Holography, Information and Meaning:

Hidden symmetries of Matrix Physics.

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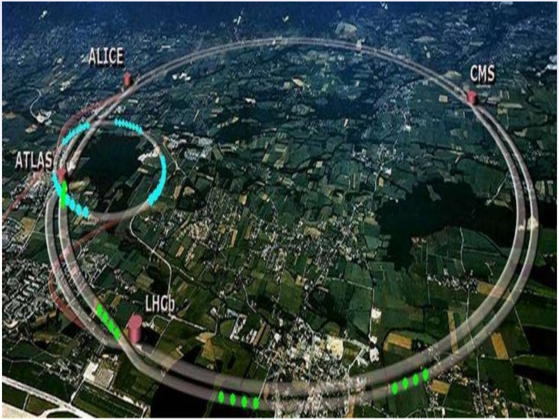
Psi Star talk, Feb. 2019

At the Large Hadron Collider Experiment in Geneva



Protons travelling at 99.9999990 percent of the speed of light have been collided to discover a new fundamental particle (Higgs particle).

The Colliding protons

are made of constituent elementary particles : quarks and gluons ...

Quarks, gluons, electrons, photons, Higgs are among the elementary particles in the standard model of particle physics.

The theory of the standard model is Quantum Field Theory. There is a set of quantum fields for each type of particle.

$$\text{Higgs scalar} \rightarrow \Phi(x^0, x^1, x^2, x^3)$$

$$\text{Photon} \rightarrow \begin{array}{l} A_0(x^0, x^1, x^2, x^3) \\ A_1(x^0, x^1, x^2, x^3) \\ A_2(x^0, x^1, x^2, x^3) \\ A_3(x^0, x^1, x^2, x^3) \end{array}$$

$A_\mu(x^\nu) : \mu, \nu \in \{0, 1, 2, 3\}$.

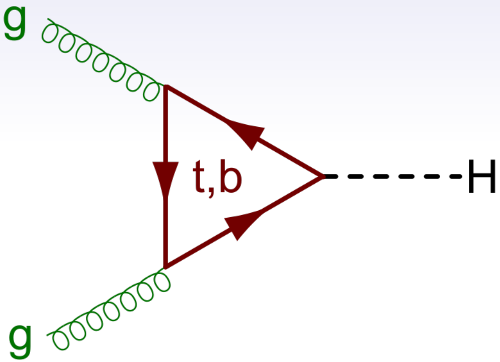
Quantum field theory

of $A_\mu(x^\nu)$ and other quantum fields ...

gives a set of rules for calculating the probabilities of outcomes (final particle states) for given initial particle states

Diagrammatic Rules (Feynman Rules) : For any given process (specified in and out-states), there is a sum over Feynman diagrams.

A Feynman diagram relevant to Higgs production



Some data specifying a Quantum Field theory ...

- A space-time, $\mathbb{R}^{3,1}$ for the standard model :

$$-\infty < x^\mu < \infty$$

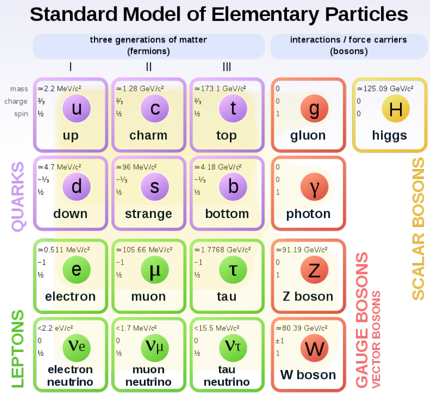
$$\mu \in \{0, 1, 2, 3, \}$$

- a notion of Lorentz-invariant distance ($c = 1$)

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$$

  which is used to build QFTs which are Lorentz invariant.
- A set of particles : e.g. particle content of the standard model of particle physics.

Standard model particles ..



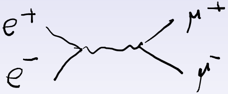
Force particles ...

The photons, gluons, W and Z are force particles.
In QFT, forces arise from exchange of particles.

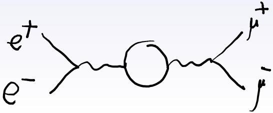
photons $\rightarrow$ electromagnetic force
gluons $\rightarrow$ strong force
W and Z $\rightarrow$ weak force

Calculations in QFT are done in "perturbation theory". Coupling constants (e.g. electric charge) can be treated as small quantities.

in Standard Model:



$\sim e^2$



$\sim e^4$

$$\mathcal{F} \sim \mathcal{F}_0 + e\,\mathcal{F}_1 + e^2\,\mathcal{F}_2 + e^3\,\mathcal{F}_3 + \dots$$

QFT Perturbation Theory

Force of gravity ?

in a unified quantum theory for all forces, should contain gravitons.

The table has no graviton.

If we attempt to directly add another quantum field (or set of quantum fields) for the graviton into the standard model ..

And start calculating using Feynman rules of the extended QFT ...

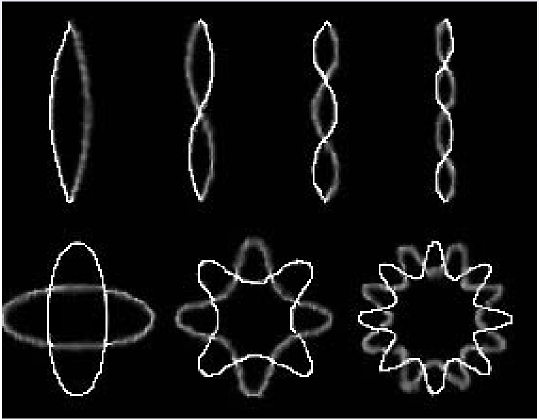
We get ...

$$\infty$$

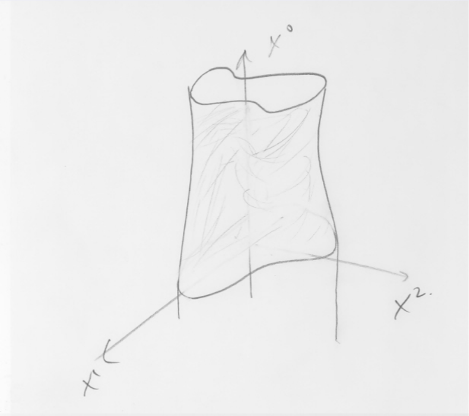
String theory manages to ...

- Produce "Generalized/Stringy Feynman Rules" for calculating interactions including gravitons and particles like the photons, quarks, gluons etc.
- All particles - including gravitons - are excitations of a string.

Strings and particles



String worldsheets



String perturbation theory

$$\mathcal{F} = \mathcal{F}_0 + g_{st.}^2 \mathcal{F}_1 + g_{st}^4 \mathcal{F}_2 + \dots$$

Features of string theory

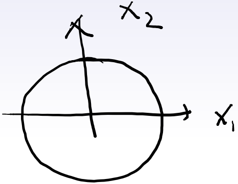
Involves a 2-dimensional surface, called the world-sheet of the string.

Quantum field theory on the world-sheet.

What sort of quantum fields live on this 2D space-time ?

Answer comes from geometry.

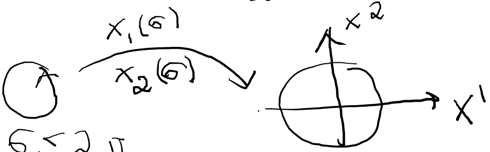
Two ways to think about a circle



$$x_1^2 + x_2^2 = 1$$

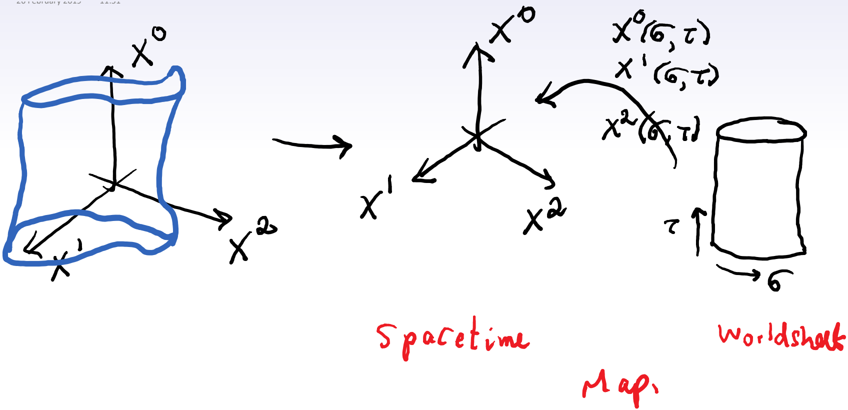
OR:

$$x_1(\sigma) = \cos \sigma$$
$$x_2(\sigma) = \sin \sigma$$



$$0 \leq \sigma < 2\pi$$

Maps from world-sheet to space-time



Some clever quirks of string theory

- Quantum fields on the string worldsheet are space-time coordinates $X^{\nu}(\sigma, \tau)$.
- $0 \leq \nu \leq (D-1)$, for classical string theory in $\mathbb{R}^{D-1,1}$.
- Interaction probabilities for photons, gluons, gravitons in D-dimensions ... are constructed as objects in two-dimensional QFT of maps!!
- Bypass the D-dimensional quantum fields $A(X^{\nu})$.

Additional Features of String Theory ...

- Quantum string theory requires the strings to exist in ten dimensions.
- $\mathbb{R}^{3,1} \times X$, where X is a CY geometry.
- $2D$ string worldsheet QFTs **are dual** to 10D space-time QFTs.
- A novel property of duality in the world of quantum field theories (a theoretical physics conjecture).

Outline of rest of talk

- A new kind of duality in string theory was discovered in 1997: Holographic duality or AdS/CFT correspondence (a theoretical physics conjecture).
- Holographic dictionary and Matrix combinatorics.
- A Mathematical Duality involving hidden symmetries, for matrix combinatorics.
- This interplay has surprising applications : computational linguistics and quantum information theory.
- Open questions in AdS/CFT and beyond ...

More about Gluons ...

The quantum fields describing gluons have the form

$$(A_\mu)^i_j(x^0, x^1, x^2, x^3)$$

where i, j can take values $\{1, 2, 3\}$. The nine possibilities can be arranged in a 3×3 matrix.

Correspondingly quarks come in 3 colors.

The matrix structure is used to write the interactions e.g.

$$\sum_{\mu,\nu} tr[A_\mu, A_\nu][A^\mu, A^\nu]$$

Gluon theory ...

poses some hard unsolved mathematical problems, notably confinement (a Millenium math problem worth 1 million dollars).

As an approach to the problem, 't Hooft (1973) generalized the 3-color theory (called $U(3)$ theory) to N-color theory ($U(N)$ theory) and proposed to study the $1/N$ expansion as an approximation.

He discovered hints of a link to string theory... worldsheets .... but not clear which string theory ? which space-time ?

't Hooft's observation led to AdS/CFT ....

Take N-color theory, called $U(N)$ Yang-Mills theory, add some Higgs-like quantum fields $\Phi^i_j(x)$ and some (spin-half) fermion fields $\Lambda^i_j(x)$.

There is a unique such non-gravitational theory in four dimensions which has maximal supersymmetry : maximal number of transformations which convert bosons to fermions.

Important : there are 6 matrix-Higgs-like fields (each an $N \times N$ matrix). $\Phi^1, \Phi^2, \cdots, \Phi^6$.

This $4D$ QFT is called $\mathcal{N} = 4$ SYM. Has the special property of conformal symmetry. Hence it is called a CFT.

10-dimensional string theory in curved spaces : part I

The surface of a ball of unit radius is described by

$$(x^1)^2 + (x^2)^2 + (x^3)^2 = 1$$

This is $S^2 \in \mathbb{R}^3$.

S^5 is a 5-dimensional geometry described by

$$(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 + (x^5)^2 + (x^6)^2 = 1$$

Inherits a constant positive curvature metric from its embedding in $\mathbb{R}^6$.

10-dimensional string theory in curved spaces : part II

A negative curvature version of S^5 is AdS_5, a five dimensional geometry described by the equation

$$(y^1)^2 + (y^2)^2 + (y^3)^2 + (y^4)^2 - (y^5)^2 - (y^6)^2 = 1$$

A very symmetric 10-dimensional space is $AdS_5 \times S^5$.

$$(x^1, x^2, x^3, x^4, x^5, x^6, y^1, y^2, y^3, y^4, y^5, y^6)$$

String theory in $AdS_5 \times S^5$ is a quantum gravitational theory. The excitations of the string include the graviton.

The AdS/CFT correspondence is the duality ..

$$\text{STRING THEORY IN ADS}_5 \times S^5$$
$$\updownarrow$$
$$\text{The } N\text{-gluon (} U(N) \text{) theory : } \mathcal{N} = 4 \text{ SYM}$$

discovered by Maldacena in 1997.

More about $U(N)$

$U(N)$ is a set of matrices U which obey the condition

$$UU^\dagger = 1$$

These matrices are invertible, have an associative multiplication – form a group.

"$U(N)$ is a gauge-symmetry" of $\mathcal{N} = 4$ SYM theory.

Everything in the CFT has well-defined transformation properties under $U(N)$ (just like A_μ indicates 4-vector transformations under Lorentz group).

Everything physical is in some sense "invariant under $U(N)$ transformations".

There is a Holographic Dictionary

Quantum states in the 10-dimensional string theory correspond to quantum states in the 4D CFT.

CFT means that there is combinatoric description of the quantum states.

Polynomial functions of elementary fields, which are $U(N)$ invariant ... a well-defined class of math problems.

Quantum states can be specified by charges

Charges correspond to symmetries of the theory (Physical Dynamics module).

In the S^5 geometry described by $(x^1, x^2, \cdots, x^6)$ coordinates, rotations in the (12) plane form part of the symmetries. And give a charge.

Gravitons in AdS_5 come with a (12) charge.

Gravitons of small (12) charge ..

are fluctuations of the metric-field in $AdS_5 \times S^5$ (see STG-module).

correspond to very simple polynomial functions of $(\Phi^1 + i\Phi^2) = Z$, by the holographic dictionary.

trZ^J is a quantum state in the supersymmetry-multiplet of a graviton of charge J.

Multi-gravitons are multi-traces $trZ^{J_1} trZ^{J_2} \cdots$.

Gravitons of large (12) charge ..

are extended objects, called "3-brane giant gravitons" (discovered by Mc Greevy, Susskind, Toumbas, 2000)

It was soon found that there were actually TWO kinds of such giant gravitons.

S^3 inside the S^5 or S^3 inside the AdS_5.

Question: How do the multi-trace polynomials get organised into these two kinds of branes ?

Key Math Observation .... A Mathematical Duality

A powerful way to approach a number of combinatoric problems (arising in quantum field theory) regarding polynomial invariants of matrix variables, involves making use of hidden symmetries in the problem.

When considering polynomials of degree k in matrix variables, we can exploit the symmetric group of all permutations of k distinct objects.

Associated math technology : representation theory.

Applying this philosophy to the puzzle of giant gravitons ...

The multi-trace invariants form a Hilbert space, with an inner product coming from $\mathcal{N} = 4$ SYM. An orthonormal basis for this inner product is labelled by Young diagrams.

This leads to an answer to the puzzle (Corley, Jevicki, Ramgoolam, 2001).

Young diagrams:

$k = 3$:

$K = 4$:

(r_1, r_2, r_3, r_4) : $r_1 \geq r_2 \geq r_3 \geq r_4$

$$
\left.\begin{aligned}
4 &= 4 \\
&= 3+1 \\
&= 2+2 \\
&= 2+1+1 \\
&= 1+1+1+1
\end{aligned}\right\} \text{partitions of } 4.
$$

$Q_{12} = L$: Large S^3 in S^5

$\updownarrow$

CFT Quantum State labelled by

$Q_{12} = L$: Large S^3 in AdS_5

$\updownarrow$

CFT Quantum State

Identifying these giant gravitons ..

involved combinatoric questions regarding $U(N)$ invariant polynomials in one matrix variable Z.

Exciting these giant gravitons by attaching strings to them involves multi-matrix invariants made out of Z, Y.

The Key Mathematical Duality – use of hidden permutation symmetries and representation theory – is applicable to these problems as well.

Results in a detailed spectroscopy of excited giant gravitons involving non-trivial comparisons of CFT calculations with AdS calculations ... work at QMUL, UC Santa Barbara (USA), Wits University (South Africa) .. 2008-2016.

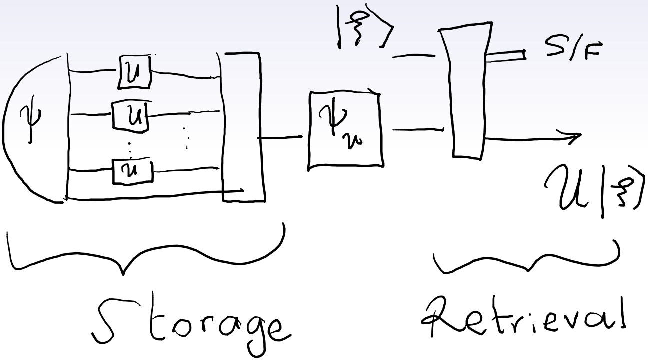
The Key Mathematical Duality ... Quantum Information Theory.

The connection between the hidden permutation symmetries and the 2-matrix problem of Z, Y invariants

has recently been used to prove new identities regarding Young diagrams, which were needed in a problem of quantum information theory.

A problem of optimizing the probability of success of retrieval of a unitary operator using quantum devices (Ramgoolam and Sedlak, 2018).

Storage/Retrieval problem in Quantum Information Theory



The Key Mathematical Duality ... Another twist

The giant graviton problem involved a 1-matrix problem, where hidden S_k permutation symmetries of k objects and representation theory of S_k, were used to solve a problem of $U(N)$ invariants for polynomials of degree k.

Take a 1-matrix problem, where the $U(N)$ symmetry is replaced by a discrete S_N symmetry.

Turns out that this has applications in the study of language.

Distributional Semantics

Distributional semantics looks at bodies of written language. E.g. all the English books in a big library (a corpus).

Associates vectors to words by looking at how the words occur in the corpus – how frequently words occur in the vicinity of other words.

Firth : You shall know a word by the company it keeps.

Identify a set of context words, e.g. in a simple model – two context words: pet, feed.

Distributional Semantics - Model example

Obtain vectors from words, by looking at frequencies with which they occur, in the vicinity of the context words.
E.g. In a single small book, you might have

$$\begin{aligned} & (PET, FEED) \\ Cat &= (200, 215) \\ Dog &= (198, 221) \\ Baby &= (2, 200) \end{aligned}$$

AI task : computer can use these freqencies to estimate the similarity in the meaning of words.

Distributional Semantics ... Compositional

originates in the work of Harris and Firth. (linguists working in the 50’s).

Recently, 2010, Computer scientists at Oxford (Sadrzadeh, Coecke, Clark) asked : How do we encode the meanings of sentences in vectors .. and the way the meanings of words combine to form the meanings of sentences?

Distributional Semantics ... Compositional

The ourcome of their research was that you need vectors, matrices (as well as higher tensors)

$$v_i, \Phi_{ij}, T_{ijk}, \cdots$$

in order to model the composition of meanings.

Nouns are vectors. Adjectives are matrices.

Concrete implementation of these ideas ..

Given a corpus, you have – for a list of 500 adjectives, a matrix for each, of size 2000×2000.

2000 is the number of context words.

Observation : You should be able to permute the context words without changing the meaning encoded by the word vectors (and word-matrices)

So we end up with a matrix problem of $N \times N$ matrices with S_N symmetry. (Kartsaklis, Ramgoolam, Sadrzadeh, "Linguistic Matrix Theory" 2017).

Large amount of data ...

How do you characterize the statistics of these random matrices ?

500 matrices each of size 2000×2000.

2×10^9 random variables.

QFT and symmetry ...

A secret about QFT : It is a generalization of statistics !!

Feynman: All the observables in a QFT (calculated by summing many Feynman diagrams) come from calculating an appropriate path integral.

$$\int d\Phi(x^0, x^1, x^2, x^3) e^{-S(\Phi)} \mathcal{O}_Q$$

For each physical question Q there is an observable $\mathcal{O}_Q$.

In zero dimensions

$$\int d\Phi e^{\mu\Phi - \lambda\Phi^2 + \cdots}$$

Means and variances are calculated by $\mathcal{O}_Q \rightarrow \Phi, \Phi^2$.

QFT and symmetry ...

By using these ideas from QFT, in LMT-2017, we used the data of permutation invariant means and variances to fix the parameters of a Gaussian statistical model having permutation symmetry.

The predictions of the Matrix Model for a set of observables $\mathcal{O}$ were compared with the data (with some limited success).

The most general Gaussian model compatible with permutation symmetry has been developed. Will be applying to data in the near future.

String theory challenges ...

String theory faces some challenging problems in fulfilling its original ambitions.

We said string theory predicts 10 dimensions. We can take $\mathbb{R}^{3,1} \times X$. Which X ? Many X's, many possible universes ...

Seems to lead to a loss of predictivity ... How do you test such a theory ? Is it testable ?

But the mathematics abstracted from string theory can have unexpected applications. And can be tested !!

As a Wise Elder

once said :

With abstraction, comes great power.

Future Research Directions ...

The giant gravitons related to the 1-matrix problem are special giant gravitons, called half-BPS. Less-supersymmetric (sixteenth-BPS) giant gravitons can form bound states which are black holes. Now we have to make the $U(N)$ invariants from $Z, DZ, DDZ, \cdots$.

Longer term Goal: Approach the puzzles of black hole physics in quantum gravity (in AdS) using the maths of the sixteenth-BPS sector.

Explore applications of string mathematics to language, in the framework of compositional distributional semantics.

To information processing problems in the context of quantum computing.

To ponder : Do theories of information and meaning have anything to say about the original goals of string theory : Unifying the four forces of nature in a predictive framework ... ?