education could also inform households about the benefits of investing in broadly-diversified funds, such as mutual funds and ETFs, that do not require familiarity with particular assets.

A third alternative is to introduce financial regulation to limit the tendency of households to bias portfolios toward a few familiar assets. For example, financial regulation could be introduced to prohibit companies from providing employees own-company stock when matching the pension contributions of employees. Financial regulation could also prohibit the use of own-company stock in 401(k) plans. Similarly, one could require mutual funds to simplify investment procedures in order to lower the barrier to entry and increase investments in these diversified assets.

A Appendix

In this appendix, we provide all derivations for the results in the main text. The title of each subsection below indicates the particular equation(s) derived in that subsection. To make it easier to read this appendix without having to go back and forth to the main text, we rewrite any equations from the main text that are needed; these equations are assigned the same number as the one in the main text.

A.1 The certainty equivalent in (6)

For clarity, we rewrite (6) as the following Lemma.

Lemma 1 *The date-t certainty equivalent of investor h's date-$t+dt$ utility is given by*

$$\mu_t[U_{h,t+dt}] = E_t[U_{h,t+dt}] - \frac{1}{2}\,\gamma\, U_{h,t}\, E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]. \tag{6}$$

Proof of Lemma 1

The definition of the certainty equivalent in (4) implies that

$$\mu_t[U_{h,t+dt}] = E_t\left[U_{h,t+dt}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}.$$

Therefore

$$\mu_t[U_{h,t+dt}] = E_t\left[U_{h,t+dt}^{1-\gamma}\right]^{\frac{1}{1-\gamma}} = E_t\left[U_{h,t}^{1-\gamma} + d(U_{h,t}^{1-\gamma})\right]^{\frac{1}{1-\gamma}}.$$

Applying Ito's Lemma, we obtain

$$\begin{aligned} d(U_{h,t}^{1-\gamma}) &= (1-\gamma)U_{h,t}^{-\gamma}dU_{h,t} - \frac{1}{2}(1-\gamma)\gamma U_{h,t}^{-\gamma-1}(dU_{h,t})^2 \\ &= (1-\gamma)U_{h,t}^{1-\gamma}\left[\frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2}\gamma\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]. \end{aligned}$$

Therefore

$$\begin{aligned} \mu_t[U_{h,t+dt}] = E_t\left[U_{h,t+dt}^{1-\gamma}\right]^{\frac{1}{1-\gamma}} &= U_{h,t}\left(E_t\left[1+(1-\gamma)\left[\frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2}\gamma\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right]\right)^{\frac{1}{1-\gamma}} \\ &= U_{h,t}\left(1+(1-\gamma)\left[E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right]\right)^{\frac{1}{1-\gamma}} \end{aligned}$$

$$= U_{h,t}\left(1+(1-\gamma)\left[E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right]-\frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right]\right)^{\frac{1}{1-\gamma}}.$$

Hence,

$$\mu_t[U_{h,t+dt}] = U_{h,t}\left(1+E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right]-\frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right)+o(dt).$$

Therefore, in the continuous-time limit, we obtain the expression in (6).

A.2 The familiarity-biased certainty equivalent in (8)

While (8), giving the familiarity-biased certainty equivalent, is given as a definition within the main text of the paper, we can derive it from more primitive assumptions. To do so, we shall need some additional definitions and lemmas.

Our approach differs from Uppal and Wang (2003), because of our assumption that investors subject to behavioral biases *cannot* extract the orthogonal factor structure underlying returns and infer how familiarity with respect to a particular firm translates into familiarity with respect to factors and hence the returns of other firms.

We start by defining the measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$.

Definition 1 *The probability measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ is defined by*

$$\mathbb{Q}^{\boldsymbol{\nu}_h}(A) = E[1_A \xi_{h,T}],$$

where E is the expectation under $\mathbb{P}$, A is an event and $\xi_{h,t}$ is the exponential martingale (under the reference probability measure $\mathbb{P}$)

$$\frac{d\xi_{h,t}}{\xi_{h,t}} = \frac{1}{\sigma}\boldsymbol{\nu}_{h,t}^\top \Omega^{-1} d\boldsymbol{Z}_t.$$

Recall that when an investor is less familiar with a particular firm, she adjusts its expected return, which is equivalent to changing the reference measure to a new measure, denoted by $\mathbb{Q}^{\nu_h}$. Applying Girsanov's Theorem, we see that under the new measure $\mathbb{Q}^{\nu_h}$, the evolution of firm n's capital stock is given by

$$dK_{n,t} = [(\alpha+\nu_{hn,t})K_{n,t} - D_{n,t}]dt + \sigma K_{n,t} dZ^{\boldsymbol{\nu}_h}_{n,t},$$

where $Z^{\boldsymbol{\nu}_h}_{n,t}$ is a standard Brownian motion under $\mathbb{Q}^{\boldsymbol{\nu}_h}$, such that

$$dZ^{\boldsymbol{\nu}_h}_{n,t} dZ^{\boldsymbol{\nu}_h}_{m,t} = \begin{cases} dt, & n = m. \\ \rho dt, & n \neq m. \end{cases}$$

Before motivating the definition of the penalty function, we make the following additional definition.

Definition 2 *The probability measure* $\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}$ *is defined by*

$$\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}(A) = E[1_A \xi_{h,n,T}],$$

where E *is the expectation under* $\mathbb{P}$*,* A *is an event and* $\xi_{h,n,t}$ *is the exponential martingale (under the reference probability measure* $\mathbb{P}$*)*

$$\frac{d\xi_{h,n,t}}{\xi_{h,n,t}} = \frac{1}{\sigma}\nu_{h,n,t} dZ_{n,t}.$$

The probability measure $\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}$ is just the probability measure associated with familiarity bias with respect to firm n. Familiarity bias along this factor is equivalent to using $\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}$ instead of $\mathbb{P}$, which leads to a loss in information. The information loss stemming from familiarity bias with respect to firm n can be quantified via the date-t conditional Kullback-Leibler divergence between $\mathbb{P}$ and $\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}$, given by

$$D^{KL}_{t,u}[\mathbb{P}|\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}] = E_t^{\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}}\left[\ln\left(\frac{\xi_{h,n,u}}{\xi_{h,n,t}}\right)\right].$$

We can now think about how to measure the total information loss from familiarity biases with respect to all N firms. Assuming households are not sufficiently sophisticated enough to extract the orthogonal factor structure underlying returns and infer how familiarity with respect to a particular firm translates into familiarity with respect to factors and hence the returns of other firms, we can form a simple weighted sum of the squares of the date-t conditional Kullback-Leibler divergences for familiarity bias with respect to each individual firm, i.e.

$$\hat{L}_{h,t} = \sum_{n=1}^{N} \mathcal{W}_{h,n}(D^{KL}_{t,u}[\mathbb{P}|\mathbb{Q}^{\boldsymbol{\nu}_{h,n}}])^2,$$

where $\mathcal{W}_{h,n}$ is a household specific weighting matrix. We can think of the matrix $\mathcal{W}_{h,n}$ as a set of weights for information losses, analogous to the weights used in the generalized method of moments.

The choice of weighting matrix depends on how a household weights information losses, which we assume depends on her level of familiarity bias. For illustration, consider the simple case where $\mathcal{W}_{h,n} = \frac{f_{h,n}}{1-f_{h,n}}$, $\rho = 0$ so shocks to firm-level returns are mutually orthogonal, and the household h is completely unfamiliar with all firms save firm 1. In this case,

$$\mathcal{W}_{h,n} = \begin{cases} \frac{f_1}{1-f_1}, & n = 1 \\ 0, & n \neq 1 \end{cases}$$

Our expression for total information loss from familiarity biases with respect to all N firms then reduces to

$$\hat{L}_{h,t} = \frac{f_1}{1-f_1}\left(D^{KL}_{t,u}[\mathbb{P}|\mathbb{Q}^{\boldsymbol{\nu}_{h,1}}]\right)^2.$$

So, we can see that if a household is completely unfamiliar with a particular firm, the information loss associated with deviating from the reference measure $\mathbb{P}$ is assigned a weight of zero. The more familiar a household is with a firm, the greater the weight on the information loss for that firm caused by deviating from the reference measure.

Motivated by the above discussion, we now define a penalty function for using the measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ instead of $\mathbb{P}$.

Definition 3 *The penalty function for investor h associated with her familiarity biases is given by*

$$\hat{L}_{h,t} = \frac{1}{\sigma^2}\boldsymbol{\nu}_{h,t}^{\top}\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}.$$

We can see that information losses linked to the firms with which the investor is totally unfamiliar are not penalized in the penalty function. The investor is penalized only for deviating from $\mathbb{P}$ with respect to a particular firm if she has some level of familiarity with that firm. If she has full familiarity with a firm, the associated penalty becomes infinitely large, so when making decisions involving this firm, she will not deviate at all from the reference probability measure $\mathbb{P}$.

Theorem 1 *The date-t familiarity-biased certainty equivalent of date-$t+dt$ investor utility is given by*

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] + U_{h,t}L_{h,t}dt, \tag{A1}$$

where $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ is defined by

$$u_\gamma\left(\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\right) = E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}}[u_\gamma\left(U_{h,t+dt}\right)], \tag{A2}$$

and

$$L_{h,t} = \frac{1}{2\gamma}\frac{\boldsymbol{\nu}_{h,t}^{\top}\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2}, \tag{A3}$$

where $\boldsymbol{\nu}_{h,t} = (\nu_{h1,t},\ldots,\nu_{hN,t})^{\top}$ is the column vector of adjustments to expected returns, and $\Gamma_h = [\Gamma_{h,nm}]$ is the $N \times N$ diagonal matrix defined by

$$\Gamma_{h,nm} = \begin{cases} \frac{1-f_{hn}}{f_{hn}}, & n = m, \\ 0, & n \neq m, \end{cases}$$

and $f_{hn} \in [0,1]$ is a measure of how familiar the investor is with firm, n, with $f_{hn} = 1$ implying perfect familiarity, and $f_{hn} = 0$ indicating no familiarity at all.

Proof of Theorem 1

Using the penalty function given in Definition 3, the construction of the familiarity-biased certainty equivalent of date-$t + dt$ utility is straightforward—it is merely the certainty-equivalent of date-$t+dt$ utility computed using the probability measure $\mathbb{Q}^{\boldsymbol{\nu}_h}$ plus a penalty. The investor will choose her adjustment to expected returns by minimizing the familiarity-biased certainty equivalent of her date-$t + dt$ utility—the penalty stops her from making the adjustment arbitarily large by penalizing her for larger adjustments. The size of the penalty is a measure of the information she loses by deviating from the common reference measure, adjusted by her familiarity preferences, and so

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] + U_{h,t}L_{h,t}dt,$$

where $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ is defined by

$$u_\gamma\left(\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\right) = E_t^{\mathbb{Q}^{\boldsymbol{\nu}_h}}[u_\gamma\left(U_{h,t+dt}\right)],$$

and

$$L_{h,t} = \frac{1}{2\gamma}\hat{L}_{h,t}.$$

Equation (8) follows from Theorem 1, so we restate the equation formally as the following corollary before giving a proof.

Corollary 1 *The date-t familiarity-biased certainty equivalent of date-$t{+}dt$ investor utility is given by*

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \mu_t[U_{h,t+dt}] + U_{h,t} \times \left(\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\boldsymbol{\nu}_{h,t}^\top\boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma}\frac{\boldsymbol{\nu}_{h,t}^\top\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2}\right)dt, \tag{8}$$

where $U_{W_{h,t}} = \frac{\partial U_{h,t}}{\partial W_{h,t}}$ is the partial derivative of the utility of investor h with respect to her wealth.

Proof of Corollary 1

The date-t familiarity-biased certainty equivalent of date-$t + dt$ investor utility is given by (A1), (A2), and (A3). We can see that $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ is like a certainty equivalent, but with the expectation taken under $\mathbb{Q}^{\boldsymbol{\nu}_h}$ in order to adjust for familiarity bias. From Lemma 1, we know that

$$\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = U_{h,t}\left(1 + E_t^{\mathbb{Q}^{\nu_h}}\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right) + o(dt).$$

We therefore obtain from (1)

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = U_{h,t}\left(1 + E_t^{\mathbb{Q}^{\nu_h}}\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right] + L_{h,t}dt\right) + o(dt). \tag{A4}$$

Applying Ito's Lemma, we see that under $\mathbb{Q}^{\boldsymbol{\nu}_h}$

$$dU_{h,t} = W_{h,t}\frac{\partial U_{h,t}}{\partial W_{h,t}}\frac{dW_{h,t}}{W_{h,t}} + \frac{1}{2}W_{h,t}^2\frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2}\left(\frac{dW_{h,t}}{W_{h,t}}\right)^2,$$

where

$$\frac{dW_{h,t}}{W_{h,t}} = \left(1 - \sum_{n=1}^{N}\omega_{hn,t}\right)idt + \sum_{n=1}^{N}\omega_{hn,t}\left((\alpha + \nu_{h,t})dt + \sigma dZ^{\mathbb{Q}^{\nu_h}}_{n,t}\right) - \frac{C_{h,t}}{W_{h,t}}dt.$$

Hence, from Girsanov's Theorem, we have

$$E_t^{\mathbb{Q}^{\nu_h}}\left[\frac{dU_{h,t}}{U_{h,t}}\right] = E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] + \frac{W_{h,t}}{U_{h,t}}\frac{\partial U_{h,t}}{\partial W_{h,t}}\boldsymbol{\omega}_{h,t}^\top\boldsymbol{\nu}_{h,t}dt.$$

We can therefore rewrite (A4) as

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = U_{h,t}\left(1 + E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right] + L_{h,t}dt + \frac{W_{h,t}}{U_{h,t}}\frac{\partial U_{h,t}}{\partial W_{h,t}}\boldsymbol{\omega}_{h,t}^\top\boldsymbol{\nu}_{h,t}dt\right) + o(dt).$$

Using (6) we obtain

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \mu_t[U_{h,t+dt}] + U_{h,t}\left(\frac{W_{h,t}}{U_{h,t}}\frac{\partial U_{h,t}}{\partial W_{h,t}}\boldsymbol{\omega}_{h,t}^\top\boldsymbol{\nu}_{h,t} + \frac{1}{2\gamma}\frac{\boldsymbol{\nu}_{h,t}^\top\Gamma_h^{-1}\boldsymbol{\nu}_{h,t}}{\sigma^2}\right)dt + o(dt),$$

and hence (8).

A.3 The familiarity-biased adjustment to expected returns in (11)

We restate (11) as the following proposition.

Proposition 1 *For a given portfolio, $\boldsymbol{\omega}_{h,t}$, adjustments to firm n's expected return are given by*

$$\nu_{hn,t} = -\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\Big(\frac{1}{f_{hn}} - 1\Big)\sigma^2\,\gamma\,\omega_{hn,t},\ n \in \{1,\ldots,N\}. \tag{11}$$

Proof of Proposition 1

From (8), we can see that

$$\inf_{\boldsymbol{\nu}_{h,t}}\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$$

is equivalent to

$$\inf_{\boldsymbol{\nu}_{h,t}} \frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^{\top} \boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma\sigma^2} \boldsymbol{\nu}_{h,t}^{\top} \Gamma_h^{-1} \boldsymbol{\nu}_{h,t}.$$

The minimum exists and is given by the FOC

$$\frac{\partial}{\partial \boldsymbol{\nu}_{h,t}} \left[\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^{\top} \boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma\sigma^2} \boldsymbol{\nu}_{h,t}^{\top} \Gamma_h^{-1} \boldsymbol{\nu}_{h,t} \right] = 0$$

Carrying out the differentiation and exploiting the fact that Γ_h^{-1} is symmetric, we obtain

$$0 = \frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\omega}_{h,t} + \frac{1}{\gamma\sigma^2} \Gamma_h^{-1} \boldsymbol{\nu}_{h,t}.$$

Hence

$$\boldsymbol{\nu}_{h,t} = -\gamma\sigma^2 \frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}} \Gamma_h \, \boldsymbol{\omega}_{h,t}.$$

Hence, we obtain (11).

A.4 Derivation of the Hamilton-Jacobi-Bellman equation in (12)

We restate the Hamilton-Jacobi-Bellman equation as the following proposition.

Proposition 2 *The utility function of an investor with familiarity biases is given by the following Hamilton-Jacobi-Bellman equation:*

$$0 = \sup_{C_{h,t}} \left(\delta \, u_{\psi} \left(\frac{C_{ht}}{U_{ht}} \right) + \sup_{\omega_t} \inf_{\nu_{h,t}} \frac{1}{U_{h,t}} \mu_{h,t}^{\boldsymbol{\nu}} \left[\frac{dU_{h,t}}{dt} \right] \right), \tag{12}$$

where the function

$$u_{\psi}(x) = \frac{x^{1-\frac{1}{\psi}} - 1}{1 - \frac{1}{\psi}}, \psi > 0,$$

and

$$\mu_{h,t}^{\boldsymbol{\nu}} \left[dU_{h,t} \right] = \mu_{h,t}^{\boldsymbol{\nu}} \left[U_{h,t+dt} - U_{h,t} \right] = \mu_{h,t}^{\boldsymbol{\nu}} \left[U_{h,t+dt} \right] - U_{h,t},$$

with $\mu_{h,t}^{\boldsymbol{\nu}} \left[U_{h,t+dt} \right]$ given in (8).

Proof of Proposition 2

Writing out (10) explicitly gives

$$U_{h,t}^{1-\frac{1}{\psi}} = (1 - e^{-\delta dt}) C_{h,t}^{1-\frac{1}{\psi}} + e^{-\delta dt} \left(\mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}] \right)^{1-\frac{1}{\psi}},$$

where for ease of notation sup and inf have been suppressed. Now

$$\left(\mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}] \right)^{1-\frac{1}{\psi}} = \left(U_{h,t} + \mu_{h,t}^{\boldsymbol{\nu}}[dU_{h,t}] \right)^{1-\frac{1}{\psi}}$$

$$= U_{h,t}^{1-\frac{1}{\psi}} \left(1 + \mu_{h,t}^{\boldsymbol{\nu}} \left[\frac{dU_{h,t}}{U_{h,t}}\right]\right)^{1-\frac{1}{\psi}}$$

$$= U_{h,t}^{1-\frac{1}{\psi}} \left(1 + \left(1 - \frac{1}{\psi}\right) \mu_{h,t}^{\boldsymbol{\nu}} \left[\frac{dU_{h,t}}{U_{h,t}}\right]\right) + o(dt).$$

Hence

$$U_{h,t}^{1-\frac{1}{\psi}} = \delta C_{h,t}^{1-\frac{1}{\psi}} dt + U_{h,t}^{1-\frac{1}{\psi}} \left(1 + \left(1 - \frac{1}{\psi}\right) \mu_{h,t}^{\boldsymbol{\nu}} \left[\frac{dU_{h,t}}{U_{h,t}}\right]\right) - \delta U_{h,t}^{1-\frac{1}{\psi}} dt + o(dt),$$

from which we obtain (12).

A.5 Mean-variance portfolio choice with familiarity bias in (13) and (14)

We collect Equations (13) and (14) in the following proposition.

Proposition 3 *The investor's optimization problem consists of two parts, a mean-variance optimization*

$$\sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}),$$

and an intertemporal consumption choice problem

$$0 = \sup_{C_{h,t}} \left(\delta\, u_\psi \left(\frac{C_{ht}}{U_{ht}}\right) - \frac{C_{ht}}{W_{ht}} + \sup_{\boldsymbol{\omega}_t} \inf_{\boldsymbol{\nu}_{h,t}} MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t})\right), \tag{13}$$

where

$$MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) = i + (\alpha - i)\mathbf{1}^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2}\gamma\sigma^2 \boldsymbol{\omega}_{h,t}^\top \Omega \boldsymbol{\omega}_{h,t} + \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma} \frac{\boldsymbol{\nu}_{h,t}^\top \Gamma_h^{-1} \boldsymbol{\nu}_{h,t}}{\sigma^2}, \tag{14}$$

and $\mathbf{1}$ *denotes the* $N \times 1$ *unit vector.*

Proof of Proposition 3

Assuming a constant risk-free rate, homotheticity of preferences combined with constant returns to scale for production implies that we have $U_{h,t} = \kappa_h W_{h,t}$, for some constant κ_h. Equations (13) and (14) are then direct consequences of (8) and (12).

A.6 Adjustment to expected returns and portfolio choice in (15), (17)–(20)

Proposition 4 *The optimal adjustment to expected returns made by a household with familiarity bias is given in* (17) *below:*

$$\boldsymbol{\nu}_h = (\alpha - i)[(I + \Gamma_h \Omega^{-1})^{-1} - I], \tag{17}$$

where $\boldsymbol{f}_h$ *is the vector of familiarity coefficients* $\boldsymbol{f}_h = (f_{h1}, \ldots, f_{hN})^\top$. *The vector of optimal portfolio weights is*

$$\boldsymbol{\omega}_h = \boldsymbol{q}_h \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2}, \tag{18}$$

where

$$\boldsymbol{q}_h = (\Omega + \Gamma_h)^{-1} \boldsymbol{f}_h.$$

The capital allocation decision, i.e. the proportion of wealth the household allocates to risky assets is given by

$$\pi_h = \frac{1}{\gamma} \frac{SR_{\boldsymbol{x}_h}}{\sigma_{\boldsymbol{x}}} \frac{1}{1 + b_h},$$

where $\boldsymbol{x}_h = \frac{\boldsymbol{\omega}_h}{\mathbf{1}^\top \boldsymbol{\omega}_h}$ *is the portfolio of risky assets, given by*

$$\boldsymbol{x}_h = \frac{\boldsymbol{q}_h}{\mathbf{1}^\top \boldsymbol{q}_h},$$

and the variance of the above portfolio is $\sigma^2_{\boldsymbol{x}_h} = \sigma^2 \boldsymbol{x}_h^\top \Omega\, \boldsymbol{x}_h,$, $SR_{\boldsymbol{x}_h}$ *is the Sharpe ratio of the portfolio* $\boldsymbol{x}_h$ *and*

$$b_h = 1 - \frac{\mathbf{1}^\top \boldsymbol{q}_h}{\boldsymbol{q}_h^\top \Omega \boldsymbol{q}_h}.$$

With familiarity bias, the optimized *portfolio-choice objective function, i.e. optimal decision utility, can be expressed as:*

$$MV^d = \sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) = i + \frac{1}{2}\frac{1}{\gamma} \left(\frac{\alpha - i}{\sigma_{\boldsymbol{x}_h}} \right)^2 \frac{1}{1 + b_h} = i + \frac{1}{2\gamma} SR^2_{\boldsymbol{x}_h} \frac{1}{1 + b_h}.$$

Proof of Proposition 4

Minimizing (14) with respect to $\nu_{h,t}$ gives (15). Substituting (15) into (14) and simplifying gives

$$MV_h = i + (\alpha - i)\pi_h - \frac{1}{2}\gamma \pi_h^2 \sigma^2 \boldsymbol{x}_h^\top (\Omega + \Gamma_h) \boldsymbol{x}_h, \tag{A5}$$

where π_h is the proportion of household h's wealth held in risky assets,

$$\pi_h = \mathbf{1}^\top \boldsymbol{\omega}_h,$$

and $\boldsymbol{x}_h$ is the vector of risky asset weights,

$$\boldsymbol{x}_h = \frac{\boldsymbol{\omega}_h}{\pi_h}.$$

We deliberately analyze the portfolio choice problem in this way, because it allows us to separate capital allocation, that is the choice of π, from risky portfolio selection, which is the choice of $\boldsymbol{x}$.

We find $\boldsymbol{x}_h$ by minimizing $\sigma^2 \boldsymbol{x}_h^\top(\Omega + \Gamma_h)\boldsymbol{x}_h$, so we can see that $\boldsymbol{x}_h$ is household h's minimum-variance portfolio adjusted for familiarity bias. The minimization we wish to perform is

$$\min \frac{1}{2} \boldsymbol{x}_h^\top (\Omega + \Gamma_h)\, \boldsymbol{x}_h$$

subject to the constraint

$$\mathbf{1}^\top \boldsymbol{x}_h = 1.$$

The Lagrangian for this problem is

$$L_h = \frac{1}{2} \boldsymbol{x}_h^\top (\Omega + \Gamma_h)\, \boldsymbol{x}_h + \lambda_h (1 - \mathbf{1}^\top \boldsymbol{x}_h),$$

where λ_h is the Lagrange multiplier. The first order condition with respect to $\boldsymbol{x}_h$ is

$$(\Omega + \Gamma_h)\, \boldsymbol{x}_h = \lambda_h \mathbf{1}.$$

Hence

$$\boldsymbol{x}_h = \lambda_h (\Omega + \Gamma_h)^{-1} \mathbf{1} = \lambda_h (\Omega + \Gamma_h)^{-1} \boldsymbol{f}_h.$$

The first order condition with respect to λ_h gives us the constraint

$$\mathbf{1}^\top \boldsymbol{x}_h = 1,$$

which implies that

$$\lambda_h = \left[\mathbf{1}^\top (\Omega + \Gamma_h)^{-1} \mathbf{1} \right]^{-1}.$$

Therefore, we have

$$\boldsymbol{x}_h = \frac{(\Omega + \Gamma_h)^{-1} \mathbf{1}}{\mathbf{1}^\top (\Omega + \Gamma_h)^{-1} \mathbf{1}}.$$

Substituting the optimal choice of $\boldsymbol{x}_h$ back into $\boldsymbol{x}_h^\top (\Omega + \Gamma_h) \boldsymbol{x}_h$ gives

$$\boldsymbol{x}_h^\top (\Omega + \Gamma_h) \boldsymbol{x}_h = \lambda_h. \tag{A6}$$

Therefore, to find the optimal π, we need to minimize

$$MV_h = i + (\alpha - i)\pi_h - \frac{1}{2} \gamma_h \pi_h^2 \sigma^2 \lambda_h.$$

Hence

$$\pi_h = \frac{1}{\lambda_h} \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2},$$

which gives us the result in (18). We can rewrite the expression for $\boldsymbol{\omega}_h$ in (18) in terms of the familiarity-biased adjustment made to expected returns:

$$\boldsymbol{\omega}_h = \frac{1}{\gamma}\Omega^{-1}\frac{\alpha\mathbf{1} + \boldsymbol{\nu}_h - i\mathbf{1}}{\sigma^2},$$

from which we get (17):

$$\boldsymbol{\nu}_h = (\alpha - i)[(I + \Gamma_h\Omega^{-1})^{-1} - I]. \tag{17}$$

We now use (18) to derive an expression for ω_{hn}, that is, the n'th element of $\boldsymbol{\omega}_n$. For all $n \in \{1, \ldots, N\}$, define $\boldsymbol{e}_n$, the $N \times 1$ column vector, with a one in the n'th entry and zeros everywhere else. Clearly $\{\boldsymbol{e}_1, \ldots, \boldsymbol{e}_N\}$ is the standard basis for $\mathbb{R}^N$ and the proportion of investor h's wealth invested in firm n is given by

$$\omega_{hn} = \boldsymbol{e}_n^\top \boldsymbol{\omega}_h.$$

We define

$$q_{hn} = \boldsymbol{e}_n^\top(\Omega + \Gamma_h)^{-1}\boldsymbol{f}_h,$$

and so

$$\boldsymbol{q}_h = (\Omega + \Gamma_h)^{-1}\boldsymbol{f}_h. \tag{A7}$$

The optimal portfolio decision is given by

$$\omega_{hn} = q_{hn}\frac{1}{\gamma}\frac{\alpha - i}{\sigma^2}, \tag{A8}$$

or equivalently

$$\boldsymbol{\omega}_h = \boldsymbol{q}_h\frac{1}{\gamma}\frac{\alpha - i}{\sigma^2}.$$

It follows from (A8) that

$$\pi_h = \frac{1}{\gamma}\frac{\alpha - i}{\sigma^2}\sum_{n=1}^{N} q_{hn} = \frac{1}{\gamma}\frac{\alpha - i}{\sigma^2}\mathbf{1}^\top\boldsymbol{q}_h. \tag{A9}$$

It also follows from (A8) that the n'th element of investor h's portfolio of risky assets is given by

$$x_{hn} = \frac{\omega_{hn}}{\sum_{n=1}^{N}\omega_{hn}} = \frac{q_{hn}}{\sum_{n=1}^{N} q_{hn}},$$

or equivalently

$$\boldsymbol{x}_h = \frac{\boldsymbol{q}_h}{\mathbf{1}^\top\boldsymbol{q}_h}. \tag{A10}$$

Substituting the expression for the optimal portfolio weight into the decision utility mean-variance objective function gives the optimized decision utility mean-variance objective function:

$$MV^d = i + \frac{1}{2}\frac{1}{\lambda_h \gamma}\left(\frac{\alpha - i}{\sigma}\right)^2, \tag{A11}$$

Using (A6) and (A7), we obtain

$$\lambda_h^{-1} = \mathbf{1}^\top \boldsymbol{q}_h,$$

and so

$$MV^d = i + \frac{1}{2}\frac{1}{\gamma}\left(\frac{\alpha - i}{\sigma}\right)^2 \mathbf{1}^\top \boldsymbol{q}_h.$$

We shall now derive an alternative expression for MV^d, which shall allow us to disentangle the effects of the capital allocation and risky portfolio choices. We can write π_h as

$$\pi_h = \frac{1}{\gamma}\frac{\alpha - i}{\sigma_{\boldsymbol{x}_h}^2}\frac{1}{\lambda_h}\frac{\sigma_{\boldsymbol{x}_h}^2}{\sigma^2},$$

where

$$\sigma_{\boldsymbol{x}_h}^2 = \sigma^2 \boldsymbol{x}_h^\top \Omega \boldsymbol{x}_h.$$

The Sharpe ratio of the risky portfolio $\boldsymbol{x}_h$ is given by

$$SR_{\boldsymbol{x}_h} = \frac{\alpha - i}{\sigma_{\boldsymbol{x}_h}},$$

and so

$$\pi_h = \frac{1}{\gamma} SR_{\boldsymbol{x}_h} \frac{1}{\lambda_h}\frac{\sigma_{\boldsymbol{x}_h}^2}{\sigma^2}.$$

Now

$$\frac{\sigma_{\boldsymbol{x}_h}^2}{\sigma^2} = \boldsymbol{x}_h^\top \Omega \boldsymbol{x}_h = (\mathbf{1}^\top \boldsymbol{q}_h)^{-2} \boldsymbol{q}_h^\top \Omega \boldsymbol{q}_h,$$

and so

$$\frac{1}{\lambda_h}\frac{\sigma_{\boldsymbol{x}_h}^2}{\sigma^2} = \boldsymbol{x}_h^\top \Omega \boldsymbol{x}_h \, \mathbf{1}^\top \boldsymbol{q}_h = \frac{\boldsymbol{q}_h^\top \Omega \boldsymbol{q}_h}{\mathbf{1}^\top \boldsymbol{q}_h}.$$

Hence, the capital allocation decision is given by

$$\pi_h = \frac{1}{\gamma}\frac{SR_{\boldsymbol{x}_h}}{\sigma_{\boldsymbol{x}_h}}\frac{1}{1 + b_h},$$

where

$$b_h = \frac{\mathbf{1}^\top \boldsymbol{q}_h}{\boldsymbol{q}_h^\top \Omega \boldsymbol{q}_h} - 1.$$

We now express a household's optimized decision utility, given in (A11), in terms of b_h and $SR_{\boldsymbol{x}_h}$.

$$
\begin{aligned}
MV^d &= i + \frac{1}{2}\frac{1}{\lambda_h\gamma}\left(\frac{\alpha - i}{\sigma}\right)^2 \\
&= i + \frac{1}{2}\frac{1}{\gamma}\left(\frac{\alpha - i}{\sigma_{\boldsymbol{x}_h}}\right)^2 \frac{1}{\lambda_h}\frac{\sigma^2_{\boldsymbol{x}_h}}{\sigma^2} \\
&= i + \frac{1}{2}\frac{1}{\gamma}\left(\frac{\alpha - i}{\sigma_{\boldsymbol{x}_h}}\right)^2 \frac{\boldsymbol{q}_h^\top \Omega \boldsymbol{q}_h}{\mathbf{1}^\top \boldsymbol{q}_h} \\
&= i + \frac{1}{2}\frac{1}{\gamma}\left(\frac{\alpha - i}{\sigma_{\boldsymbol{x}_h}}\right)^2 \frac{1}{1 + b_h} \\
&= i + \frac{1}{2\gamma} SR^2_{\boldsymbol{x}_h} \frac{1}{1 + b_h}.
\end{aligned}
$$

A.7 Experienced Mean-Variance Utility in (21) and (22)

The following proposition summarizes results on how familiarity bias impacts a household's experienced mean-variance utility.

Proposition 5 *If a household makes both her capital allocation decision, i.e.* π_h, *and her risky portfolio decision, i.e.* $\boldsymbol{x}_h$, *based on her mean-variance decision utility, then her mean-variance experienced utility is given by*

$$
MV^e_{\boldsymbol{x}_h} = i + \frac{1}{2\gamma} SR^2_{\boldsymbol{x}_h}(1 - d_h^2), \tag{21}
$$

where

$$
d_h = \frac{b_h}{1 + b_h}.
$$

If a household makes her capital allocation decision correctly, i.e. she maximizes her experienced utility, but chooses her risky portfolio based on her mean-variance decision utility, then her mean-variance experienced utility is given by

$$
MV^e = i + \frac{1}{2\gamma} SR^2_{\boldsymbol{x}_h}.
$$

If a household makes both her capital allocation and risky portfolio decisions correctly, then her mean-variance experienced utility is given by

$$
MV^e_{1/N} = i + \frac{1}{2\gamma} SR^2_{1/N}, \tag{22}
$$

where $SR_{1/N}$ *is the Sharpe ratio of the equally-weighted* $(1/N)$ *portfolio of risky assets, i.e.*

$$
SR_{1/N} = \frac{\alpha - i}{\sigma_{1/N}},
$$

and $\sigma_{1/N}$ is the volatility of the $1/N$ portfolio, i.e.

$$\sigma_{1/N} = \sigma \sqrt{\frac{1}{N} + \left(1 - \frac{1}{N}\right) \rho}.$$

Assuming the risk-free interest is constant, removing familiarity bias increases a household's experienced mean-variance utility by the following amount

$$\frac{1}{2\gamma}\left(SR^2_{1/N} - SR^2_{\boldsymbol{x}_h}(1 - d_h^2)\right) = \frac{1}{2\gamma}\left(SR^2_{1/N} - SR^2_{\boldsymbol{x}_h} + SR^2_{\boldsymbol{x}_h} d_h^2\right).$$

Proof of Proposition 5

From (A5), we can write the experienced utility mean-variance objective function as

$$MV^e = i + (\alpha - i)\pi_h - \frac{1}{2}\gamma \pi_h^2 \sigma^2 \boldsymbol{x}_h^\top \Omega \boldsymbol{x}_h.$$

Substituting (20) into the above expression gives (21) below, with (22) being a special case of this:

$$\begin{aligned}
MV^e_{\boldsymbol{x}_h} &= i + (\alpha - i)\frac{1}{\gamma}\frac{SR_{\boldsymbol{x}_h}}{\sigma_{\boldsymbol{x}_h}}\frac{1}{1+b_h} - \frac{1}{2}\gamma\left(\frac{1}{\gamma}\frac{SR_{\boldsymbol{x}_h}}{\sigma_{\boldsymbol{x}_h}}\frac{1}{1+b_h}\right)^2 \sigma^2 \boldsymbol{x}_h^\top \Omega \boldsymbol{x}_h \\
&= i + \frac{1}{\gamma} SR^2_{\boldsymbol{x}_h}\frac{1}{1+b_h} - \frac{1}{2\gamma} SR^2_{\boldsymbol{x}_h}\frac{1}{(1+b_h)^2} \\
&= i + \frac{1}{2\gamma} SR^2_{\boldsymbol{x}_h}\left[1 - \left(1 - \frac{1}{1+b_h}\right)^2\right] \\
&= i + \frac{1}{2\gamma} SR^2_{\boldsymbol{x}_h}(1 - d_h^2),
\end{aligned}$$

where $d_h = \frac{b_h}{1+b_h}$.

If a household makes her capital allocation decision correctly, but makes her risky portfolio decision incorrectly, using her decision utility, then

$$\pi^d = \frac{1}{\gamma}\frac{SR_{\boldsymbol{x}_h}}{\sigma_{\boldsymbol{x}_h}},$$

where $\boldsymbol{x}_h$ is given by (A10), and so

$$MV^e_{\boldsymbol{x}_h} = i + \frac{1}{2\gamma} SR^2_{\boldsymbol{x}}.$$

If a household makes both her capital allocation and risky portfolio decisions correctly, then her risky portfolio is the equally-weighted $1/N$ portfolio, i.e. $N^{-1}\mathbf{1}$. The variance of her risky portfolio is therefore given by

$$\sigma_{1/N}^2 = \sigma^2 \frac{1}{N}^2 \mathbf{1}_h^\top \Omega \mathbf{1}_h = \sigma^2 \left[\frac{1}{N} + \left(1 - \frac{1}{N}\right) \rho \right],$$

and so the Sharpe ratio of the equally-weighted portfolio is

$$SR_{1/N} = \frac{\alpha - i}{\sigma_{1/N}}.$$

If familiarity biases are removed, a household's mean-variance experienced utility increases by

$$\frac{1}{2\gamma} \left(SR_{1/N}^2 - SR_{\boldsymbol{x}}^2 (1 - d_n^2) \right) = \frac{1}{2\gamma} \left(SR_{1/N}^2 - SR_{\boldsymbol{x}}^2 + SR_{\boldsymbol{x}}^2 d_n^2) \right).$$

A.8 Optimal consumption in (24)–(25)

The following proposition summarizes results on optimal consumption choice.

Proposition 6 *An household's optimal consumption-to-wealth ratio is given by*

$$\frac{C_{h,t}}{W_{h,t}} = \psi\delta + (1-\psi) \left(\left[i + \left(\alpha \mathbf{1} + \frac{1}{2} \boldsymbol{\nu}_{h,t} - i \mathbf{1} \right)^\top \boldsymbol{\omega}_{h,t} \right] - \frac{1}{2} \gamma \sigma^2 \boldsymbol{\omega}_{h,t}^\top \Omega \boldsymbol{\omega}_{h,t} \right) \tag{24}$$

$$= \psi\delta + (1-\psi) \left(i + \frac{1}{2\gamma} SR_{\boldsymbol{x}_h}^2 \frac{1}{1 + b_h} \right). \tag{25}$$

Proof of Proposition 6

From the Hamilton-Jacobi-Bellman equation in (12), the first-order condition with respect to consumption is

$$\delta \left(\frac{C_{ht}}{U_{ht}} \right)^{-\frac{1}{\psi}} = \frac{U_{ht}}{W_{h,t}}. \tag{A17}$$

Substituting the above first-order condition into the Hamilton-Jacobi-Bellman equation allows us to solve for investor utility, and hence, optimal consumption in (24), which upon further algebraic simplification leads to (25).

The following proposition summarizes how familiarity bias distorts a household's optimal consumption-wealth ratio.

Proposition 7 *Assuming a partial equilibrium perspective, where the risk-free interest rate, i, is held fixed, familiarity bias distorts a household's intertemporal consumption choice as follows*

$$\left.\frac{C_{h,t}}{W_{h,t}}\right|_{1/N} - \left.\frac{C_{h,t}}{W_{h,t}}\right|_{\boldsymbol{x}_h} = (1-\psi) \frac{1}{2\gamma} \left(SR_{1/N}^2 - SR_{\boldsymbol{x}_h}^2 + SR_{\boldsymbol{x}_h}^2 d_h^2 + SR_{\boldsymbol{x}_h}^2 d_h (1 - d_h) \right).$$

Proof of Proposition 7

We know that with no familiarity bias, the risky portfolio has the Sharpe ratio $SR_{1/N}$ and $b_h = 0$. It therefore follows from Proposition 6, that with no familiarity bias, a household's optimal consumption-wealth ratio is given by

$$\frac{C_{h,t}}{W_{h,t}} = \psi\delta + (1-\psi)\left(i + \frac{1}{2\gamma}SR^2_{1/N}\right).$$

Hence

$$\begin{aligned}
&\left.\frac{C_{h,t}}{W_{h,t}}\right|_{1/N} - \left.\frac{C_{h,t}}{W_{h,t}}\right|_{\boldsymbol{x}_h} \\
&= (1-\psi)\frac{1}{2\gamma}\left(SR^2_{1/N} - SR^2_{\boldsymbol{x}_h}\frac{1}{1+b_h}\right) \\
&= (1-\psi)\frac{1}{2\gamma}\left(SR^2_{1/N} - SR^2_{\boldsymbol{x}_h} + SR^2_{\boldsymbol{x}_h}d_h^2 + SR^2_{\boldsymbol{x}_h}(1-d_h^2) - SR^2_{\boldsymbol{x}_h}\frac{1}{1+b_h}\right) \\
&= (1-\psi)\frac{1}{2\gamma}\left(SR^2_{1/N} - SR^2_{\boldsymbol{x}_h} + SR^2_{\boldsymbol{x}_h}d_h^2 + SR^2_{\boldsymbol{x}_h}(1-d_h^2) - SR^2_{\boldsymbol{x}_h}(1-d_h)\right) \\
&= (1-\psi)\frac{1}{2\gamma}\left(SR^2_{1/N} - SR^2_{\boldsymbol{x}_h} + SR^2_{\boldsymbol{x}_h}d_h^2 + SR^2_{\boldsymbol{x}_h}d_h(1-d_h)\right).
\end{aligned}$$

A.9 Experienced Utility

The following proposition shows how familiarity bias impacts lifetime experienced utility.

Proposition 8 *If a household is subject to familiarity bias, her lifetime experienced utility level, $U^e_{h,t}$, is given by*

$$\left.U^e_{h,t}\right|_{\boldsymbol{x}_h} = \left[\frac{\psi\delta + (1-\psi)\left(i + \frac{1}{2\gamma}SR^2_{\boldsymbol{x}_h}\frac{1}{1+b_h}\right)}{\delta\psi}\right]^{\frac{1}{1-\psi}} W_{h,t},$$

The lifetime experienced utility level of a household that does not suffer from familiarity bias is given by the following expression,

$$\left.U^e_{h,t}\right|_{1/N} = \left[\frac{\psi\delta + (1-\psi)\left(i + \frac{1}{2\gamma}SR^2_{1/N}\right)}{\delta\psi}\right]^{\frac{1}{1-\psi}} W_{h,t}.$$

Proof of Proposition 8

From (A17) we obtain

$$\frac{U_{h,t}}{W_{h,t}} = \delta\left(\frac{C_{h,t}}{W_{h,t}}\frac{W_{h,t}}{U_{h,t}}\right)^{-\frac{1}{\psi}}$$
$$\frac{U_{h,t}}{W_{h,t}} = \left(\frac{C_{h,t}/W_{h,t}}{\delta\psi}\right)^{\frac{1}{1-\psi}}.$$

Using Proposition 6 we thus obtain the experienced utility-wealth ratio for a household with familiarity bias

$$\left.\frac{U^e_{h,t}}{W_{h,t}}\right|_{1/N} = \left(\frac{\delta\psi + (1-\psi)\left(i + \frac{1}{2\gamma}SR^2_{\boldsymbol{x}}\frac{1}{1+b_h}\right)}{\delta\psi}\right)^{\frac{1}{1-\psi}}.$$

A.10 Condition for no aggregate familiarity bias across investors

We start by formally stating the "no aggregate bias condition."

Definition 4 *Suppose investor h's risky portfolio weight for firm n is given by*

$$x_{hn} = \frac{1}{N} + \epsilon_{hn},$$

where $\frac{1}{N}$ *is the unbiased portfolio weight and* ϵ_{hn} *is the bias of investor h's portfolio when investing in firm n. The biases* ϵ_{hn} *"cancel out in aggregate" if*

$$\forall n, \ \ \frac{1}{H}\sum_{h=1}^{H}\epsilon_{hn} = 0.$$

The following proposition gives the symmetry condition, which implies that the no-aggregate bias condition holds.

Proposition 9 *For every investor* $h \in \{1,\dots,H\}$*, define the adjusted-familiarity vector* $(q_{h1},\dots,q_{hN})$*, where* q_{hn} *is defined in* (19)*. If the following symmetry condition holds:*

1. *given an investor* $h \in \{1,\dots,H\}$*, for all investors* $h' \in \{1,\dots,H\}$*, there exists a permutation* $\tau_{h'}$ *such that* $\tau_{h'}(q_{h'1},\dots,q_{h'N}) = (q_{h1},\dots,q_{hN})$*; and,*
2. *given a firm* $n \in \{1,\dots,N\}$*, for all firms* $n' \in \{1,\dots,N\}$*, there exists a permutation* $\tau_{n'}$ *such that* $\tau_{n'}(q_{1n'},\dots,q_{Hn'}) = (q_{1n},\dots,q_{Hn})$*,*

then there is no aggregate bias.

Proof of Proposition 9

Observe that the no aggregate bias condition is equivalent to (28), reproduced below:

$$\frac{1}{H}\sum_{h=1}^{H} q_{hn} = \frac{1}{N}\sum_{n=1}^{N} q_{hn}. \tag{28}$$

Now define a $H \times N$ familiarity matrix,

$$Q = [q_{hn}].$$

The permutations described in the symmetry condition imply that one can obtain all the rows of the matrix by rearranging any particular row, and one can obtain all the columns of the matrix by rearranging any particular column, which implies that (28) is satisfied.

A.11 Equilibrium interest rate in (30)

The following proposition summarizes the equilibrium interest rate.

Proposition 10 *The equilibrium risk-free interest rate is given by the constant*

$$i = \alpha - \gamma\, \sigma_p^2, \tag{30}$$

where $\sigma_p^2 = \frac{\sigma^2}{\hat{q}}$ *is the variance of the portfolio held by each investor with an adjustment for familiarity bias and* $\hat{q}$ *is defined by* $\hat{q} = \hat{q}_h, \forall h$, *with* $\hat{q}_h = \sum_{n=1}^{N} q_{hn}$.

Proof of Proposition 10

We start by observing the symmetry condition in Theorem 9 implies that

$$\forall n,\, \forall h,\ \frac{1}{H}\sum_{h=1}^{H} q_{hn} = \frac{1}{N}\sum_{n=1}^{N} q_{hn},$$

which is equivalent to

$$\forall n,\, \forall h,\ \frac{1}{H}\hat{q}_n = \frac{1}{N}\hat{q}_h, \tag{A18}$$

where

$$\hat{q}_n = \sum_{h=1}^{H} q_{hn},\ \hat{q}_h = \sum_{n=1}^{N} q_{hn}.$$

From (A18) we can also see that $\hat{q}_n$ and $\hat{q}_h$ must be independent of n and h, respectively.

We now prove that the condition that $\hat{q}_h$ is independent of h implies that the risk-free interest rate is the constant given by (30). Market clearing in the bond market implies that

$$\sum_{h=1}^{H} B_{h,t} = 0., \tag{A19}$$

where the amount of wealth held in the bond by investor h is given by

$$B_{h,t} = (1 - \pi_{h,t})W_{h,t}.$$

Using the expression for $\pi_{h,t}$ given in (A9), we can rewrite the market clearing condition (A19) as

$$\sum_{h=1}^{H} \left(1 - \frac{1}{\gamma}\frac{\alpha - i}{\sigma^2}\left(\sum_{n=1}^{N} q_{hn}\right)\right) W_{h,t} = 0.$$

Hence,

$$\begin{aligned}
0 &= \sum_{h=1}^{H} \left(W_{h,t} - \hat{q}_h \frac{1}{\gamma}\frac{\alpha - i}{\sigma^2} W_{h,t}\right) \\
\sum_{h=1}^{H} W_{h,t} &= \frac{1}{\gamma}\frac{\alpha - i}{\sigma^2} \sum_{h=1}^{H} \hat{q}_h W_{h,t} \\
i &= \alpha - \frac{\sum_{h=1}^{H} W_{h,t}}{\sum_{h=1}^{H} \hat{q}_h W_{h,t}} \gamma\sigma^2 \\
&= \alpha - \frac{1}{\hat{q}}\gamma\sigma^2,
\end{aligned}$$

which is the expression for the equilibrium interest rate in (30). This also implies that in equilibrium $B_{h,t} = 0$; that is, each household invests solely in risky firms.

A.12 Equilibrium macroeconomic quantities in (31)–(33)

Proposition 11 *The general equilibrium economy-wide consumption-wealth ratio is given by*

$$\frac{C_t^{agg}}{W_t^{agg}} = c = \alpha - g = \psi\delta + (1 - \psi)\Big(\alpha - \frac{1}{2}\gamma\sigma_p^2\Big), \tag{31}$$

where g, the aggregate growth rate of the economy, is equal to the aggregate investment-capital ratio, which is given by

$$g = \frac{I_t^{agg}}{K_t^{agg}} = \alpha - c = \psi(\alpha - \delta) - \frac{1}{2}(\psi - 1)\gamma\sigma_p^2. \tag{33}$$

Proof of Proposition 11

Substituting the equilibrium interest rate in (30) into the expression in (25) for the consumption-wealth ratio for each individual gives the general-equilibrium consumption-wealth ratio:

$$\frac{C_{h,t}}{W_{h,t}} = c,$$

where

$$c = \psi\delta + (1-\psi)\Big(\alpha - \frac{1}{2}\gamma\sigma_p^2\Big).$$

Observe that in the expression above, all the terms on the right-hand side are constants, implying that the consumption-wealth ratio is the same across investors. Exploiting the fact that the consumption-wealth ratio is constant across investors allows us to obtain the ratio of aggregate consumption-to-wealth ratio, where aggregate consumption is $C_t^{\text{agg}} = \sum_{h=1}^{H} C_{h,t}$ and aggregate wealth is $W_t^{\text{agg}} = \sum_{h=1}^{H} W_{h,t}$:

$$\frac{C_t^{\text{agg}}}{W_t^{\text{agg}}} = c,$$

which is the result in (31).

Equation (1) implies

$$\sum_{n=1}^{N} Y_{n,t} = \alpha \sum_{n=1}^{N} K_{n,t},$$

and Equation (2) implies

$$d\left(E_t\left[\sum_{n=1}^{N} K_{n,t}\right]\right) = E_t\left[d\sum_{n=1}^{N} K_{n,t}\right] = \alpha\sum_{n=1}^{N} K_{n,t} - \sum_{n=1}^{N} D_{n,t}dt.$$

In equilibrium $\sum_{n=1}^{N} K_{n,t} = W_t^{\text{agg}}$ and $\sum_{n=1}^{N} D_{n,t} = C_t^{\text{agg}}$. Therefore,

$$\frac{dW_t^{\text{agg}}}{W_t^{\text{agg}}} = \left(\alpha - \frac{C_t^{\text{agg}}}{W_t^{\text{agg}}}\right)dt.$$

We also know that

$$\frac{dW_t^{\text{agg}}}{W_t^{\text{agg}}} = \frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}}$$

and so

$$g\,dt = E_t\left[\frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}}\right] = \left(\alpha - \frac{C_t^{\text{agg}}}{W_t^{\text{agg}}}\right)dt.$$

Therefore

$$c = \alpha - g.$$

From (30) it follows that

$$c = i + \gamma\sigma_p^2 - g. \tag{A20}$$

We now derive the aggregate investment-capital ratio. The aggregate investment flow must be equal to aggregate output flow less the aggregate consumption flow:

$$I_t^{\text{agg}} = \alpha K_t^{\text{agg}} - C_t^{\text{agg}}.$$

It follows that the aggregate investment-capital ratio is given by

$$\frac{I_t^{\text{agg}}}{K_t^{\text{agg}}} = \alpha - \frac{C_t^{\text{agg}}}{K_t^{\text{agg}}} = \alpha - c,$$

which is the expression in (33).

Trend output growth is given by $E_t\left[\frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}}\right]$. Observe that $Y_t^{\text{agg}} = \alpha K_t^{\text{agg}} = \alpha W_t^{\text{agg}} = \frac{\alpha}{c} C_t^{\text{agg}}$. It follows that trend output growth equals the growth rate of aggregate consumption:

$$g = E_t\left[\frac{dY_t^{\text{agg}}}{Y_t^{\text{agg}}}\right].$$

We now relate trend output growth to aggregate investment. Firms all have constant returns to scale and differ only because of shocks to their capital stocks. Therefore, the aggregate growth rate of the economy is the aggregate investment-capital ratio:

$$g = \frac{I_t^{\text{agg}}}{K_t^{\text{agg}}},$$

which gives us the expression for g in (33).

A.13 The aggregate price-dividend ratio in (36)

Proposition 12 *The aggregate price-dividend ratio is given in terms of the endogenous expected growth rate of aggregate output, g, and the perceived volatility of investor portfolios, σ_p by*

$$p_t^{agg} = \frac{1}{\delta + \left(\frac{1}{\psi} - 1\right)\left(g - \frac{1}{2}\gamma\sigma_p^2\right)}. \tag{36}$$

Proof of Proposition 12

The aggregate price-dividend ratio is defined as

$$p_t^{\text{agg}} = \frac{K_t^{\text{agg}}}{C_t^{\text{agg}}} = \frac{W_t^{\text{agg}}}{C_t^{\text{agg}}}.$$

Noting that this is the inverse of the aggregate consumption-wealth ratio in (A20), we get that

$$p_t^{\text{agg}} = \frac{1}{i + \gamma\sigma_p^2 - g}.$$

Substituting the expression from (30) for the interest rate gives:

$$p_t^{\text{agg}} = \frac{1}{\alpha - g}. \tag{A21}$$

Finally, substituting for g in (36) and simplifying the resulting expression shows that it is identical to (A21).

A.14 Disentangling the micro-level internality and the macro-level general-equilibrium effect

Proposition 13 *If familiarity bias is removed the decrease in risk at the household level, i.e. the micro-level volatility internality, is given by*

$$\Delta\sigma_p^2 = \sigma_{1/N}^2 - \sigma_{\boldsymbol{x}}^2(1 + b_h) = -(\sigma_{\boldsymbol{x}}^2 - \sigma_{1/N}^2) - \sigma_{\boldsymbol{x}}^2 b_h,$$

is the decrease in risk at the household level, i.e. the micro-level volatility internality, and the change in the economy-wide growth rate, i.e. the macro-level growth effect, is given by

$$\Delta g = -\frac{1}{2}(\psi - 1)\gamma\Delta\sigma_p^2.$$

The percentage increase in social welfare per unit capital stock stemming from a removal of familiarity bias is given by

$$\frac{d\ln\left(\frac{U_t^{agg}}{K_t^{agg}}\right)}{d\ln(\sigma_p^2)} = -\frac{1}{2}\gamma\, p_t^{agg}\, \sigma_p^2 \Bigg(\underbrace{1 - \frac{1}{\psi}}_{\substack{\textit{macro-level} \\ \textit{general-equilibrium effect}}} + \underbrace{\frac{1}{\psi}}_{\substack{\textit{micro-level} \\ \textit{interality}}} \Bigg).$$

Proof of Proposition 13

We start by observing that the capital asset allocation decision of a household with familiarity bias is given by

$$\pi_h = \frac{1}{\gamma}\frac{\alpha - i}{\sigma_{\boldsymbol{x}}^2}\frac{1}{1 + b_h}.$$

The symmetry condition implies that b_h is the same for all household's, and so market clearing implies $\pi^d = 1$, and so

$$i = \alpha - \gamma\sigma_{\boldsymbol{x}}^2(1 + b_h).$$

Hence

$$\sigma_p^2 = \sigma_{\boldsymbol{x}}^2(1 + b_h).$$

With no familiarity bias the above expression reduces to

$$\sigma_p^2 = \sigma_{1/N}^2.$$

Therefore, eliminating familiarity bias changes σ_p^2, which is the micro-level volatility internality. The change is can be decomposed as follows

$$\Delta\sigma_p^2 = \sigma_{1/N}^2 - \sigma_{\boldsymbol{x}}^2(1 + b_h) = -(\sigma_{\boldsymbol{x}}^2 - \sigma_{1/N}^2) - \sigma_{\boldsymbol{x}}^2 b_h,$$

where $-(\sigma_{\boldsymbol{x}}^2 - \sigma_{1/N}^2)$ is the reduction in variance due to improved diversification and $-\sigma_{\boldsymbol{x}}^2 b_h$ is the reduction in variance due to improved capital allocation.

The reduction in risk at the household-level impacts growth, which is the macro-level general-equilibrium effect. From (33) in Proposition 11, we can see that the resultant change in the aggregate growth rate of the economy is given by

$$\Delta g = -\frac{1}{2}(\psi - 1)\gamma\Delta\sigma_p^2.$$

Total differentiation of the social welfare per unit capital stock gives (37). Observing that

$$\frac{\partial g}{\partial(\sigma_p^2)} = -\frac{1}{2}(\psi - 1)\gamma$$

and using (35) and (36) to compute $\frac{\partial \ln\left(\frac{U_t^{\text{agg}}}{K_t^{\text{agg}}}\right)}{\partial \ln g}$ and $\frac{\partial \ln\left(\frac{U_t^{\text{agg}}}{K_t^{\text{agg}}}\right)}{\partial \ln(\sigma_p^2)}$ allows us to obtain (38) from (37).

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